Received: 15 September 2019 | Revised: 14 February 2021 | Accepted: 29 May 2021

DOI: 10.1111/jori.12353

ORIGINAL ARTICLE

Journal of Risk and Insurance

Comparative risk aversion in two periods: An application to self-insurance and self-protection

Tobias Huber

Institute for Risk Management and Insurance, Munich Risk and Insurance Center, Ludwig-Maximilians-Universität München, Munich, Germany

Correspondence
Tobias Huber, Institute for Risk Management and Insurance, Munich Risk and Insurance Center, Ludwig-Maximilians-Universität München, Geschwister-Scholl-Platz 1, 80539 Munich, Germany.
Email: tobias.huber@lmu.de

Abstract

Risk management decisions provide a means to elicit individuals' risk preferences empirically. In such a context, the literature often presumes that the decision to invest in risk management and the benefit of this investment occur contemporaneously. There is, however, no consensus in the theoretical literature that one-period results can be transferred to intertemporal settings. To address this gap, we study the effect of an increase in risk aversion on the demand for risk management in a two-period context. Our findings reproduce the one-period results and, thus, support the focus of previous empirical literature on the structure of the risk rather than on the timing of investments and benefits. We also contrast our results with those obtained by employing widely used but limited preferences to examine risk aversion in intertemporal settings (standard additive expected utility setting, Selden, Epstein and Zin).

KEYWORDS

comparative risk aversion, saving, self-insurance, self-protection

JEL CLASSIFICATION

D61, D81, D91

1 | INTRODUCTION

The decision to invest in risk management involves the consideration of risk and uncertainty. Individuals can prevent car accidents by driving carefully or by applying what is learned in a defensive driving course, reduce the risk of a fire by installing sprinkler systems or lightning conductors, and eat well and exercise regularly to reduce health risks. Companies invest in cybersecurity to prevent successful cyberattacks, install burglar alarms to reduce the risk of theft, and, in the case of farming, store extra water in the event of a drought. Moreover, both individuals and companies obtain insurance to mitigate financial losses. Given the numerous examples of risk management activities, how can we use observed behavior to draw inferences about the underlying risk preferences of decision makers?

Mitigation decisions usually involve activities to reduce either the size or the probability of loss. We follow Ehrlich and Becker (1972) and call the former self-insurance and the latter self-protection.[1] While previous research supports to employ self-insurance decisions to infer risk preferences, it identifies intricacies to draw inferences from self-protection activities. Theoretical findings show that greater risk aversion raises optimal self-insurance but has an ambiguous effect on optimal self-protection in one-period models (Briys & Schlesinger, 1990; Dionne & Eeckhoudt, 1985). In this setting, the investment in risk management and the induced risk reduction are assumed to occur contemporaneously, but, often, for example, when buying insurance or when making safety investments, they occur temporally separated.

This paper employs a two-period model to incorporate the time structure of investments in risk management and their benefits. The objective is to study how optimal risk management changes with increased risk aversion. This comes with the conceptual challenge of how to compare risk preferences in a two-period model. Kihlstrom and Mirman (1974) demonstrate that comparing agents in terms of their risk aversion requires keeping ordinal preferences unchanged. Preferences also need to fulfill a monotonicity property to ensure that agents do not prefer first-order stochastically dominated lotteries (Bommier et al., 2012, 2017). Theoretical research, however, widely employs risk preferences that are either not consistent with ordinal preferences (standard additive expected utility setting) or not necessarily monotone (Epstein & Zin, 1989; Selden, 1978). In contrast, our approach to comparative risk aversion in intertemporal settings fulfills both requirements.

First, we find that increased risk aversion unambiguously raises optimal self-insurance, using a framework provided by Bommier et al. (2012), which works only with ordinal preferences and does not assume a particular representation of risk preferences. To study the interaction between saving and self-insurance, we further employ the preferences proposed by Kihlstrom and Mirman (1974). In this setting, we demonstrate that a more risk-averse agent invests more in self-insurance with and without endogenous saving. Second, we study self-protection decisions. While Bommier et al.'s approach remains silent about the effect of greater risk aversion on self-protection, Kihlstrom and Mirman's preferences enable us to draw conclusions about this effect. Both with and without endogenous saving, we show that, if the loss probability is sufficiently small, a more risk-averse agent invests more in self-protection. We further contrast our results with those obtained by employing widely used but limited

[1]Insurance is, for example, a special case of self-insurance and has similar comparative statics (Courbage et al., 2013). Additionally, note that decision makers in Ehrlich and Becker's (1972) setting are assumed to have precise information about the benefits of risk mitigation. In contrast, Li and Peter (2021) analyze self-insurance and self-protection activities where the effectiveness of risk mitigation is subject to exogenous environmental factors.

preferences to examine risk aversion in intertemporal settings (standard additive expected utility setting, Selden, Epstein and Zin).

Our results contribute to both theoretical and empirical literature. Bommier et al. (2012) show that standard additive expected utility settings as well as Selden's (1978) and Epstein and Zin's (1989) preferences are not well ordered in terms of risk aversion. We explore how the lack thereof affects optimal risk management under an increase in risk aversion. Without the consistency of ordinal preferences (standard additive expected utility setting) or the lack of monotonicity (Selden, Epstein and Zin), we find that two-period results for risk management hinge on the consideration of endogenous saving. Agents who optimize risk management and saving separately (employing two mental accounts) would then behave differently than would agents, who optimize risk management and saving jointly (employing only one mental account). Using preferences that are well ordered in terms of risk aversion, however, we reproduce the one-period results in our two-period models independent of the consideration of endogenous saving. Our findings are thus good news for empiricists. Empirical studies often ignore the intricacies of the time structure when analyzing mitigation decisions and simply assume all costs and benefits to appear contemporaneously. They thus make the implicit assumption that results of one-period models can be transferred to intertemporal settings. Our analysis, therefore, supports the focus of the empirical literature on the structure of the risk rather than on the timing of investments and benefits.

Risk management in two-period settings has received only scant attention in the literature. Menegatti (2009) was the first to study self-protection in a two-period setting, finding that prudence is positively related to investments in self-protection. By introducing endogenous saving, Peter (2017) reproduces the one-period result and explains his finding through the substitution effect between self-protection and saving (Menegatti & Rebessi, 2011). The paper most closely related to ours is Hofmann and Peter (2016), who study self-insurance and self-protection in a two-period expected utility framework with and without saving. In the absence of saving, the authors find that, if first-period consumption is sufficiently high, a more concave utility function raises optimal self-insurance and optimal self-protection. They show that, in the presence of saving, if the loss probability is sufficiently low, higher concavity unambiguously increases optimal self-insurance and optimal self-protection. Increasing the concavity of the utility function, however, changes ordinal preferences in a standard additive expected utility setting, which makes it impossible to study the effect of risk aversion on optimal self-insurance and optimal self-protection. We address this gap and contribute to the literature by focusing on the role of greater risk aversion on optimal risk management in isolation.

In Section 2, we first summarize Bommier et al.'s (2012) framework of comparative risk aversion. We then examine the effect of greater risk aversion on self-insurance and self-protection decisions, using monotone preferences in Section 3 and employing nonmonotone preferences in Section 4. We discuss our findings in view of the extant literature in Section 5. The paper ends with concluding remarks and directions for future research in Section 6. All proofs appear in Appendix A.

2 | COMPARATIVE RISK AVERSION IN TWO PERIODS

2.1 | Overview

According to comparative Arrow-Pratt risk aversion, an agent A is more risk-averse than agent B if A's utility function is more concave than B's (Arrow, 1963; Pratt, 1964). While greater concavity of an agent's utility function represents higher risk aversion in a one-period expected utility setting, it changes ordinal preferences in a standard additive expected utility setting with two periods. This makes it impossible to compare agents in terms of their risk aversion in this setting (Kihlstrom & Mirman, 1974).[2]

Recent theoretical literature presents approaches to examine comparative risk aversion in intertemporal settings. These approaches ensure that ordinal preferences are preserved and that agents do not prefer first-order stochastically dominated lotteries, consistent with monotonicity. For example, Bommier et al. (2012) develop a framework that works only with ordinal preferences and does not assume a particular representation of risk preferences when analyzing the effect of risk aversion on behavior. Further, they show that, while Kihlstrom and Mirman (1974) preferences are well ordered in terms of risk aversion, those of Selden (1978) and Epstein and Zin (1989) are not. In the following, we present Bommier et al.'s (2012) framework of comparative risk aversion for intertemporal settings.

2.2 | Bommier et al.'s (2012) framework of comparative risk aversion

We focus on agents who invest in risk management today and face a potential loss tomorrow. Agents, thus, face a certain period-one consumption and an uncertain period-two consumption. This consumption stream is modeled by lotteries over "certain × uncertain" consumption pairs $(c_1, \tilde{c}_2)$, which pay off in the state space $C = \mathbb{R}_+^2$. Their payoffs are, therefore, of the form (c_1, c_2) with $c_1, c_2 \geq 0$. The uncertainty with respect to the lottery outcome is represented by a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where Ω denotes the set of states of the world θ, $\mathcal{F}$ denotes the σ-algebra of sets in Ω, and $\mathbb{P}$ is the probability measure. Further, $\mathcal{L}(C)$ denotes the set of lotteries with payoffs in the state space C. Each lottery $l \in \mathcal{L}(C)$, $l\colon \Omega \to C$, is a mapping from states of the world into the state space. In such a two-period model, we define comparative risk aversion according to Yaari (1969).

> **Definition 1** (Yaari, 1969). Agent A is at least as risk-averse as agent B if all increases in risk that are acceptable to A are also acceptable to B.

Yaari (1969) shows that his definition of comparative risk aversion is equivalent to comparative Arrow-Pratt risk aversion in the case of a univariate utility function. Bommier et al. (2012) work with Yaari's (1969) definition of comparative risk aversion by (1) assuming certain regularity conditions on *ordinal preferences* as well as *risk preferences* and (2) introducing a relation to compare the *riskiness of lotteries*, which will then be used to define increases in risk.

First, we introduce the regularity conditions on preferences. Agents rank consumption pairs (c_1, c_2) according to a common *ordinal preference* relation $\succcurlyeq$, defined on the state space C. For simplicity, we assume that this ordinal preference relation can be represented by a continuous lifetime utility function $u\colon C \to \mathbb{R}$.[3] Agents also rank uncertain consumption pairs $(c_1, \tilde{c}_2)$ according to their individual *risk preference*. We define an agent's risk preference over subsets H of the set of lotteries $\mathcal{L}(C)$, where $\{\delta_c | c \in C\} \subset H \subset \mathcal{L}(C)$, meaning that H contains the degenerate lotteries δ_c, which pay off

[2]See also Gollier (2001, Section 15.3) for a discussion on this issue regarding the consumption-saving problem.
[3]Debreu (1954) shows the existence of such a continuous utility function if the preference relation is complete, transitive and continuous.

$c := (c_1, c_2)$ with certainty. To make their risk preferences comparable, we require all risk preferences to be consistent with a common ordinal preference relation.

Definition 2 (Consistency with ordinal preferences). Let $\succcurlyeq$ be an ordinal preference and $\succcurlyeq^A$ be the risk preference of agent A. The risk preference $\succcurlyeq^A$ is consistent with the ordinal preference $\succcurlyeq$, if, for $c', c'' \in C$ the relation $c' \succcurlyeq c'' \Leftrightarrow \delta_{c'} \succcurlyeq^A \delta_{c''}$ holds true.

Agents A and B, for example, whose risk preferences $\succcurlyeq^A$ and $\succcurlyeq^B$ are consistent with the ordinal preference relation $\succcurlyeq$, rank degenerate lotteries as the ordinal preference relation ranks consumption pairs. This makes A's and B's risk preference comparable in the absence of risk, which is required for their comparability in the presence of risk.

All risk preferences shall further satisfy the assumption of monotonicity so that agents do not prefer first-order stochastically dominated lotteries.

Definition 3 (Monotonicity). Let $\succcurlyeq$ be an ordinal preference and $\succcurlyeq^A$ be the risk preference of agent A. The risk preference $\succcurlyeq^A$ satisfies monotonicity if, for any lotteries $l_1, l_2 \in H$, the following holds true:

$$(\forall\ \theta \in \Omega\colon\ l_1(\theta) \succcurlyeq l_2(\theta)) \Rightarrow l_1 \succcurlyeq^A l_2. \tag{1}$$

Monotonicity ensures that decision makers never choose an action that provides less lifetime utility in all states of the world. In intertemporal choice models without monotonicity, agents may desire to reduce the difference in lifetime utilities although this may decrease lifetime utility in all states of the world (Bommier & Le Grand, 2018; Bommier et al., 2017).

Second, we introduce the notion of a p-spread to compare the *riskiness of lotteries*. Given a lottery $l \in H$ and its corresponding cumulative distribution function $F_l(z) = \mathbb{P}\{\theta \in \Omega\colon u(l(\theta)) \leq z\}$, this "riskier than" relation is defined as follows.

Definition 4 (Bommier et al., 2012). For fixed $p \in (0, 1)$ lottery l_1 is a p-spread of lottery l_2 (in notation: $l_1 \vdash_p l_2$), if there exists an $u_0 \in \mathbb{R}$ such that:

1. $p \geq F_{l_1}(z) \geq F_{l_2}(z)$ for all $z < u_0$,
2. $p \leq F_{l_1}(z) \leq F_{l_2}(z)$ for all $z \geq u_0$.

In this case, we say that lottery l_1 is an increase in risk of lottery l_2.

Figure 1 provides an example of a p-spread. Here, lottery l_1 is a p-spread of lottery l_2 for $p = 0.5$ and $u_0 = 3$. The pair (u_0, p) allows the separation of utility levels into good and bad states of the world. The bad states of the world ($z < u_0$) occur with probability p and the good states of the world ($z \geq u_0$) with probability $1 - p$. Conditional on the bad states of the world lottery, l_2 first-order dominates lottery l_1 and, conditional on the good states of the world, lottery l_1 first-order dominates lottery l_2. P-spreads, thus, satisfy single-crossing between the lotteries involved.

Given the above p-spread relation, we can define comparative risk aversion in the sense of Yaari (1969).

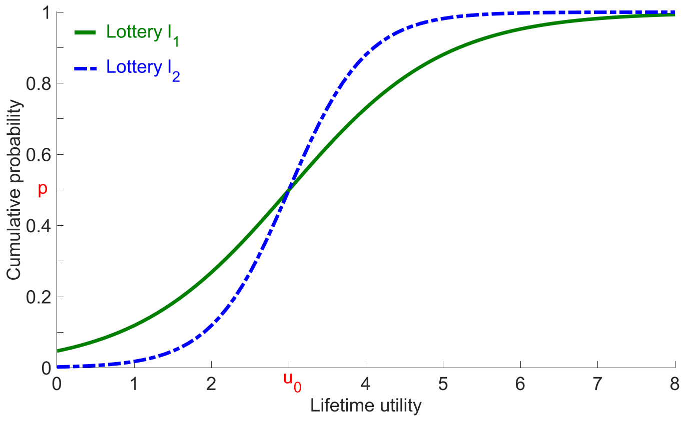


FIGURE 1 The concept of the p-spread relation is displayed. Lottery l_2 first-order dominates lottery l_1 for utility levels below u_0 and lottery l_1 first-order dominates lottery l_2 for utility levels above u_0. Lottery l_1, thus, has a higher chance than l_2 of outcomes that are ordinally more extreme in the sense that l_1 has more probability mass on ordinally favorable and unfavorable outcomes than l_2. Lottery l_1 is, therefore, riskier than lottery l_2 [Color figure can be viewed at wileyonlinelibrary.com]

Definition 5 (Bommier et al., 2012). Agent A is at least as risk-averse as agent B with respect to the p-spread relation if for all $l_1, l_2 \in H$:

$$(l_1 \vdash_p l_2) \wedge (l_1 \succcurlyeq^A l_2) \Rightarrow l_1 \succcurlyeq^B l_2. \tag{2}$$

The intuition behind Bommier et al.'s (2012) definition of comparative risk aversion is as follows. If lottery l_1 is riskier than lottery l_2 according to the p-spread notion, l_1 has a higher chance than l_2 of outcomes that are ordinally more extreme in the sense that l_1 has more probability mass on ordinally favorable and unfavorable outcomes than l_2 (see Figure 1). Consequently, if l_1 is preferred to l_2 by the more risk-averse agent, it is only consequential that it is also preferred over l_2 by the less risk-averse agent.

2.3 | Increased risk aversion and optimal behavior

Lottery $l \in H$ provides an agent with a consumption stream $(c_1, \tilde{c}_2)$. The agent can take an action $y \in J \subset \mathbb{R}_+$ that alters the cumulative distribution function F_l of the lottery l. After the implementation of action y (e.g., investing in self-insurance), we denote the new lottery by l_y and its corresponding cumulative distribution function by F_{l_y}. The objective is then to study optimal actions under an increase in risk aversion. If agent A is at least as risk-averse as agent B with respect to the p-spread relation for all $p \in (0, 1)$, we can use Bommier et al.'s (2012) theorem to compare the optimal actions of agents A and B. This theorem requires the action y to fulfill certain assumptions. First, the action y shall have a nonzero effect on the cumulative distribution function of the lottery l.

Assumption 1 (Nonconstant). Let $y_1 \in J$ and $y_2 \in J$ be two actions. If $l_{y_1}(\theta) \sim l_{y_2}(\theta)$ for all $\theta \in \Omega$, then $y_1 = y_2$.

If an agent is indifferent toward the implementation of action y_1 and action y_2 for all states of the world, the above assumption ensures that these actions are identical. Second, an optimal action y_θ shall exist for every state of the world $\theta \in \Omega$. In addition, given an optimal action y_θ for state $\theta \in \Omega$, moving closer to the optimal action shall improve the lottery in terms of the ordinal preference.

Assumption 2 (Single-peakedness). Let $y, y_1, y_2, y_3, y_4 \in J$ be actions. For all $\theta \in \Omega$, it holds true that:

- There exists an optimal action y_θ so that $\forall\, y \in J$: $l_{y_\theta}(\theta) \succsim l_y(\theta)$.
- For $y_1 \leq y_2 \leq y_\theta \leq y_3 \leq y_4$ we have $l_{y_\theta}(\theta) \succsim l_{y_2}(\theta) \succsim l_{y_1}(\theta)$ and $l_{y_\theta}(\theta) \succsim l_{y_3}(\theta) \succsim l_{y_4}(\theta)$.

Third, actions are assumed not to change the initial ordering of lottery outcomes with respect to the states of the world. In the case of only two states, this means that outcomes in the good state are always preferred to outcomes in the bad state, regardless of the action y implemented.

Assumption 3 (Comonotonicity). Let $\theta_1, \theta_2 \in \Omega$ be two states of the world. If $l_{y_1}(\theta_1) \succsim l_{y_1}(\theta_2)$ for some $y_1 \in J$, then it follows that $l_y(\theta_1) \succsim l_y(\theta_2)$ for all $y \in J$.

For two states $\theta_1, \theta_2 \in \Omega$, we can thus write $\theta_1 \geq \theta_2$ if $l_y(\theta_1) \succsim l_y(\theta_2)$ for some action $y \in J$. The ordering of the states is well defined due to the assumption of comonotonicity. For the action $y = 0$, we obtain the initial ordering of lottery outcomes. According to the above assumption, any other action $y \in J$ does not alter this ordering. The assumption of comonotonicity, therefore, enables us to order the states of the world according to the lottery outcomes.

Assumption 4 (Action order). Applying a suitable bijection ν: $\mathbb{R} \to \mathbb{R}$ leads to $y_{\theta_1} \leq y_{\theta_2}$ whenever $\theta_1 \geq \theta_2$.

Thus, the better the state of the world, the smaller the optimal action. Because we already have defined an ordering of the states of the world, we can always find a bijection so that Assumption 4 is fulfilled. Given Assumptions (1)–(4), we can compare optimal actions under an increase in risk aversion.

Proposition 1 (Bommier et al., 2012). *Consider two agents, A and B, who choose an action y that provides them with a lottery l_y so that Assumptions (1)–(4) are fulfilled. Suppose that the risk preferences of agents A and B are consistent with the same ordinal preference relation and satisfy the assumption of monotonicity. Further, let their respective risk preference define optimal actions y_A^* and y_B^*. If agent A is at least as risk-averse as is agent B with respect to the p-spread relation for all $p \in (0, 1)$, then it follows that $y_A^* \geq y_B^*$.*

According to the assumption of action order, the worse the state of the world θ, the higher the optimal action y_θ in that state. Using agent B's optimal action y_B^* enables us to call states θ with $y_\theta < y_B^*$ good states and those with $y_\theta > y_B^*$ bad states. Agent A, who is more risk-averse than B, is more concerned about the bad states. The assumption of single-peakedness guarantees that actions in the bad states are better the closer they are to the optimal action in this state. It is, therefore, only consequential that the more risk-averse agent chooses a higher action than the less risk-averse agent. This ensures an increase in welfare in the bad states of the world by decreasing welfare in the good states of the world.

3 | RESULTS WITH MONOTONE RISK PREFERENCES

3.1 | Overview

This section provides an examination of the effect of increased risk aversion on optimal self-insurance and self-protection with and without endogenous saving. First, we apply Bommier et al.'s (2012) approach of comparative risk aversion to self-insurance and self-protection decisions. This is parsimonious because it dispenses with the implicit assumptions that typically come with the choice of a preference representation. While more general, such a framework may not permit definitive conclusions about the effect of greater risk aversion on risk management decisions in some situations, such as in the presence of saving. Ordinal preferences rank consumption bundles (c_1, c_2) but are unable to distinguish between two actions, each affecting period-one and period-two consumption. Second, we thus employ a particular representation of risk preferences. Kihlstrom and Mirman's (1974) preferences are well ordered in terms of risk aversion and enable us to establish a link between risk aversion and optimal actions in those situations where ordinal preferences do not provide enough structure to derive specific results. In our subsequent analyses, we distinguish between three instruments to manage risk.

Definition 6 (Instruments).

1. Self-insurance: Agents invest an amount y in self-insurance in the first period to reduce the size of a random loss $\widetilde{L}$ with realizations of 0 and $L(y)$ in the second period. The size of loss $L(y)$ is convex in self-insurance. Whenever we employ a particular representation of risk preferences, we further assume that $L(y)$ is a twice differentiable function of y with $L'(y) < 0$ and $L''(y) \geq 0$.
2. Self-protection: In the first period, agents invest an amount x in self-protection to reduce the probability $p(x) p'(x) < 0 \, p''(x) \geq 0$ of a loss of size L in the second period. The probability of loss $p(x)$ is convex in self-protection. Whenever we employ a particular representation of risk preferences, we further assume that $p(x)$ is a twice differentiable function of x with $p'(x) < 0$ and $p''(x) \geq 0$.
3. Saving: Agents can save an amount s in the first period to receive income of $Rs, R := 1 + r > 1$, in the second period.

Self-insurance, self-protection, and saving reduce first-period income to increase expected second-period consumption, but these instruments act differently in achieving this outcome. Whereas saving increases second-period wealth in both states of the world, self-insurance increases second-period wealth in the loss state and keeps it constant in the no-loss state. Further, self-protection keeps second-period wealth constant in both states of the world but shifts probability mass from the loss state to the no-loss state.

3.2 | No particular representation of risk preferences

3.2.1 | Setting

We consider an agent who is endowed with initial wealth of w_1 in the first period and w_2 in the second period. In the second period, either a loss of size L occurs with probability p (state θ_2) or no loss occurs with probability $1 - p$ (state θ_1). The agent faces the income profile $(w_1, w_2 - L)$ if a loss occurs and the income profile of (w_1, w_2) if no loss occurs. We write $l := (w_1, w_2 - \widetilde{L}\,) \in H$ for the agent's uncertain income profile with $\widetilde{L}$ as the random variable that designates the size of the loss. Consider agents A and B with risk preferences $\succcurlyeq^i$, $i = A, B$. Let agent A be more risk-averse than B with respect to the p-spread relation for every $p \in (0, 1)$. Using Proposition 1, we aim to study whether agent A invests more in mitigating the loss in the second period than B.

3.2.2 | Self-insurance

In the first period, the agent can invest in self-insurance to reduce the size of the loss in the second period. The agent gives up wealth from the certain first-period income to increase expected consumption in the second period and to decrease the riskiness of second-period consumption. By investing in self-insurance, the agent changes the payoff of the lottery l. For a self-insurance level y, we denote the modified lottery by $l_y := (w_1 - y, w_2 - \widetilde{L}\,(y)) \in H$.

Figure 2 provides an example of how self-insurance alters the distribution function over lifetime utilities. Self-insurance decreases lifetime utility in the good state and increases it in the bad state. Assume an agent with an optimal self-insurance level y^* and two self-insurance levels y_1 and y_2 $(0 \leq y_1 < y_2 \leq y^*)$. If an agent raises her investment in self-insurance from y_1 to y_2, this makes lottery $l_2 := l_{y_2}$ first-order dominate lottery $l_1 := l_{y_1}$ for utility levels smaller than u_0 $(p \geq F_{l_1}(z) \geq F_{l_2}(z)$ for all $z < u_0)$. For utility levels above u_0, l_1 first-order dominates lottery l_2 $(p \leq F_{l_1}(z) \leq F_{l_2}(z)$ for all $z \geq u_0)$. Following the definition of a p-spread, lottery l_1 is a p-spread of lottery l_2 for $p = 0.2$ and $u_0 = 9$.[4]

Investing in self-insurance changes lottery l in a way that enables us to compare optimal self-insurance levels under greater risk aversion according to Bommier et al. (2012). The assumptions of Proposition 1 are satisfied if we assume normality of first-period consumption and convexity of ordinal preferences.[5] Thus, if agent A is at least as risk-averse as B with respect to the p-spread relation for all $p \in (0, 1)$ and the size of loss is decreasing and convex in investments in self-insurance, we can use Proposition 1 to show:

Proposition 2. *An increase in risk aversion raises optimal self-insurance.*

The proof of this result uses only ordinal preferences. The result, thus, is neither restricted to expected utility, nor is it restricted to a particular representation of risk preferences.

3.2.3 | Self-protection

In the first period, the agent can invest in self-protection to reduce the probability of a loss in the second period. In the second period, either a loss of size L occurs with probability $p(x)$ or no loss occurs with probability $1 - p(x)$. By investing in self-protection, the agent alters the

[4]Lottery l_1 is a p-spread of lottery l_2 for each choice $u_0 \in [5, 9]$ and probability $p = 0.2$.

[5]An ordinal preference relation has the property of normality of first-period consumption if optimal first-period consumption rises with an increase in income. Additionally, convexity of ordinal preferences can be interpreted by a diminishing marginal rate of substitution of consumption in period one and consumption in period two.

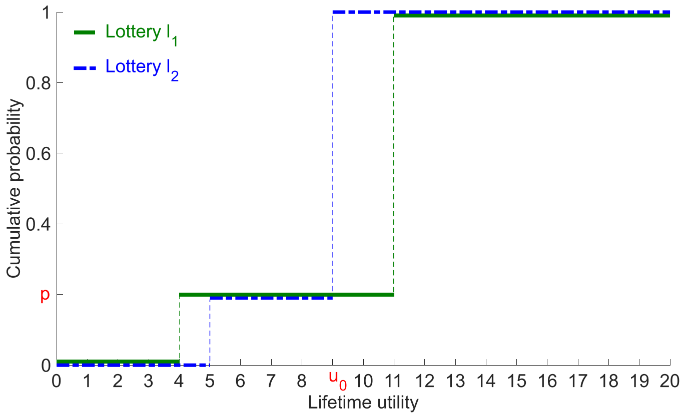


FIGURE 2 Effect of self-insurance on the cumulative distribution function over lifetime utilities. Without self-insurance, the agent faces the income profile $(w_1, w_2 - L)$ if a loss occurs and (w_1, w_2) if no loss occurs. Let y denote the cost of self-insurance in the first period. By investing in self-insurance, the agent changes the payoff of the lottery $l \coloneqq (w_1, w_2 - \widetilde{L})$. For a self-insurance level y, we denote the modified lottery by $l_y \coloneqq (w_1 - y, w_2 - \widetilde{L}(y))$. If an agent raises her investment in self-insurance from y_1 to y_2, this makes lottery $l_2 \coloneqq l_{y_2}$ first-order dominate lottery $l_1 \coloneqq l_{y_1}$ for utility levels smaller than u_0. For utility levels above u_0, lottery l_1 first-order dominates lottery l_2. Lottery l_1, thus, is riskier than lottery l_2 according to the p-spread notion for $p = 0.2$ and $u_0 = 9$ [Color figure can be viewed at wileyonlinelibrary.com]

probability distribution over payoffs of the lottery $l \coloneqq (w_1, w_2 - \widetilde{L})$. For a self-protection level x, we denote the modified lottery by $l_x \coloneqq (w_1 - x, w_2 - \widetilde{L})$.

Figure 3 illustrates how self-protection changes the cumulative distribution function over lifetime utilities. Assume an agent with an optimal self-protection level x^* and two self-protection levels x_1 and x_2 $(0 \leq x_1 < x_2 \leq x^*)$. If an agent raises her investment in self-protection from x_1 to x_2, this makes the cumulative distribution functions of lottery $l_2 \coloneqq l_{x_2}$ and $l_1 \coloneqq l_{x_1}$ cross twice. The intuition is that, in both the loss state and the no-loss state, agents derive higher utility from foregoing an investment in self-protection. The agent nevertheless benefits from investing in self-protection as she shifts probability mass from the loss state to the no-loss state. The agent gives up wealth from the certain first-period income to increase expected consumption in the second period and to decrease the riskiness of second-period consumption. Because the cumulative distribution functions F_{l_1} and F_{l_2} cross twice, neither is lottery l_1 a p-spread of lottery l_2, nor is lottery l_2 a p-spread of lottery l_1. Hence, Bommier et al.'s (2012) approach remains silent about the effect of greater risk aversion on self-protection.

While the notion of a p-spread is parsimonious because it only depends on ordinal preferences, it does not allow us to draw any conclusions about the effect of greater risk aversion on self-protection. This is because self-protection does not reduce risk in the sense of Rothschild and Stiglitz (1970).[6] There also is no hope of finding results for self-protection by replacing the p-spread notion with other notions of dispersion because any other notion (e.g., higher-order stochastic dominance) would require a particular representation of risk preferences (Bommier et al., 2012, footnote 6).

[6]Self-protection does not reduce risk in the sense of Rothschild and Stiglitz (1970) because in both the loss state and the no-loss state agents derive higher utility from foregoing an investment in self-protection (Briys & Schlesinger, 1990).

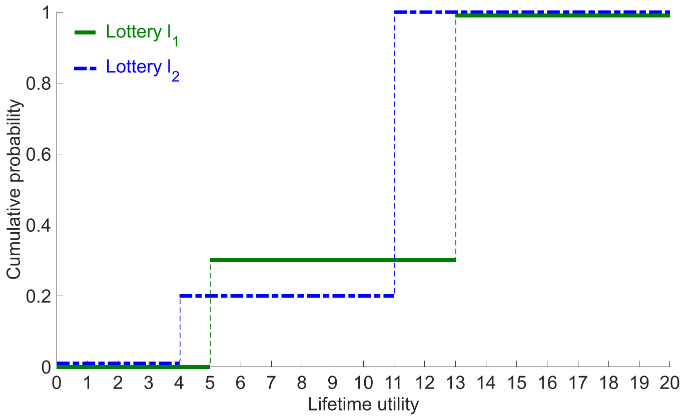


FIGURE 3 Effect of self-protection on the cumulative distribution function over lifetime utilities. Without self-protection, the agent faces the income profile $(w_1, w_2 - L)$ if a loss occurs and (w_1, w_2) if no loss occurs. Let x denote the cost of self-protection in the first period. By investing in self-protection, the agent alters the probability distribution over payoffs of the lottery $l := (w_1, w_2 - \widetilde{L})$. For a self-protection level x, we denote the modified lottery by $l_x := (w_1 - x, w_2 - \widetilde{L})$. If an agent raises her investment in self-protection from x_1 to x_2, this makes the cumulative distribution functions of lottery $l_2 := l_{x_2}$ and $l_1 := l_{x_1}$ cross twice. Because the cumulative distribution functions F_{l_1} and F_{l_2} cross twice, neither is lottery l_1 a p-spread of lottery l_2, nor is lottery l_2 a p-spread of lottery l_1 [Color figure can be viewed at wileyonlinelibrary.com]

3.3 | Kihlstrom–Mirman preferences

3.3.1 | Setting

Using particular representations of risk preferences enables us to establish a link between risk aversion and optimal actions in situations in which we cannot draw conclusions using Bommier et al.'s (2012) framework of comparative risk aversion (e.g., self-protection, risk management in the presence of endogenous saving). The Kihlstrom–Mirman framework (1974) provides a way to analyze the role of risk aversion in a two-period expected utility framework (Bommier et al., 2012; Bommier & Le Grand, 2014). For a given consumption stream, where c_1 denotes certain first-period consumption and $\tilde{c}_2$ denotes uncertain second-period consumption, the agent derives welfare according to the following expected utility objective:

$$U(c_1, \tilde{c}_2) = \mathbb{E}[\varphi(u(c_1) + v(\tilde{c}_2))]. \tag{3}$$

In the above expression φ, u and v are twice-differentiable real-valued functions that are increasing and concave. u denotes the agent's utility function in the first period and v denotes the agent's utility function in the second period. Given u and v, agent A is more risk-averse than agent B if A's φ-function is more concave than B's (Kihlstrom & Mirman, 1974). The transformation φ does not have an impact in the absence of risk and, thus, does not change ordinal preferences. Increasing the concavity of φ raises the agent's risk aversion in the sense of Yaari (1969) and is the only way to study risk aversion while remaining in an expected utility framework (Bommier et al., 2012; Kihlstrom & Mirman, 1974).

3.3.2 | Self-insurance, Kihlstrom–Mirman preferences

The agent's optimal demand for self-insurance maximizes

$$\max_{y} U(y) = (1-p)\cdot\varphi(u(w_1-y)+v(w_2)) + p\cdot\varphi(u(w_1-y)+v(w_2-L(y))). \tag{4}$$

For the sake of notational simplicity, we set $u_F := u(w_1-y), v_L := v(w_2-L(y))$, and $v_N := v(w_2)$. Here, u_F denotes the utility derived from the first period, v_L denotes the utility derived from the second period in the loss state, and v_N denotes the utility derived from the second period in the no-loss state ($v_L \le u_F \le v_N$). An interior solution[7] for optimal self-insurance y^* is given by the first-order condition:

$$\begin{aligned}\frac{\partial}{\partial y}U(y) = -[(1-p)\cdot\varphi'(u_F+v_N)+p\cdot\varphi'(u_F+v_L)]u_F' \\ -pL'(y)\cdot\varphi'(u_F+v_L)v_L' = 0.\end{aligned} \tag{5}$$

The optimal level of self-insurance y^* balances the marginal cost (first term in Equation 5) and benefit of the investment (second term in Equation 5). Consider two agents, A and B, where $\varphi_A = k(\varphi_B)$ with a twice-differentiable real-valued function k which is increasing and concave. Then, agent A is more risk-averse than agent B (Kihlstrom & Mirman, 1974). Evaluating A's first-order condition at B's optimal level of self-insurance, shows that the more risk-averse A invests more in self-insurance than the less risk-averse B.

Proposition 3. *An increase in risk aversion raises optimal self-insurance.*

This result is a special case of Proposition 2 under the assumption of an expected utility setting endowed with Kihlstrom and Mirman (1974) preferences.

3.3.3 | Self-protection, Kihlstrom–Mirman preferences

The agent optimizes her expected utility over investments in self-protection:

$$\max_{x} U(x) = (1-p(x))\cdot\varphi(u(w_1-x)+v(w_2)) + p(x)\cdot\varphi(u(w_1-x)+v(w_2-L)). \tag{6}$$

For the sake of notational simplicity, we again use the above abbreviations and set $v_L := v(w_2-L)$ ($v_L \le u_F \le v_N$). An interior solution[8] for optimal self-protection x^* can be obtained from the first-order condition:

$$\begin{aligned}\frac{\partial}{\partial x}U(x) = -[(1-p(x))\cdot\varphi'(u_F+v_N)+p(x)\cdot\varphi'(u_F+v_L)]u_F' \\ -p'(x)[\varphi(u_F+v_N)-\varphi(u_F+v_L)] = 0.\end{aligned} \tag{7}$$

[7] The second-order condition is satisfied, that means $U''(y) = (1-p)\cdot\varphi''(u_F+v_N)(-u_F')^2 + (1-p)\cdot\varphi'(u_F+v_N)\cdot u_F'' + p\cdot\varphi''(u_F+v_L)(-u_F'-L'(y)\cdot v_L')^2 + p\cdot\varphi'(u_F+v_L)(u_F''-L''(y)\cdot v_L' + (L'(y))^2\cdot v_L'') < 0$.

[8] The second-order condition is not necessarily satisfied. It is given by $U''(x) = p''(x)[\varphi(u_F+v_L)-\varphi(u_F+v_N)] + 2p'(x)\cdot\varphi'(u_F+v_N)u_F' - 2p'(x)\cdot\varphi'(u_F+v_L)u_F' + (1-p(x))\cdot[\varphi''(u_F+v_N)(-u_F')^2 + \varphi'(u_F+v_N)u_F'' + p(x)[\varphi'(u_F+v_L)(-u_F')^2 + \varphi'(u_F+v_L)u_F'']$.

The optimal level of self-protection x^* balances the marginal cost (first term in Equation 7) and the marginal benefit of the investment (second term in Equation (7)). Using A's and B's first-order conditions, we find that the more risk-averse A invests more in self-protection than the less risk-averse B if and only if the loss probability is small enough.

Proposition 4. *An increase in risk aversion raises optimal self-protection if and only if the probability of loss is below an endogenous threshold.*

This ambiguity is also shown in a one-period setting (Briys & Schlesinger, 1990; Dionne & Eeckhoudt, 1985; Jullien et al., 1999). The intuition is that self-protection does not reduce risk in the sense of Rothschild and Stiglitz (1970). It induces a mean-preserving spread at low wealth levels and a mean-preserving contraction at high wealth levels.

3.3.4 | Self-insurance with endogenous saving, Kihlstrom–Mirman preferences

Beyond investing in self-insurance, agents can engage in saving. Referring to the concept of mental accounting (Thaler, 1999), the agent can optimize self-insurance and saving either separately or jointly. If the agent employs two separate mental accounts for these instruments, Proposition 3 shows that greater risk aversion increases optimal self-insurance. If the agent employs only one mental account, she jointly optimizes her expected utility over investments in self-insurance and saving:

$$\begin{aligned}\max_{y,s} U(y,s) = (1-p)\cdot\varphi(u(w_1-y-s)+v(w_2+Rs))\\ +p\cdot\varphi(u(w_1-y-s)+v(w_2-L(y)+Rs)).\end{aligned} \tag{8}$$

For the sake of notational simplicity, we set $u_F := u(w_1-y-s)$, $v_L := v(w_2-L(y)+Rs)$, and $v_N := v(w_2+Rs)$. Interior solutions[9] for optimal self-insurance y^* and saving s^* are characterized by the first-order conditions:

$$\begin{aligned}\frac{\partial}{\partial y}U(y,s) = -[(1-p)\cdot\varphi'(u_F+v_N)+p\cdot\varphi'(u_F+v_L)]u_F'\\ -pL'(y)\cdot\varphi'(u_F+v_L)v_L' = 0.\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\partial}{\partial s}U(y,s) = -[(1-p)\cdot\varphi'(u_F+v_N)+p\cdot\varphi'(u_F+v_L)]u_F'\\ +[(1-p)\cdot\varphi'(u_F+v_N)v_N'+p\cdot\varphi'(u_F+v_L)v_L']R = 0.\end{aligned} \tag{10}$$

The pair (y^*, s^*) jointly balances the marginal cost and the marginal benefit of self-insurance and saving. To investigate the effect of increased risk aversion on optimal self-insurance, we recap the following lemma as given in Hofmann and Peter (2016, Lemma 1).

Lemma 1. *Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be a concave function in the variables (y,s), that is, $f_{yy} < 0$ and $f_{yy}f_{ss} - f_{ys}^2 > 0$, which is maximal at (y^*, s^*). Let $\bar{y} \in \mathbb{R}$ be a value we want to*

[9]The Hessian matrix is negative definite. Thus, the objective function is concave in the parameters (y,s). The second-order conditions are given in the Supporting Information Appendix.

compare y^* with. Then, $y^* > \bar{y}$ if and only if $f_y(\bar{y}, \hat{s}) > 0$ where $\hat{s}$ is the value that maximizes $f(\bar{y}, s)$.

Comparing the optimal self-insurance level y_A^* to y_B^*, where agent A is more risk-averse than agent B, Lemma 1 instructs to evaluate A's first-order condition at $(y_B^*, \hat{s})$, where $\hat{s}$ is the value that maximizes A's expected utility evaluated at y_B^*. Using A's and B's first-order conditions, we can show that A's optimal investment in self-insurance is greater than B's.

Proposition 5. *Under endogenous saving, an increase in risk aversion raises optimal self-insurance.*

A more risk-averse agent, thus, invests more in self-insurance not only without but also with endogenous saving.

3.3.5 | Self-protection with endogenous saving, Kihlstrom-Mirman preferences

The agent utilizes investments in self-protection and saving to optimize her expected utility. According to Proposition 4, an agent who optimizes self-protection and saving separately increases her optimal self-protection under greater risk aversion if the loss probability is below an endogenous threshold. In contrast, if the agent employs only one mental account, she considers investments in self-protection and saving jointly in her decision making:

$$\max_{x,s} U(x,s) = (1 - p(x)) \cdot \varphi(u(w_1 - x - s) + v(w_2 + Rs)) \\ + p(x) \cdot \varphi(u(w_1 - x - s) + v(w_2 - L + Rs)). \tag{11}$$

We again define u_F and v_N as above and set $v_L := v(w_2 - L + Rs)$. We characterize interior solutions[10] for optimal self-protection x^* and saving s^* by using the first-order conditions:

$$\frac{\partial}{\partial x} U(x,s) \quad -[(1 - p(x)) \cdot \varphi'(u_F + v_N) + p(x) \cdot \varphi'(u_F + v_L)]u_F' \\ -p'(x)[\varphi(u_F + v_N) - \varphi(u_F + v_L)] = 0, \tag{12}$$

$$\frac{\partial}{\partial s} U(x,s) = -[(1 - p(x)) \cdot \varphi'(u_F + v_N) + p(x) \cdot \varphi'(u_F + v_L)]u_F' \\ +[(1 - p(x)) \cdot \varphi'(u_F + v_N)v_N' + p(x) \cdot \varphi'(u_F + v_L)v_L']R = 0. \tag{13}$$

The pair (x^*, s^*) jointly balances the marginal cost and marginal benefit of self-protection and saving. We apply Lemma 1 to study the effect of increased risk aversion on optimal self-protection. Using agent A's and agent B's first-order conditions, we find that the more risk-averse A invests more in self-protection than the less risk-averse B if and only if the loss probability is small enough.

[10]The Hessian matrix is not necessarily negative definite. Thus, we cannot conclude that the objective function is necessarily concave in the parameters (x, s). The second-order conditions are given in the Supporting Information Appendix.

Proposition 6. *Under endogenous saving, an increase in risk aversion raises optimal self-protection if and only if the probability of loss is below an endogenous threshold.*

A more risk-averse agent, therefore, invests more in self-protection with and without endogenous saving if the loss probability is sufficiently low.

4 | RESULTS WITH NONMONOTONE RISK PREFERENCES

4.1 | Overview

In this section, we employ the widely used preferences proposed by Selden (1978) and Epstein and Zin (1989) to study optimal self-insurance and self-protection. While Kihlstrom and Mirman's (1974) preferences are monotone, Selden's (1978) and Epstein and Zin's (1989) preferences lack this property. This enables us to explore how the monotonicity property of preferences affects optimal risk management under an increase in risk aversion.

4.2 | Selden preferences

4.2.1 | Setting

For a given consumption stream where c_1 denotes certain first-period consumption and $\tilde{c}_2$ denotes uncertain second-period consumption, the agent derives welfare according to the following recursive utility objective (Selden, 1978):

$$U(c_1, \tilde{c}_2) = u(c_1) + v(M) = u(c_1) + v(\psi^{-1}(\mathbb{E}[\psi(\tilde{c}_2)])). \tag{14}$$

In this equation, u, v, and ψ are twice-differentiable real-valued functions that are increasing and concave. u denotes the agent's utility function in the first period, and v denotes the agent's utility function in the second period. Given u and v, agent A is more risk-averse than agent B if A's ψ-function is more concave than B's (Selden, 1978). In addition, we abbreviate the certainty equivalent of future consumption by M.

The above model encompasses two well-known risk preferences. We obtain the standard additive expected utility model for $\psi = v$ and the isoelastic representation of Epstein and Zin's (1989) preferences for $u(x) = v(x) = \frac{x^{1-\rho}}{1-\rho}$ and $\psi(x) = x^{1-\gamma}$. In a standard additive expected utility setting, risk preferences are not consistent with ordinal preferences but monotone. In contrast, Selden's (1978) and Epstein and Zin's (1989) preferences are consistent with ordinal preferences but are not necessarily monotone.

Further, in a standard additive expected utility framework, the agent is indifferent to the timing of the resolution of uncertainty. This is no longer the case in a setting with Selden's (1978) preferences. The relation between the second-period utility function that reflects consumption smoothing (v) and the one that reflects risk aversion (ψ) can be interpreted as the preference for an early or late resolution of uncertainty (Gollier, 2001).

Definition 7 (Preferences for the resolution of uncertainty).

1. An agent prefers an *early resolution of uncertainty*, if v is less concave than ψ.
2. An agent prefers a *late resolution of uncertainty*, if v is more concave than ψ.

If an agent exhibits a preference for an early resolution of uncertainty, she prefers to observe random second-period consumption in the first period rather than in the second period. This is beneficial because early information can lead to better decision making.

We now turn our focus to the effect of greater risk aversion on optimal risk management decisions. Unfortunately, objective functions in a Selden (1978) framework are not necessarily well behaved even if u, v, and ψ are concave functions. Subsequently, we assume that interior solutions exist. We refer to Gollier (2001) for conditions to ensure that second-order conditions are fulfilled.

4.2.2 | Self-insurance, Selden preferences

The agent's optimal demand for self-insurance maximizes

$$\begin{aligned}\max_y U(y) &= u(w_1 - y) + v(M(y)) \\ &= u(w_1 - y) + v(\psi^{-1}(p \cdot \psi(w_2 - L(y)) + (1 - p) \cdot \psi(w_2))).\end{aligned} \tag{15}$$

An interior solution for optimal self-insurance y^* is given by the first-order condition:

$$\frac{\partial}{\partial y} U(y) = -u'(w_1 - y) - pL'(y) \cdot \psi'(w_2 - L(y)) \cdot \frac{v'(M(y))}{\psi'(M(y))} = 0. \tag{16}$$

The optimal level of self-insurance y^* balances the marginal cost (first term in Equation 16) and benefit of the investment (second term in Equation 16). Consider two agents, A and B, where $\psi_A = k(\psi_B)$ with a twice-differentiable real-valued function k, which is increasing and concave. Then, agent A is more risk-averse than agent B (Selden, 1978). Evaluating A's first-order condition at B's optimal level of self-insurance leads to the following proposition.

Proposition 7. *An increase in risk aversion either increases or decreases optimal self-insurance. If the agent prefers to resolve uncertainty late, then increased risk aversion raises optimal self-insurance.*

This result is identical to Proposition 1 in Berger (2010), who studies optimal self-insurance and self-protection without endogenous saving. While Kihlstrom and Mirman's (1974) preferences are monotone, Selden's preferences lack this property. This explains why we obtain a different result with Kihlstrom and Mirman's preferences.

4.2.3 | Self-protection, Selden preferences

The agent optimizes her expected utility over investments in self-protection:

$$\begin{aligned}\max_x U(x) &= u(w_1 - x) + v(M(x)) \\ &= u(w_1 - x) + v(\psi^{-1}(p(x) \cdot \psi(w_2 - L) + (1 - p(x)) \cdot \psi(w_2))).\end{aligned} \tag{17}$$

An interior solution for optimal self-protection x^* can be obtained from the first-order condition:

$$\frac{\partial}{\partial x}U(x) = -u'(w_1 - x) - p'(x)\cdot(\psi(w_2) - \psi(w_2 - L))\cdot\frac{v'(M(x))}{\psi'(M(x))} = 0. \tag{18}$$

The optimal level of self-protection x^* balances the marginal cost (first term in Equation 18) and the marginal benefit of the investment (second term in Equation 18). Using A's and B's first-order conditions, we show that the more risk-averse A invests more in self-protection than the less risk-averse B if the loss probability is small enough.

Proposition 8. *An increase in risk aversion either increases or decreases optimal self-protection. If the agent prefers to resolve uncertainty late and the probability of loss is below an endogenous threshold, then increased risk aversion raises optimal self-protection.*

Again, the result with Kihlstrom and Mirman's preferences differs from that with Selden's preferences.

4.2.4 | Self-insurance with endogenous saving, Selden preferences

Beyond investing in self-insurance, agents can engage in saving. Proposition 7 shows that, if the agent employs two separate mental accounts for these instruments, greater risk aversion either increases or decreases optimal self-insurance. If the agent employs only one mental account, she jointly optimizes her expected utility over investments in self-insurance and saving:

$$\begin{aligned}\max_{y,s} U(y,s) &= u(w_1 - y - s) + v(M(y,s))\\ &= u(w_1 - y - s) + v(\psi^{-1}(p\cdot\psi(w_2 - L(y) + Rs) + (1-p)\cdot\psi(w_2 + Rs))).\end{aligned} \tag{19}$$

Interior solutions for optimal self-insurance y^* and saving s^* are characterized by the first-order conditions:

$$\frac{\partial}{\partial y}U(y,s) = -u'(w_1 - y - s) - pL'(y)\cdot\psi'(w_2 - L(y) + Rs)\cdot\frac{v'(M(y,s))}{\psi'(M(y,s))} = 0, \tag{20}$$

$$\begin{aligned}\frac{\partial}{\partial s}U(y,s) &= -u'(w_1 - y - s)\\ &\quad + R\cdot(p\cdot\psi'(w_2 - L(y) + Rs) + (1-p)\cdot\psi'(w_2 + Rs))\cdot\frac{v'(M(y,s))}{\psi'(M(y,s))} = 0.\end{aligned} \tag{21}$$

The pair (y^*, s^*) jointly balances the marginal cost and the marginal benefit of self-insurance and saving. Applying Lemma 1 and using A's and B's first-order conditions, we can show the following relation between A's and B's optimal investment in self-insurance.

Proposition 9. *Under endogenous saving, an increase in risk aversion raises optimal self-insurance.*

With endogenous saving, greater risk aversion increases optimal self-insurance independent of the agent's preference for the resolution of uncertainty.

4.2.5 | Self-protection with endogenous saving, Selden preferences

The agent utilizes self-protection and saving to optimize her expected utility. According to Proposition 8, an agent who optimizes self-protection and saving separately either increases or decreases her optimal self-protection when her level of risk aversion increases. In contrast, if the agent employs only one mental account, she considers investments in self-protection and saving jointly in her decision making:

$$\begin{aligned}\max_{x,s} U(x,s) &= u(w_1 - x - s) + v(M(x,s)) \\ &= u(w_1 - x - s) + v(\psi^{-1}(p(x)\cdot\psi(w_2 - L + Rs) + (1 - p(x))\cdot\psi(w_2 + Rs))).\end{aligned} \tag{22}$$

We characterize interior solutions for optimal self-protection x^* and saving s^* by using the first-order conditions:

$$\frac{\partial}{\partial x}U(x,s) = -u'(w_1 - x - s) - p'(x)\cdot(\psi(w_2 + Rs) - \psi(w_2 - L + Rs))\cdot\frac{v'(M(x,s))}{\psi'(M(x,s))} = 0, \tag{23}$$

$$\begin{aligned}\frac{\partial}{\partial s}U(x,s) &= -u'(w_1 - x - s) \\ &\quad + R\cdot(p(x)\cdot\psi'(w_2 - L + Rs) + (1 - p(x))\cdot\psi'(w_2 + Rs))\cdot\frac{v'(M(x,s))}{\psi'(M(x,s))} = 0.\end{aligned} \tag{24}$$

The pair (x^*, s^*) jointly balances the marginal cost and marginal benefit of self-protection and saving. We apply Lemma 1 to study the effect of increased risk aversion on optimal self-protection. Using agent A's and agent B's first-order conditions, we find that the more risk-averse A invests more in self-protection than the less risk-averse B if and only if the loss probability is small enough.

Proposition 10. *Under endogenous saving, an increase in risk aversion raises optimal self-protection if and only if the probability of loss is below an endogenous threshold.*

With endogenous saving and if the loss probability is small enough, greater risk aversion raises optimal self-protection independent of the agent's preference for the resolution of uncertainty.

4.3 | Epstein–Zin preferences

4.3.1 | Setting

One famous example of Selden's (1978) preferences is the isoelastic representation of Epstein and Zin (1989). For a given consumption stream, where c_1 denotes certain first-period consumption and $\tilde{c}_2$ denotes uncertain second-period consumption, the agent derives welfare according to the following recursive utility objective (Epstein & Zin, 1989):

$$U(c_1, \tilde{c}_2) = \frac{c_1^{1-\rho}}{1-\rho} + \frac{1}{1-\rho}\left(\mathbb{E}\left[\tilde{c}_2^{\,1-\gamma}\right]\right)^{\frac{1-\rho}{1-\gamma}}. \tag{25}$$

In Equation (25), $\rho \neq 1$ and $\gamma \neq 1$ are positive scalars. The agent's risk aversion is represented by γ. A higher value of γ leads to a more risk-averse agent. The agent's elasticity of intertemporal substitution *EIS* is equal to the inverse of ρ. Consequently, the larger the parameter ρ, the smaller the agent's *EIS*, which can be interpreted as response of consumption growth to changes in the interest rate r. In the following, we employ these preferences to illustrate Propositions (7)–(10), using a numerical simulation.

4.3.2 | Self-insurance, Epstein–Zin preferences

Agents are endowed with a wealth of $w = 100$ in each period and face a loss of size $L = 50$ with a probability $p = 0.3$ in the second period. We consider two instruments. First, agents can engage in saving ($R = 1$).[11] Second, investments in self-insurance reduce the size of the loss:

$$L(y) = \frac{2}{3} \cdot L \cdot \exp(-0.06y) + \frac{1}{3} \cdot L.$$

We consider several relations between self-insurance and saving decisions: (1) agents optimize saving separately (Equation 26), (2) agents optimize self-insurance separately (Equation 27), and (3) agents optimize self-insurance and saving jointly (Equation 28).

$$\max_s U(s) = \frac{(w-s)^{1-\rho}}{1-\rho} + \frac{1}{1-\rho}\left[(1-p)(w+Rs)^{1-\gamma} + p(w-L+Rs)^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}}, \tag{26}$$

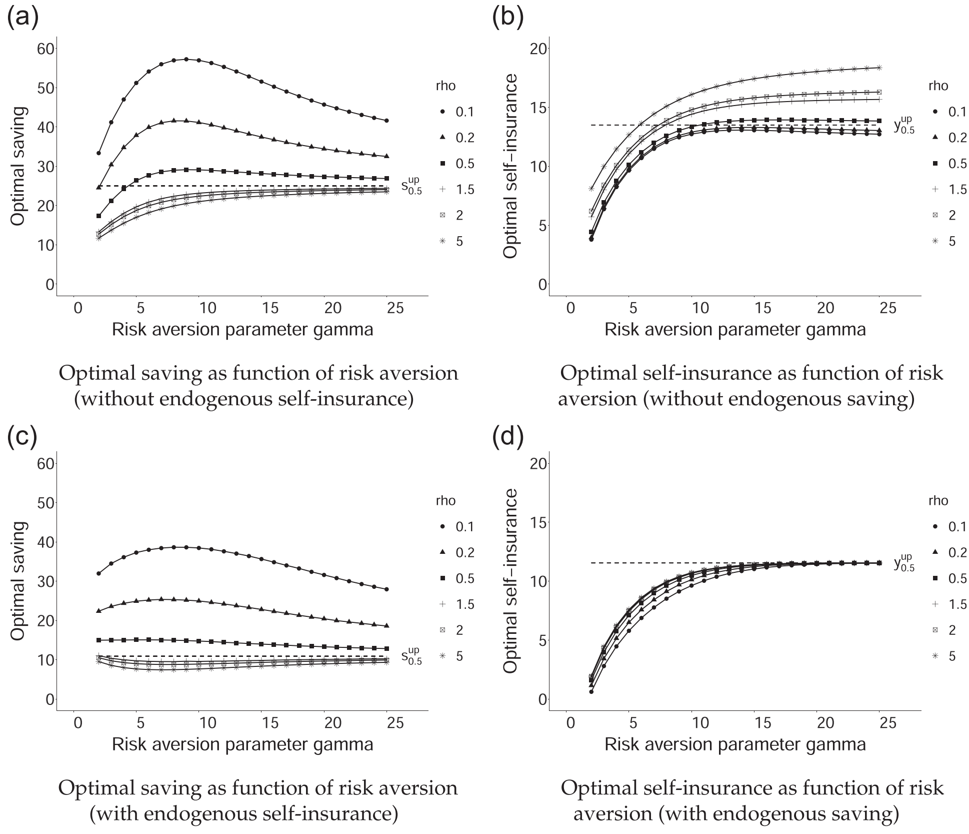
$$\max_y U(y) = \frac{(w-y)^{1-\rho}}{1-\rho} + \frac{1}{1-\rho}\left[(1-p)w^{1-\gamma} + p(w-L(y))^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}}, \tag{27}$$

$$\max_{y,s} U(y,s) = \frac{(w-y-s)^{1-\rho}}{1-\rho} + \frac{1}{1-\rho}\left[(1-p)(w+Rs)^{1-\gamma} + p(w-L(y)+Rs)^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}}. \tag{28}$$

Figure 4 provides a plot of optimal saving and self-insurance as functions of $\gamma = 2, \ldots 25$ and $\rho \in \{0.1, 0.2, 0.5, 1.5, 2, 5\}$. Panel (a) shows optimal saving. The lower ρ and thus the lower the desire for consumption smoothing, the higher the amount of saving. Further, we observe that increased risk aversion either increases or decreases optimal saving. To derive some intuition, we assume $\rho = 0.5$. We can calculate the optimal saving level given that the good state ($s_{0.5}^{\mathrm{up}}$, dashed line in Panel (a)) or the bad state of the world ($s_{0.5}^{\mathrm{low}}$, not depicted here) occurs for certain. The monotonicity requirement of risk preferences ensures that optimal saving lies always in the interval $[s_{0.5}^{\mathrm{low}}, s_{0.5}^{\mathrm{up}}]$ (Bommier et al., 2017). According to Proposition 1, the more risk-averse the agent, the closer optimal saving to $s_{0.5}^{\mathrm{up}}$. Because saving acts as a means of consumption smoothing and risk management in this setting, increased risk aversion would raise optimal saving if preferences were monotone. Epstein and Zin's (1989) preferences, however, do not fulfill the assumption of monotonicity and, thus, optimal saving can lie above $s_{0.5}^{\mathrm{up}}$. Starting at $\gamma = 10$, greater risk aversion decreases optimal saving, leading to a value closer to the optimal saving level $s_{0.5}^{\mathrm{up}}$.

The same intuition applies to optimal self-insurance. We derive the optimal level of self-insurance in the bad state of the world for $\rho = 0.5$, denoted by $y_{0.5}^{\mathrm{up}}$. Due to the nonmonotonicity of Epstein and Zin's (1989) preferences, optimal self-insurance can increase above the optimal level of self-insurance in the bad state. Optimal self-insurance increases with greater risk aversion until $\gamma = 17$ and decreases afterward. Panel (b) further shows that, even for small values of risk aversion, there are marked differences between optimal self-insurance levels. Without saving, self-insurance acts as

[11] We also assume the discount factor to be equal to 1.



(a) Optimal saving as function of risk aversion (without endogenous self-insurance)

(b) Optimal self-insurance as function of risk aversion (without endogenous saving)

(c) Optimal saving as function of risk aversion (with endogenous self-insurance)

(d) Optimal self-insurance as function of risk aversion (with endogenous saving)

FIGURE 4 Saving and self-insurance decisions with Epstein–Zin preferences. Optimal saving and self-insurance as functions of risk aversion γ and the desire to smooth consumption ρ are plotted. (a) Agents optimize saving separately, (b) agents optimize self-insurance separately, and (c) and (d) agents optimize saving and self-insurance jointly

instrument of consumption smoothing and risk management. The lower ρ, the lower the desire for consumption smoothing. Self-insurance, is thus, used for reducing risk rather than for smoothing consumption over time. This explains why small values of ρ lead to smaller optimal self-insurance.

Panels (c) and (d) show the situation with joint optimization over saving and self-insurance. Saving is now employed to smooth consumption over time, and self-insurance is used to reduce risk. Optimal saving and optimal self-insurance thus decrease when compared with the situation portrayed in Panels (a) and (b). Further, the differences between optimal self-insurance for different ρ decline compared with Panel (b). We also find that optimal self-insurance lies always below the optimal level of the loss state $y_{0.5}^{\mathrm{up}}$, and optimal self-insurance is increasing in γ. We conclude that the presence of saving replicates the monotonic relationship between risk aversion and self-insurance, which would be obtained in a setting with monotone risk preferences.

Our results are in line with Propositions 7 and 9. Without endogenous saving, greater risk aversion either increases or decreases optimal self-insurance. In addition, our simulation shows that risk aversion always raises optimal self-insurance if ρ is larger than γ. In this situation, the function that reflects consumption smoothing is more concave than is the one that reflects risk aversion, and agents prefer to resolve uncertainty late (in line with Proposition 7). With endogenous saving, higher risk aversion monotonically increases optimal self-insurance (in line with Proposition 9).

4.3.3 | Self-protection, Epstein–Zin preferences

We again consider two instruments. First, agents can engage in saving. Second, investments in self-protection reduce the probability of the loss in the second period:

$$p(x) = \frac{2}{3}\cdot p\cdot \exp(-0.9x) + \frac{1}{3}\cdot p. \tag{29}$$

We distinguish three cases: (1) Agents optimize saving separately (Equation 30), (2) agents optimize self-protection separately (Equation 31), and (3) agents optimize self-protection and saving jointly (Equation 32).

$$\max_{s} U(s) = \frac{(w-s)^{1-\rho}}{1-\rho} + \frac{1}{1-\rho}[(1-p(x))(w+Rs)^{1-\gamma} + p(x)(w-L+Rs)^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}}, \tag{30}$$

$$\max_{x} U(x) = \frac{(w-x)^{1-\rho}}{1-\rho} + \frac{1}{1-\rho}[(1-p(x))w^{1-\gamma} + p(x)(w-L)^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}}, \tag{31}$$

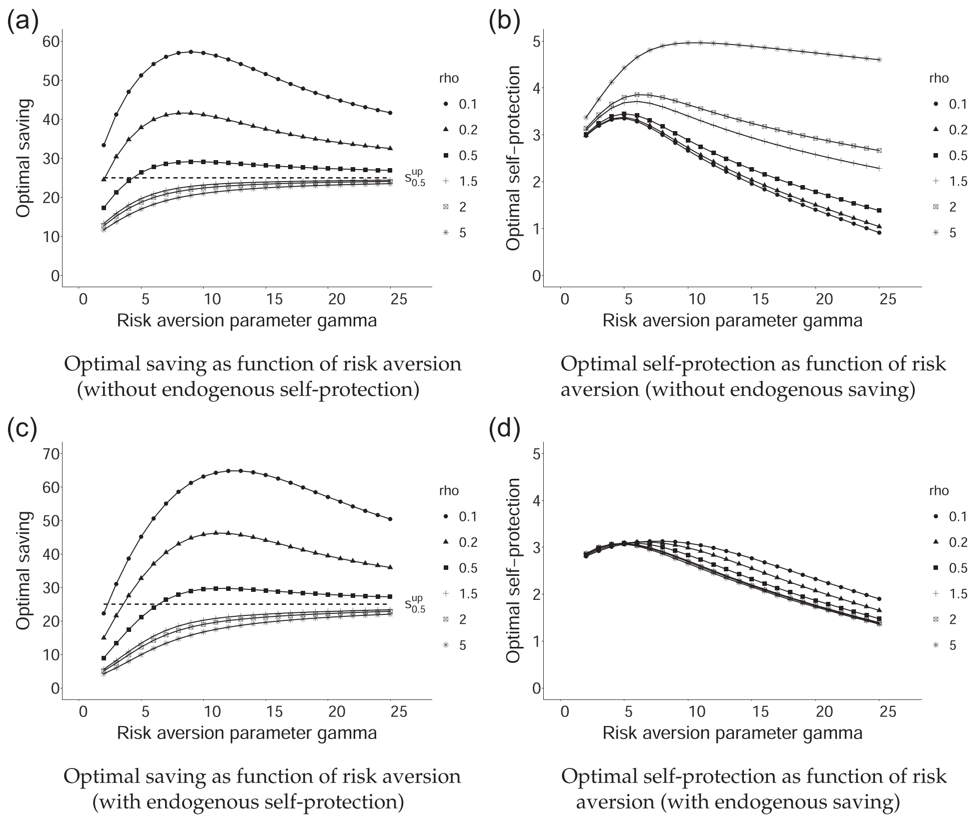
$$\max_{x,s} U(x,s) = \frac{(w-x-s)^{1-\rho}}{1-\rho} + \frac{1}{1-\rho}[(1-p(x))(w+Rs)^{1-\gamma} + p(x)(w-L+Rs)^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}}. \tag{32}$$

Figure 5 displays optimal saving and self-protection as functions of γ and ρ. Panel (a) is identical to Panel (a) of Figure 4. Again, we denote the optimal level of saving in the bad state of the world for $\rho = 0.5$ by $s_{0.5}^{\text{up}}$. While $s_{0.5}^{\text{up}}$ is greater than zero, optimal self-protection ($x_{0.5}^{\text{up}}$, not depicted here), in this situation, is zero. Panel (b) shows that the lower ρ, the lower the optimal self-protection. In addition, even for small values of risk aversion, we observe marked differences between optimal self-protection levels. Self-protection, thus, not only reduces second-period risk but also smooths consumption over time. In line with Proposition 8, greater risk aversion either increases or decreases optimal self-protection. Panel (b) further shows that, if ρ is larger than γ and the probability of loss is sufficiently small, an increase in risk aversion raises optimal self-protection.

Panels (c) and (d) depict the situation with joint optimization of saving and self-protection. Saving is now employed to smooth consumption over time, and self-protection is used to reduce risk. Optimal self-protection, thus, decreases when compared with the situation in Panel (b). Further, the differences between optimal self-protection for different ρ decline compared with Panel (b). While Proposition 10 identifies a monotonically increasing relation between risk aversion and optimal self-protection if the loss probability is small enough, Panel (d) shows that greater risk aversion either increases or decreases optimal self-protection. The proposition and Panel (d) do not contradict each other because the endogenous threshold of the loss probability is dependent on the utility function. Increasing γ thus changes the endogenous threshold of the loss probability. Therefore, we cannot observe the finding of Proposition 10 for all depicted values of γ in this illustration.

5 | DISCUSSION

We now return to the question of whether we can use observed risk management behavior to draw inferences about the underlying risk preferences of decision makers. First, we find that increased risk aversion unambiguously raises optimal self-insurance without using a particular representation of risk preferences. To study the interaction between saving and self-insurance, we further employ Kihlstrom and Mirman's (1974) preferences. In this setting, we demonstrate



Optimal saving as function of risk aversion (without endogenous self-protection)

Optimal self-protection as function of risk aversion (without endogenous saving)

Optimal saving as function of risk aversion (with endogenous self-protection)

Optimal self-protection as function of risk aversion (with endogenous saving)

FIGURE 5 Saving and self-protection decisions with Epstein–Zin preferences. Optimal saving and self-protection as functions of risk aversion γ and the desire to smooth consumption ρ are plotted. (a) Agents optimize saving separately, (b) agents optimize self-protection separately, and (c) and (d) agents optimize saving and self-protection jointly

that a more risk-averse agent invests more in self-insurance with and without endogenous saving. This result is in line with the findings for a one-period expected utility setting (Briys & Schlesinger, 1990; Dionne & Eeckhoudt, 1985).

Second, we study self-protection decisions. Because there is no hope of finding general results with Bommier et al.'s (2012) framework, we employ Kihlstrom and Mirman's (1974) preferences and demonstrate that a more risk-averse agent invests more in self-protection with and without endogenous saving if the loss probability is sufficiently small. This ambiguity is also shown in a one-period setting (Briys & Schlesinger, 1990; Dionne & Eeckhoudt, 1985; Jullien et al., 1999). The intuition for this result is that investing in self-protection does reduce risk in the sense of Rothschild and Stiglitz (1970). The investment reduces welfare for the agent in the loss state and, therefore, deteriorates her situation compared with not investing in self-protection.

In contrast to our results, Hofmann and Peter (2016) show that two-period results for risk management hinge on the consideration of endogenous saving. In the absence of endogenous saving, the authors find that a more concave utility function raises optimal self-insurance and optimal self-protection if first-period consumption is sufficiently high. In the presence of endogenous saving, they show that an increase in concavity unambiguously increases optimal self-insurance and optimal self-protection if the loss probability is sufficiently low. Increasing the concavity of the utility function, however, changes ordinal preferences in a standard additive expected utility setting, which makes it impossible to study the effect of risk aversion on optimal risk management in isolation. This explains why we obtain a different result.

Selden (1978) and Epstein and Zin (1989) propose preferences that disentangle risk from time preferences and, thus, leave ordinal preferences unchanged. Without endogenous saving, we find that greater risk aversion either increases or decreases optimal self-insurance and optimal self-protection. With endogenous saving, we demonstrate that increased risk aversion unambiguously increases optimal self-insurance and optimal self-protection if the loss probability is sufficiently small. While Kihlstrom and Mirman's (1974) preferences are monotone, Selden's and Epstein and Zin's preferences lack this property. Thus, the results obtained with Kihlstrom and Mirman's preferences differ from those obtained with Selden's or Epstein and Zin's preferences.

Our results contribute to both theoretical and empirical literature. Bommier et al. (2012) show that standard additive expected utility settings as well as Selden's (1978) and Epstein and Zin's (1989) preferences are not well ordered in terms of risk aversion. We explore how the lack thereof affects optimal risk management under an increase in risk aversion. Without consistency of ordinal preferences (standard additive expected utility setting) or the lack of monotonicity (Selden, Epstein and Zin), we find that two-period results for risk management hinge on the consideration of endogenous saving. Using preferences that are well ordered in terms of risk aversion, however, we reproduce the one-period results in our two-period models, which is good news for empiricists.

6 | CONCLUSION

This paper analyzes the effect of increased risk aversion on self-insurance and self-protection in situations in which an agent invests in risk management today and benefits from this investment tomorrow. We demonstrate that increased risk aversion is monotonically linked to optimal self-insurance and show that an increase in risk aversion raises optimal self-protection if and only if the loss probability is below an endogenous threshold. Interestingly, our findings are independent of whether the agent considers endogenous saving in her decision making. This is surprising, as previous literature shows that two-period results on risk management hinge on the consideration of endogenous saving (Hofmann & Peter, 2016; Peter, 2017). Mental accounting (Thaler, 1999), thus, plays no role in the effect of greater risk aversion on optimal risk management.

This is good news for the empirical literature. Agents who optimize risk management and saving separately (employing two mental accounts) behave in the same way as agents who optimize risk management and saving jointly (employing only one mental account). Because we reproduce the one-period results in our two-period models, our findings support the focus of the extant empirical literature on the structure of the risk rather than on the timing of investments and benefits.

Our work provides some direction for further research. First, theoretical research should develop a framework to analyze self-protection without employing a particular representation of risk preferences. There is no hope of finding such a result using Bommier et al.'s (2012) framework. Second, future research could establish an exogenous rather than endogenous loss probability threshold in the self-protection result with a particular representation of risk preferences. Peter (2021) uses risk-neutral probabilities to remove the endogeneity of the probability threshold and thus provides a promising starting point for this analysis. Third, it would be interesting to see whether Bommier et al.'s (2012) framework can be extended to two-dimensional actions. Finally, replicating our results with risk-sensitive preferences proposed by Bommier et al. (2017) seems to be a worthwhile next step.

ACKNOWLEDGMENTS

I have received helpful suggestions from the Editor and two anonymous referees. In addition, I am thankful for thoughtful comments from participants at the 2017 Frankfurt Insurance Research Workshop, the 2018 annual meeting of the American Risk and Insurance Association, the 2018 annual meeting of the European Group of Risk and Insurance Economists, the 2018 CEAR/MRIC Behavioral Insurance Workshop, and seminar participants at the University of Georgia and St. John's University. I am indebted to Frederike Bernhöft, Claudio Bonilla, Johannes Jaspersen, Tara Kuruvila, Joëlle Näger, Richard Peter, and Andreas Richter for valuable comments. Open access funding enabled and organized by Projekt DEAL.

ORCID

Tobias Huber http://orcid.org/0000-0002-9894-9710

REFERENCES

Arrow, K. J. (1963). *Liquidity preference*. In *Lecture VI in Lecture Notes for Economics 285: The Economics of Uncertainty* (pp. 33–53). Stanford University.

Berger, L. (2010). *Two-period self-insurance and self-protection models* (working paper). https://sites.google.com/site/loicbberger/research

Bommier, A., Chassagnon, A., & LeGrand, F. (2012). Comparative risk aversion: A formal approach with applications to saving behavior. *Journal of Economic Theory*, *147*(4), 1614–1641.

Bommier, A., Kochov, A., & LeGrand, F. (2017). On monotone recursive preferences. *Econometrica*, *85*(5), 1433–1466.

Bommier, A., & LeGrand, F. (2014). Too risk averse to purchase insurance? *Journal of Risk and Uncertainty*, *48*(2), 135–166.

Bommier, A., & LeGrand, F. (2018). Risk aversion and precautionary savings in dynamic settings. *Management Science*, *65*(3), 1386–1397.

Briys, E., & Schlesinger, H. (1990). Risk aversion and the propensities for self-insurance and self-protection. *Southern Economic Journal*, *57*(2), 458–467.

Courbage, C., Rey, B., & Treich, N. (2013). Prevention and precaution. In G. Dionne (Ed.), *Handbook of insurance* (pp. 185–204). Springer.

Debreu, G. (1954). Representation of a preference ordering by a numerical function. In C. Thrall, R. Coombs, & R. Davies (Eds.), *Decision processes* (pp. 159–165). Wiley.

Dionne, G., & Eeckhoudt, L. (1985). Self-insurance, self-protection and increased risk aversion. *Economics Letters*, *17*(1-2), 39–42.

Ehrlich, I., & Becker, G. S. (1972). Market insurance, self-insurance, and self-protection. *Journal of Political Economy*, *80*(4), 623–648.

Epstein, L. G., & Zin, S. E. (1989). Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, *57*(4), 937–969.

Gollier, C. (2001). *The economics of risk and time*. The MIT Press.

Hofmann, A., & Peter, R. (2016). Self-insurance, self-protection, and saving: On consumption smoothing and risk management. *Journal of Risk and Insurance*, *83*(3), 719–734.

Jullien, B., Salanié, B., & Salanié, F. (1999). Should more risk-averse agents exert more effort? *The Geneva Papers on Risk and Insurance Theory*, *24*(1), 19–28.

Kihlstrom, R. E., & Mirman, L. J. (1974). Risk aversion with many commodities. *Journal of Economic Theory*, *8*(1), 361–388.

Li, L., & Peter, R. (2021). Should we do more when we know less? The effect of technology risk on optimal effort. *Journal of Risk and Insurance*, (forthcoming).

Menegatti, M. (2009). Optimal prevention and prudence in a two-period model. *Mathematical Social Sciences*, *58*(3), 393–397.

Menegatti, M., & Rebessi, F. (2011). On the substitution between saving and prevention. *Mathematical Social Sciences*, *62*(3), 176–182.

Peter, R. (2017). Optimal self-protection in two periods: On the role of endogenous saving. *Journal of Economic Behavior & Organization*, *137*, 19–36.
Peter, R. (2021). Who should exert more effort? Risk aversion, downside risk aversion and optimal prevention. *Economic Theory* (forthcoming). *71* 1259–1281.
Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica*, *32*(1/2), 122–136.
Rothschild, M., & Stiglitz, J. E. (1970). Increasing risk: I. A definition. *Journal of Economic Theory*, *2*(3), 225–243.
Selden, L. (1978). A new representation of preferences over "certain x uncertain" consumption pairs: The "ordinal certainty equivalent" hypothesis. *Econometrica*, *46*(5), 1045–1060.
Thaler, R. H. (1999). Mental accounting matters. *Journal of Behavioral Decision Making*, *12*(3), 183–206.
Yaari, M. E. (1969). Some remarks on measures of risk aversion and on their uses. *Journal of Economic Theory*, *1*(3), 315–329.

SUPPORTING INFORMATION

Additional Supporting Information may be found online in the supporting information tab for this article.

How to cite this article: Huber, T. (2022). Comparative risk aversion in two periods: An application to self-insurance and self-protection. *Journal of Risk and Insurance*, 89, 97–130. https://doi.org/10.1111/jori.12353

APPENDIX A

Proof of Proposition 2. Consider two agents, A and B, with risk preferences $\succcurlyeq^i$, $i = A, B$. Let agent A be more risk-averse than B with respect to the p-spread relation for every $p \in (0, 1)$. We apply Proposition 1 to show that agent A invests more in self-insurance than agent B. Thus, we need to show the assumptions given in this proposition:

- *Nonconstant*: We need to show that, if for all states of the world an agent is indifferent between actions y_1 and y_2, y_1 must be equal to y_2. Choosing a self-insurance level of y has a real effect on the lottery outcome in the loss state and in the no-loss state. If we have for all $\theta \in \Omega$ and investments y_1 and y_2 that the lottery outcome $l_{y_1}(\theta)$ equals $l_{y_2}(\theta)$, then the investment y_1 equals y_2.
- *Comonotonicity*: We need to show that the agent's action does not change the initial ordering of lottery outcomes that is implied by the states of the world. Independent of the self-insurance level y, the agent always prefers the no-loss state to the loss-state. This allows us to write $\theta_1 \succcurlyeq \theta_2$ in short for $l_y(\theta_1) \succcurlyeq l_y(\theta_2)$. Self-insurance, thus, preserves the initial ordering of lottery outcomes.
- *Action order*: We need to show that optimal actions can be ordered according to states of the world. Assuming normality of first-period consumption leads to action order. The better the state of the world (i.e. the smaller the loss), the smaller the optimal investment in self-insurance (i.e. the greater optimal first-period consumption). Therefore, we can order the optimal actions according to states of the world.
- *Single-peakedness*: We need to show that for all $\theta \in \Omega$:

1. There exists an optimal investment y_θ such that $\forall\, y$: $l_{y_\theta}(\theta) \succcurlyeq l_y(\theta)$.

2. For $y_1 \leq y_2 \leq y_\theta \leq y_3 \leq y_4$ we have $l_{y_\theta}(\theta) \succcurlyeq l_{y_2}(\theta) \succcurlyeq l_{y_1}(\theta)$ and $l_{y_\theta}(\theta) \succcurlyeq l_{y_3}(\theta) \succcurlyeq l_{y_4}(\theta)$.
First, the Bolzano-Weierstrass theorem guarantees the existence of a solution y^* *for the optimization problem* $\max_y u(w_1 - y, w_2 - L(y))$ *because the subset* $O = [0, w_1] \times [0, w_2]$ *is bounded and closed in* $\mathbb{R}^2$ *and is therefore compact due to the Heine–Borel theorem. Second, we assume convexity of the ordinal preference relation. Ordinal preferences are convex if and only if for all pairs* $(c_1, c_2), (c_1', c_2')$ *and* (c_1'', c_2'') *in the state space* C *and for all* $\lambda \in [0, 1]$ *we can conclude from* $(c_1', c_2') \succcurlyeq (c_1, c_2)$ *and* $(c_1'', c_2'') \succcurlyeq (c_1, c_2)$ *that* $(\lambda c_1' + (1-\lambda)c_1'', \lambda c_2' + (1-\lambda)c_2'') \succcurlyeq (c_1, c_2)$. *This can be interpreted by a diminishing marginal rate of substitution of consumption in period 1 and consumption in period 2. We consider* y', y'' *with* $y' < y'' < y^*$. *There exists* $\lambda \in [0, 1]$ *so that* $y'' = \lambda y^* + (1-\lambda)y'$. *Using the definition of* y'' *(first step), the convexity of* $L(y)$ *(second step), and the convexity of the ordinal preference relation (third step), we can then conclude from* $(w_1 - y^*, w_2 - L(y^*)) \succcurlyeq (w_1 - y', w_2 - L(y'))$ *and* $(w_1 - y', w_2 - L(y')) \succcurlyeq (w_1 - y', w_2 - L(y'))$ *that*

$$(w_1 - y'', w_2 - L(y'')) \succcurlyeq (w_1 - \lambda y^* - (1-\lambda)y', w_2 - L(\lambda y^* + (1-\lambda)y')) \\ \succcurlyeq (w_1 - \lambda y^* - (1-\lambda)y', w_2 - \lambda L(y^*) - (1-\lambda)L(y')) \succcurlyeq (w_1 \\ - y', w_2 - L(y')). \tag{A1}$$

If we assume y', y'' with $y^* < y'' < y'$, this completes the proof for single-peakedness in the loss state. Using the same rationale, we can also prove single-peakedness for the no-loss state.

Therefore, according to Proposition 2 in Bommier et al. (2012) we have $y_A^* \geq y_B^*$ for the optimal self-insurance levels of agent A and B. □

Proof of Proposition 3. Consider two agents, A and B, where $\varphi_A = k(\varphi_B)$ with a twice differentiable real-valued function k which is increasing and concave. Then, agent A is more risk-averse than B (Kihlstrom & Mirman, 1974). Further, let y_A^* and y_B^* denote optimal self-insurance for agents A and B. Evaluating A's first-order condition at y_B^* yields:

$$\frac{\partial}{\partial y} U_A(y_B^*) = -(1-p) \cdot k'(\varphi_B(u_F + v_N)) \cdot \varphi_B'(u_F + v_N) u_F' \\ - p \cdot k'(\varphi_B(u_F + v_L)) \cdot \varphi_B'(u_F + v_L) \cdot \left(u_F' + L'\left(y_B^*\right) \cdot v_L'\right). \tag{A2}$$

Because the first term in Equation (A2) is negative and $k'(\varphi_B(u_F + v_N)) \leq k'(\varphi_B(u_F + v_L))$, we decrease the expression in (A2) by substituting $k'(\varphi_B(u_F + v_N))$ by $k'(\varphi_B(u_F + v_L))$. Moreover, we use agent B's first-order condition and the optimality of y_B^* to get:

$$\frac{\partial}{\partial y} U_A(y_B^*) \geq k'(\varphi_B(u_F + v_L))\Big[-(1-p) \cdot \varphi_B'(u_F + v_N) u_F' \\ -p \cdot \varphi_B'(u_F + v_L) \cdot \left(u_F' + L'\left(y_B^*\right) \cdot v_L'\right)\Big] = 0. \tag{A3}$$

Because $U_A(y)$ is globally concave and $\frac{\partial}{\partial y}U_A(y_B^*) \geq 0$ we get $y_A^* \geq y_B^*$. Therefore, a more risk-averse agent invests more in self-insurance. □

Proof of Proposition 4. We make agent A more risk-averse than agent B. Thus, we have $\varphi_A = k(\varphi_B)$ with a twice differentiable real-valued function k which is increasing and concave. Further, let x_A^* and x_B^* denote optimal self-protection for agent A and B. Using agent A's and agent B's first-order conditions, we find the condition that the more risk-averse agent A invests more in self-protection than the less risk-averse agent B if and only if the loss probability is small enough.

Without loss of generality, we can set:[12]

$$\begin{aligned} \varphi_A\left(u\left(w_1 - x_B^*\right) + v(w_2)\right) &= \varphi_B\left(u\left(w_1 - x_B^*\right) + v(w_2)\right), \\ \varphi_A\left(u\left(w_1 - x_B^*\right) + v\left(w_2 - L\left(x_B^*\right)\right)\right) &= \varphi_B\left(u\left(w_1 - x_B^*\right) + v\left(w_2 - L\left(x_B^*\right)\right)\right). \end{aligned} \tag{A4}$$

Evaluating A's first-order condition at x_B^* yields:

$$\begin{aligned} \frac{\partial}{\partial x}U_A\left(x_B^*\right) = &-p'\left(x_B^*\right)\cdot[\varphi_B(u_F + v_N) - \varphi_B(u_F + v_L)] \\ &-\left(1 - p\left(x_B^*\right)\right)\cdot k'(\varphi_B(u_F + v_N))\varphi_B'(u_F + v_N)u_F' \\ &-p\left(x_B^*\right)\cdot k'(\varphi_B(u_F + v_L))\varphi_B'(u_F + v_L)u_F'. \end{aligned} \tag{A5}$$

Due to $k'' < 0$ we have $k'(\varphi_B(u_F + v_N)) < k'(\varphi_B(u_F + v_L))$. Comparing Equation (7) with Equation (A5) shows that increased risk aversion can increase or decrease optimal self-protection. We can derive a necessary and sufficient condition for an increase in optimal self-protection by substituting the first term in Equation (A5) by agent B's first-order condition:

$$\begin{aligned} \frac{\partial}{\partial x}U_A\left(x_B^*\right) = &\, p\left(x_B^*\right)\cdot\left[\varphi_B'(u_F + v_L) - \varphi_A'(u_F + v_L)\right]u_F' \\ &+\left(1 - p\left(x_B^*\right)\right)\cdot\left[\varphi_B'(u_F + v_N) - \varphi_A'(u_F + v_N)\right]u_F'. \end{aligned} \tag{A6}$$

Due to increased risk aversion we have that in (A6) the first term is negative and the second term is positive. A loss probability of one renders a negative expression in (A6) and a loss probability of zero a positive expression. Additionally, $\frac{\partial}{\partial x}U_A(x_B^*)$ is linear in $p(x_B^*)$. Therefore, there exists an endogenously defined threshold p^* so that increased risk aversion leads to higher optimal self-protection if and only if the loss probability is below this threshold. □

[12]There always exists a suitable affine transformation such that this can be achieved and this transformation does not change preferences.

Proof of Proposition 5. We make agent A more risk-averse than agent B. We evaluate A's first-order condition at $(y_B^*, \hat{s})$, where y_B^* denotes optimal self-insurance for agent B and $\hat{s}$ maximizes A's expected utility given the investment in self-insurance y_B^*. Without loss of generality we can set:[13]

$$\begin{aligned}\varphi_A\Big(u\Big(w_1 - y_B^* - \hat{s}\Big) + v\Big(w_2 - L\Big(y_B^*\Big) + R\hat{s}\Big)\Big) &= \varphi_B\Big(u\Big(w_1 - y_B^* - s_B^*\Big) \\ &\quad + v\Big(w_2 - L\Big(y_B^*\Big) + Rs_B^*\Big)\Big), \\ \varphi_A\Big(u\Big(w_1 - y_B^* - \hat{s}\Big) + v\Big(w_2 + R\hat{s}\Big)\Big) &= \varphi_B\Big(u\Big(w_1 - y_B^* - s_B^*\Big) + v\Big(w_2 + Rs_B^*\Big)\Big).\end{aligned} \tag{A7}$$

Using the shorthand notation $\hat{u}_F := u(w_1 - y_B^* - \hat{s})$, $\hat{v}_N := v(w_2 + R\hat{s})$, $\hat{v}_L := v(w_2 - L(y_B^*) + R\hat{s})$, $u_F := u(w_1 - y_B^* - s_B^*)$, $v_N := v(w_2 + Rs_B^*)$ and $v_L := v(w_2 - L(y_B^*) + Rs_B^*)$ we obtain:

$$\begin{aligned}\frac{\partial}{\partial y}U_A\Big(y_B^*, \hat{s}\Big) &= -(1-p)\cdot\varphi_A'(\hat{u}_F + \hat{v}_N)\hat{u}' \\ &\quad - p\cdot\varphi_A'(\hat{u}_F + \hat{v}_L)\hat{u}_F' - pL'(y_B^*)\cdot\varphi_A'(\hat{u}_F + \hat{v}_L)\hat{v}_L'.\end{aligned} \tag{A8}$$

Additionally, we use the optimality of (y_B^*, s_B^*) with respect to B's first-order conditions to substitute the expression $pL'(y_B^*)$ and get:

$$\begin{aligned}\frac{\partial}{\partial y}U_A\Big(y_B^*, \hat{s}\Big) &= \Big((1-p)\cdot\varphi_B'(u_F + v_N)u_F' + p\cdot\varphi_B'(u_F + v_L)u_F'\Big) \\ &\quad \cdot\left[\frac{\varphi_A'(\hat{u}_F + \hat{v}_L)\hat{v}_L'}{\varphi_B'(\hat{u}_F + v_L)v_L'} - \frac{(1-p)\cdot\varphi_A'(\hat{u}_F + \hat{v}_N)\hat{u}_F' + p\cdot\varphi_A'(\hat{u}_F + \hat{v}_L)\hat{u}_F'}{(1-p)\cdot\varphi_B'(u_F + v_N)u_F' + p\cdot\varphi_B'(u_F + v_L)u_F'}\right].\end{aligned} \tag{A9}$$

The overall expression is positive if the term in square brackets is positive. Using agent A's and B's first-order condition for saving yields:

$$\begin{aligned}&\frac{\partial}{\partial y}U_A\Big(y_B^*, \hat{s}\Big) > 0 \\ &\Leftrightarrow R\Big[\Big[(1-p)\cdot\varphi_B'(u_F + v_N)v_N' + p\cdot\varphi_B'(u_F + v_L)v_L'\Big]\varphi_A'(\hat{u}_F + \hat{v}_L)\hat{v}_L' \\ &-\Big[(1-p)\cdot\varphi_A'(\hat{u}_F + \hat{v}_N)\hat{v}_N' + p\cdot\varphi_A'(\hat{u}_F + \hat{v}_L)\hat{v}_L'\Big]\varphi_B'(u_F + v_L)v_L'\Big] > 0 \\ &\Leftrightarrow \frac{\varphi_A'(\hat{u}_F + \hat{v}_L)\hat{v}_L'}{\varphi_B'(u_F + v_L)v_L'} > \frac{\varphi_A'(\hat{u}_F + \hat{v}_N)\hat{v}_N'}{\varphi_B'(u_F + v_N)v_N'}.\end{aligned} \tag{A10}$$

According to the assumptions in (A7) and because agent A's φ-function is a concave transformation of agent B's φ-function, the first expression is always greater than one and the second expression is always smaller than one. Therefore, the inequality is satisfied and agent A invests more in self-insurance than agent B. □

[13]There always exists a suitable affine transformation such that this can be achieved and this transformation does not change preferences.

Proof of Proposition 6. We make agent A more risk-averse than agent B. We evaluate A's first-order condition at $(x_B^*, \hat{s})$, where x_B^* denotes optimal self-protection for agent B and $\hat{s}$ maximizes A's expected utility given the investment in self-protection x_B^*. Without loss of generality, we can set:[14]

$$\begin{aligned}\varphi_A\Big(u\big(w_1 - x_B^* - \hat{s}\big) + v(w_2 - L + R\hat{s})\Big) &= \varphi_B\Big(u\big(w_1 - x_B^* - s_B^*\big) + v\big(w_2 - L + Rs_B^*\big)\Big),\\ \varphi_A\Big(u\big(w_1 - x_B^* - \hat{s}\big) + v(w_2 + R\hat{s})\Big) &= \varphi_B\Big(u\big(w_1 - x_B^* - s_B^*\big) + v\big(w_2 + Rs_B^*\big)\Big).\end{aligned} \tag{A11}$$

Using the shorthand notation $\hat{u}_F := u(w_1 - x_B^* - \hat{s}), \hat{v}_N := v(w_2 + R\hat{s}), \hat{v}_L := v(w_2 - L + R\hat{s}), u_F := u(w_1 - x_B^* - s_B^*), v_N := v(w_2 + Rs_B^*)$ and $v_L := v(w_2 - L + Rs_B^*)$ we obtain:

$$\begin{aligned}\frac{\partial}{\partial x}U_A\big(x_B^*, \hat{s}\big) = &-p'\big(x_B^*\big)[\varphi_A(\hat{u}_F + \hat{v}_N) - \varphi_A(\hat{u}_F + \hat{v}_L)]\\ &-\Big(1 - p\big(x_B^*\big)\Big)\cdot\varphi_A'(\hat{u}_F + \hat{v}_N)\hat{u}_F' - p(y_B^*)\cdot\varphi_A'(\hat{u}_F + \hat{v}_L)\hat{u}_F'.\end{aligned} \tag{A12}$$

Additionally, we use the optimality of (x_B^*, s_B^*) with respect to B's first-order conditions to substitute the expression $-p'(x_B^*)$ and get:

$$\begin{aligned}\frac{\partial}{\partial x}U_A\big(x_B^*, \hat{s}\big) = &\frac{\varphi_A(\hat{u}_F + \hat{v}_N) - \varphi_A(\hat{u}_F + \hat{v}_L)}{\varphi_B(u_F + v_N) - \varphi_B(u_F + v_L)}\\ &\cdot\Big(p\big(x_B^*\big)\cdot\varphi_B'(u_F + v_L)u_F' + \Big(1 - p\big(x_B^*\big)\Big)\cdot\varphi_B'(u_F + v_N)u_F'\Big)\\ &-\Big(1 - p\big(x_B^*\big)\Big)\cdot\varphi_A'(\hat{u}_F + \hat{v}_N)\hat{u}_F' - p\big(x_B^*\big)\cdot\varphi_A'(\hat{u}_F + \hat{v}_L)\hat{u}_F'.\end{aligned} \tag{A13}$$

According to the assumptions in (A11), the above fraction is equal to 1. Further, we use agent A's and B's first-order conditions for saving and derive:

$$\begin{aligned}\frac{\partial}{\partial x}U_A\big(x_B^*, \hat{s}\big) = R\Big[&p\big(x_B^*\big)\big[\varphi_B'(u_F + v_L)v_L' - \varphi_A'(\hat{u}_F + \hat{v}_L)\hat{v}_L'\big]\\ &+\Big(1 - p\big(x_B^*\big)\Big)\big[\varphi_B'(u_F + v_N)v_N' - \varphi_A'(\hat{u}_F + \hat{v}_N)\hat{v}_N'\big]\Big].\end{aligned} \tag{A14}$$

Due to increased risk aversion, we have that the first term is negative and the second term is positive. A loss probability of one renders a negative expression and a loss probability of zero a positive expression. Additionally, $\frac{\partial}{\partial x}U_A(x_B^*, \hat{s})$ is linear in $p(x_B^*)$. Therefore, there exists an endogenously defined threshold p^* so that increased risk aversion leads to higher optimal self-protection if and only if the loss probability is below this threshold. □

Proof of Proposition 7. Consider two agents, A and B, where $\psi_A = k(\psi_B)$ with a twice differentiable real-valued function k which is increasing and concave. Then, agent A is more

[14] There always exists a suitable affine transformation such that this can be achieved and this transformation does not change preferences.

risk-averse than agent B (Selden, 1978). Further, let y_A^* and y_B^* denote optimal self-insurance for agents A and B. Evaluating A's first-order condition at y_B^* and using the shorthand notation $N(y) := \psi_B^{-1}(k^{-1}(p \cdot k(\psi_B(w_2 - L(y))) + (1-p) \cdot k(\psi_B(w_2))))$ yields:

$$\frac{\partial}{\partial y} U_A\left(y_B^*\right) = -u'\left(w_1 - y_B^*\right) - pL'\left(y_B^*\right) \cdot \psi_B'\left(w_2 - L\left(y_B^*\right)\right) \cdot \frac{v'\left(N\left(y_B^*\right)\right)}{\psi_B'\left(N\left(y_B^*\right)\right)} \cdot \frac{k'\left(\psi_B\left(w_2 - L\left(y_B^*\right)\right)\right)}{k'\left(\psi_B\left(N\left(y_B^*\right)\right)\right)}. \tag{A15}$$

Moreover, we use agent B's first-order condition to get:

$$\frac{\partial}{\partial y} U_A\left(y_B^*\right) = -pL'\left(y_B^*\right) \cdot \psi_B'\left(w_2 - L\left(y_B^*\right)\right) \cdot \left(\frac{v'\left(N\left(y_B^*\right)\right)}{\psi_B'\left(N\left(y_B^*\right)\right)} \cdot \frac{k'\left(\psi_B\left(w_2 - L\left(y_B^*\right)\right)\right)}{k'\left(\psi_B\left(N\left(y_B^*\right)\right)\right)} - \frac{v'\left(M\left(y_B^*\right)\right)}{\psi_B'\left(M\left(y_B^*\right)\right)}\right). \tag{A16}$$

This expression is non-negative if and only if:

$$\frac{\partial}{\partial y} U_A\left(y_B^*\right) \geq 0 \Leftrightarrow \frac{v'\left(N\left(y_B^*\right)\right)}{\psi_B'\left(N\left(y_B^*\right)\right)} \cdot \frac{k'\left(\psi_B\left(w_2 - L\left(y_B^*\right)\right)\right)}{k'\left(\psi_B\left(N\left(y_B^*\right)\right)\right)} \geq \frac{v'\left(M\left(y_B^*\right)\right)}{\psi_B'\left(M\left(y_B^*\right)\right)}. \tag{A17}$$

Using $k'' < 0$ we can show that $k'(\psi_B(N(y_B^*))) \leq k'(\psi_B(w_2 - L(y_B^*)))$ and employing Jensen's inequality yields $N(y) \leq M(y)$. This enables us to derive a sufficient condition for risk aversion to raise optimal self-insurance: if the second-period utility function that reflects consumption smoothing (v) is more concave than the one that reflects risk aversion (ψ), then increased risk aversion raises optimal self-insurance. □

Proof of Proposition 8. We make agent A more risk-averse than agent B. Thus, we have $\psi_A = k(\psi_B)$ with a twice differentiable real-valued function k which is increasing and concave. Further, let x_A^* and x_B^* denote optimal self-protection for agent A and B. Without loss of generality, we can set:[15]

$$\begin{aligned} \psi_A(w_2) &= \psi_B(w_2), \\ \psi_A\left(w_2 - L\left(x_B^*\right)\right) &= \psi_B\left(w_2 - L\left(x_B^*\right)\right). \end{aligned} \tag{A18}$$

Using the shorthand notation $N(x) = \psi_B^{-1}(k^{-1}(p(x) \cdot k(\psi_B(w_2 - L)) + (1 - p(x)) \cdot k(\psi_B(w_2))))$ and evaluating A's first-order condition at x_B^* yields:

[15]There always exists a suitable affine transformation such that this can be achieved and this transformation does not change preferences.

$$\frac{\partial}{\partial x}U_A\left(x_B^*\right) = -u'\left(w_1 - x_B^*\right) - p'\left(x_B^*\right)\cdot\frac{v'\left(N\left(x_B^*\right)\right)}{\psi_B'\left(N\left(x_B^*\right)\right)}\cdot\frac{k(\psi_B(w_2)) - k(\psi_B(w_2 - L))}{k'\left(\psi_B\left(N\left(x_B^*\right)\right)\right)}. \quad \text{(A19)}$$

We now use agent B's first-order condition to get:

$$\begin{aligned}\frac{\partial}{\partial x}U_A\left(x_B^*\right) &= -p'\left(x_B^*\right)(\psi_B(w_2) - \psi_B(w_2 - L))\\ &\quad\cdot\left(\frac{v'\left(N\left(x_B^*\right)\right)}{\psi_B'\left(N\left(x_B^*\right)\right)}\cdot\frac{1}{k'\left(\psi_B\left(N\left(x_B^*\right)\right)\right)} - \frac{v'\left(M\left(x_B^*\right)\right)}{\psi_B'\left(M\left(x_B^*\right)\right)}\right).\end{aligned} \quad \text{(A20)}$$

This expression is nonnegative if and only if:

$$\frac{\partial}{\partial x}U_A\left(x_B^*\right) \geq 0 \Leftrightarrow \frac{v'\left(N\left(x_B^*\right)\right)}{\psi_B'\left(N\left(x_B^*\right)\right)}\cdot\frac{1}{k'\left(\psi_B\left(N\left(x_B^*\right)\right)\right)} \geq \frac{v'\left(M\left(x_B^*\right)\right)}{\psi_B'\left(M\left(x_B^*\right)\right)}. \quad \text{(A21)}$$

Using the convexity of k^{-1} we can show that:

$$\begin{aligned}k'\left(\psi_B\left(N\left(x_B^*\right)\right)\right) &= k'\left(k^{-1}\left(p\left(x_B^*\right)\cdot k(\psi_B(w_2 - L)) + \left(1 - p\left(x_B^*\right)\right)\cdot k(\psi_B(w_2))\right)\right)\\ &\leq k'\left(p\left(x_B^*\right)\cdot\psi_B(w_2 - L) + \left(1 - p\left(x_B^*\right)\right)\cdot\psi_B(w_2)\right).\end{aligned}$$

(A22)

By equation (A18) we know that $k'(\psi_B(w_2 - L)) > 1$ and $k'(\psi_B(w_2)) < 1$. Therefore, there exists an endogenously defined threshold p^* so that $k'(\psi_B(N(x_B^*))) < 1$.

Moreover, employing Jensen's inequality yields $N(x) \leq M(x)$. This enables us to derive a sufficient condition for risk aversion to raise optimal self-protection: if the probability of loss is below an endogenous threshold and the second-period utility function that reflects consumption smoothing (v) is more concave than the one that reflects risk aversion (ψ), then increased risk aversion raises optimal self-protection. □

Proof of Proposition 9. We make agent A more risk-averse than agent B. We evaluate A's first-order condition at $(y_B^*, \hat{s})$, where y_B^* denotes optimal self-insurance for B and $\hat{s}$ maximizes A's expected utility given the investment in self-insurance y_B^*. Without loss of generality, we can set:[16]

$$\begin{aligned}\psi_A\left(w_2 - L\left(y_B^*\right) + R\hat{s}\right) &= \psi_B\left(w_2 - L\left(y_B^*\right) + Rs_B^*\right),\\ \psi_A(w_2 + R\hat{s}) &= \psi_B\left(w_2 + Rs_B^*\right).\end{aligned} \quad \text{(A23)}$$

Using the shorthand notation $N(y, s) = \psi_B^{-1}(k^{-1}(p\cdot k(\psi_B(w_2 - L(y) + Rs))+(1 - p)\cdot k(\psi_B(w_2 + Rs))))$ we obtain:

[16]There always exists a suitable affine transformation such that this can be achieved and this transformation does not change preferences.

$$\frac{\partial}{\partial y}U_A\left(y_B^*,\hat{s}\right)=-u'\left(w_1-y_B^*-\hat{s}\right)$$
$$-pL'\left(y_B^*\right)\cdot\psi_B'\left(w_2-L\left(y_B^*\right)+R\hat{s}\right)\cdot\frac{v'\left(N\left(y_B^*,\hat{s}\right)\right)}{\psi_B'\left(N\left(y_B^*,\hat{s}\right)\right)}\cdot\frac{k'\left(\psi_B\left(w_2-L\left(y_B^*\right)+R\hat{s}\right)\right)}{k'\left(\psi_B\left(N\left(y_B^*,\hat{s}\right)\right)\right)}. \tag{A24}$$

Next, we use agent B's first-order condition to substitute the expression $pL'(y_B^*)$ and get:

$$\frac{\partial}{\partial y}U_A\left(y_B^*,\hat{s}\right)=u'\left(w_1-y_B^*-s_B^*\right)\cdot\left(\frac{\frac{v'\left(N\left(y_B^*,\hat{s}\right)\right)}{\psi_B'\left(N\left(y_B^*,\hat{s}\right)\right)}\cdot\frac{k'\left(\psi_B\left(w_2-L\left(y_B^*\right)+R\hat{s}\right)\right)}{k'\left(\psi_B\left(N\left(y_B^*,\hat{s}\right)\right)\right)}}{\frac{v'\left(M\left(y_B^*,s_B^*\right)\right)}{\psi_B'\left(M\left(y_B^*,s_B^*\right)\right)}\cdot\frac{\psi_B'\left(w_2-L\left(y_B^*\right)+Rs_B^*\right)}{\psi_B'\left(w_2-L\left(y_B^*\right)+R\hat{s}\right)}}-\frac{u'\left(w_1-y_B^*-\hat{s}\right)}{u'\left(w_1-y_B^*-s_B^*\right)}\right). \tag{A25}$$

The overall expression is positive if the bracketed term is positive. Using agent A's and B's first-order condition for saving yields:

$$\begin{aligned}
&\frac{\partial}{\partial y}U_A\left(y_B^*,\hat{s}\right)>0\\
&\Leftrightarrow R\cdot\frac{v'\left(N\left(y_B^*,\hat{s}\right)\right)}{\psi_B'\left(N\left(y_B^*,\hat{s}\right)\right)}\cdot\frac{1}{k'\left(\psi_B\left(N\left(y_B^*,\hat{s}\right)\right)\right)}\cdot\frac{v'\left(M\left(y_B^*,s_B^*\right)\right)}{\psi_B'\left(M\left(y_B^*,s_B^*\right)\right)}\\
&\cdot\left(\left(p\cdot\psi_B'\left(w_2-L\left(y_B^*\right)+R\hat{s}\right)+(1-p)\cdot\psi_B'(w_2+R\hat{s})\right)\right.\\
&\quad\cdot k'\left(\psi_B\left(w_2-L\left(y_B^*\right)+R\hat{s}\right)\right)\\
&-\left(p\cdot k'\left(\psi_B\left(w_2-L\left(y_B^*\right)+Rs_B^*\right)\right)\cdot\psi_B'\left(w_2-L\left(y_B^*\right)+Rs_B^*\right)\right.\\
&\left.\left.+(1-p)\cdot k'\left(\psi_B\left(w_2+Rs_B^*\right)\right)\cdot\psi_B'\left(w_2+Rs_B^*\right)\right)\cdot\frac{\psi_B'\left(w_2-L\left(y_B^*\right)+Rs_B^*\right)}{\psi_B'\left(w_2-L\left(y_B^*\right)+R\hat{s}\right)}\right)>0.
\end{aligned} \tag{A26}$$

Rearranging the bracketed expression leads to the following inequality:

$$\begin{aligned}
&\frac{\partial}{\partial y}U_A\left(y_B^*,\hat{s}\right)>0\\
&\Leftrightarrow\frac{k'\left(\psi_B\left(w_2-L\left(y_B^*\right)+R\hat{s}\right)\right)\cdot\psi_B'\left(w_2-L\left(y_B^*\right)+R\hat{s}\right)}{\psi_B'\left(w_2-L\left(y_B^*\right)+Rs_B^*\right)}>\frac{k'\left(\psi_B\left(w_2-L\left(y_B^*\right)+R\hat{s}\right)\cdot\psi_B'(w_2+R\hat{s}\right)}{\psi_B'\left(w_2+Rs_B^*\right)}.
\end{aligned} \tag{A27}$$

According to the assumptions in (A23) and because agent A's ψ-function is a concave transformation of agent B's ψ-function, the first expression is always greater than one and the second expression is always smaller than one. Therefore, the inequality is satisfied and A invests more in self-insurance than B. □

Proof of Proposition 10. We make agent A more risk-averse than agent B. We evaluate A's first-order condition at $(x_B^*, \hat{s})$, where x_B^* denotes optimal self-protection for B and $\hat{s}$ maximizes A's expected utility given the investment in self-protection x_B^*. Without loss of generality, we can set:[17]

$$\begin{aligned} \psi_A(w_2 - L + R\hat{s}) &= \psi_B\left(w_2 - L + Rs_B^*\right), \\ \psi_A(w_2 + R\hat{s}) &= \psi_B\left(w_2 + Rs_B^*\right). \end{aligned} \tag{A28}$$

Using the shorthand notation $N(x, s) = \psi_B^{-1}(k^{-1}(p(x)\cdot k(\psi_B(w_2 - L + Rs)) + (1 - p(x))\cdot k(\psi_B(w_2 + Rs))))$ we obtain:

$$\begin{aligned} \frac{\partial}{\partial x} U_A\left(x_B^*, \hat{s}\right) = &-u'\left(w_1 - x_B^* - \hat{s}\right) \\ &-p'\left(x_B^*\right)\cdot \frac{v'\left(N\left(x_B^*, \hat{s}\right)\right)}{\psi_B'\left(N\left(x_B^*, \hat{s}\right)\right)} \cdot \frac{k(\psi_B(w_2 + R\hat{s})) - k(\psi_B(w_2 - L + R\hat{s}))}{k'\left(\psi_B\left(N\left(x_B^*, \hat{s}\right)\right)\right)}. \end{aligned} \tag{A29}$$

Additionally, we use the optimality of (x_B^*, s_B^*) with respect to B's first-order conditions to substitute the expression $-p'(x_B^*)$ and get:

$$\begin{aligned} \frac{\partial}{\partial x} U_A\left(x_B^*, \hat{s}\right) = &-u'\left(w_1 - x_B^* - \hat{s}\right) \\ &+u'\left(w_1 - x_B^* - s_B^*\right)\cdot \frac{v'\left(M\left(x_B^*, s_B^*\right)\right)}{\psi_B'\left(M\left(x_B^*, s_B^*\right)\right)} \cdot \frac{v'\left(N\left(x_B^*, \hat{s}\right)\right)}{\psi_B'\left(N\left(x_B^*, \hat{s}\right)\right)} \cdot \frac{1}{k'\left(\psi_B\left(N\left(x_B^*, \hat{s}\right)\right)\right)}. \end{aligned} \tag{A30}$$

Further, we use A's and B's first-order conditions for saving and derive:

$$\begin{aligned} \frac{\partial}{\partial x} U_A\left(x_B^*, \hat{s}\right) = R \cdot &\frac{v'\left(N\left(x_B^*, \hat{s}\right)\right)}{\psi_B'\left(N\left(x_B^*, \hat{s}\right)\right)} \cdot \frac{1}{k'\left(\psi_B\left(N\left(x_B^*, \hat{s}\right)\right)\right)} \\ &\Big(p\left(x_B^*\right)\cdot \psi_B'\left(w_2 - L + Rs_B^*\right) + \left(1 - p\left(x_B^*\right)\right)\cdot \psi_B'\left(w_2 + Rs_B^*\right) \\ &-p\left(x_B^*\right)\cdot k'(\psi_B(w_2 - L + R\hat{s}))\cdot \psi_B'(w_2 - L + R\hat{s}) \\ &-\left(1 - p\left(x_B^*\right)\right)\cdot k'(\psi_B(w_2 + R\hat{s}))\cdot \psi_B'(w_2 + R\hat{s})\Big). \end{aligned} \tag{A31}$$

Rearranging the bracketed expression leads to the following inequality:

$$\begin{aligned} &\frac{\partial}{\partial x} U_A\left(x_B^*, \hat{s}\right) > 0 \\ \Leftrightarrow &p\left(x_B^*\right)\cdot\left(\psi_B'\left(w_2 - L + Rs_B^*\right) - k'(\psi_B(w_2 - L + R\hat{s}))\cdot \psi_B'(w_2 - L + R\hat{s})\right) \\ &+\left(1 - p\left(x_B^*\right)\right)\left(\psi_B'\left(w_2 + Rs_B^*\right) - k'(\psi_B(w_2 + R\hat{s}))\cdot \psi_B'(w_2 + R\hat{s})\right) > 0. \end{aligned} \tag{A32}$$

[17]There always exists a suitable affine transformation such that this can be achieved and this transformation does not change preferences.

Due to increased risk aversion, we have that the first term is negative and the second term is positive. A loss probability of one renders a negative expression and a loss probability of zero a positive expression. Additionally, $\frac{\partial}{\partial x} U_A(x_B^*, \hat{s})$ is linear in $p(x_B^*)$. Therefore, there exists an endogenously defined threshold p^* so that increased risk aversion leads to higher optimal self-protection if and only if the loss probability is below this threshold. $\square$