

**PROPOSAL FOR THE
RECALIBRATION OF MORTALITY
AND LONGEVITY SHOCKS UNDER
THE SOLVENCY II FRAMEWORK**

Technical Note

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ABSTRACT

This Technical Note has the proposal to be a critical contrast with the EIOPA's CP of November 6, 2017 for the recalibration of longevity and mortality shock under the Solvency II framework. So, this paper is addressed to EIOPA and the European Commission for taking into consideration for a review of the shocks applied for longevity and mortality in the framework of Solvency II.

These risks of an opposite biometric nature must be considered together in order to evaluate the impact of the aforementioned shocks, although the conclusions of the resulting metrics are different.

The work confirms the actuarial technical justification of the granularity by age in every years of the projection applicable to the longevity risk valued by means of a multimodal projection of the general European population trend. On the other hand, and under the principle of proportionality, the practical application of this granularity is evidenced.

In the case referred to the mortality risk, it is confirmed that the trend risk is not applicable to calibrate the level of shock, and on the other hand the 15% shock currently required prudently overcomes the practical application of a stressed scenario. Therefore, we do not find technical evidence to justify the recalibration upwards over the current 15%.

1. EXECUTIVE SUMMARY

EIOPA asked in April 2017 about if the current calibration of the longevity shock is adequate and if a more granular approach would be necessary taking into account two relevant aspects: risk sensitivity and complexity. This recalibration also extends it to the risk of mortality (November 2017).

The insurance industry in Spain represented by UNESPA already in January 2009 drew up a document, with the help of Towers Perrin, which pursued a more granular shock (depending on the age and maturity of the policy), an approach that was not finally adopted by the European authorities. Although this report contributed, significantly, to the fact that the longevity shock was reduced, at that time, from 25% to 20% (instantaneous, unique and permanent decrease in 20% of mortality rates) that currently appears in the Article 138 of Delegated Regulation 2015/35 of Solvency II.

Some positions of academic research in actuarial sciences demonstrate the need of granularity for the longevity shock, in contrast to the legal position of the single shock of 20%. Among others, it's demonstrated by the research work *The Longevity Risk and its practical application to Solvency II: Advanced actuarial models for its management* (Julio Castelo International Award), and which confirm the **actuarial robustness of the granularity of longevity shocks that are referenced to the age and residual maturity of the insurance contract.**

The work presented in this Technical Note can be understood as a continuation of the previous investigations mentioned above, already consolidated in the actuarial literature, incorporating as the particularity that of carrying out the analysis on European population.

Therefore, the aim of this research study is to propose to EIOPA and European Commission an actuarially contrasted methodology that confirms the suitability of the granularity of the longevity shock, which contributes to the improvement in the capital charge assigned to the biometric survival risk as well as to improve the appetite of the European insurance industry to assume this risk on their balance sheet and so a better offer of annuities to the citizens.

We also believe that it is appropriate to note that the granularity in the context of efficient **risk management adjusted to capital requirements demonstrates all its benefits.**

This Technical Note also analyzes the suitability of the capital charge assigned to the mortality risk. In this case we refer exclusively to the Spanish perimeter, since in the analysis it is necessary to know the structure and amounts of the different charges contemplated by the sectorial actuarial tables of mortality. Even so, the methodology used is common for all countries.

This analysis for mortality risk demonstrate that trend risk must not be apply for the assessment of mortality shock in every European countries analyzed in this study, and **concluding that current 15% of mortality shock is definitely enough** for covering volatility and level risk.

2. LONGEVITY RISK

The dynamics of longevity risk are certainly complex. Actuaries have been improving the models trying to capture this risk from a perspective of dynamics over time.

The intrinsic uncertainty associated with human survival makes the very long-term projection of biometric behavior requires complex statistical techniques. The graduation of the risk from different statistical techniques such as parametric or non-parametric techniques provides the analyst with a set of tools that, if used together, allow to better observe it's validity over the time, and that using some of them together can mitigate what is known as model risk, as we will see throughout this Technical Note.

2.1. Longevity sub-risks

For calibrating the longevity shock from the postulates established by Solvency II Directive, it is necessary to decompose the longevity risk in the different sub-risks included on it. These sub-risks are:

- **Volatility risk:** It is refer to the random deviation of the risk on the actuarial hypotheses established in the demographic technical bases in the exercise that is subject to observation. This sub-risk is mitigated by increasing the size of the exposed portfolio, with the incorporation of lifestyle variables, or by means of risk transfer instruments such as reinsurance.
- **Level risk:** It refers to the fact that the chosen model does not capture in a structural way the dynamics of biometrics. The mitigation of this risk must be carried out by means of expert judgment criteria that assess preventively if the model to be applied is adequate according to the most prudential hypotheses.
- **Catastrophic risk:** Shocks to mortality rates during a specific year due to pandemics, wars, etc.
- **Trend risk:** This sub-risk is related to the dynamics over time of human biometrics, breaking the classic principle of stationarity in the time established in the treaties of actuarial biometrics. The difficulty of extrapolating trends over time with actuarial sufficiency and reliability, is the main gradient of the insufficiencies or underestimates of the expected values of survival established in the original technical bases.

We understand and share with the criterion of the regulator/supervisor that the longevity shock refers to the trend risk, which is a source of relevant uncertainty and therefore requires special attention in its disruptive variations.

Given the special relevance of trend risk, we understand that this must be measured as accurately as possible in accordance with the best actuarial practices, avoiding simplifications for practical reasons. That is, according to the criteria that govern actuarial science for the measurement of a risk: prediction, reliability, sufficiency and proportionality, we believe it is not advisable, as we demonstrate in this document, not to opt for a single value, because far from reducing uncertainty it increases it and we could even say that the principle of actuarial equity is broken given the significance of the values resulting from the study by age/maturity.

If we appeal to the principle of proportionality, understood as the advantages overcome the disadvantages, we understand more than justified to opt for the granularity in the longevity shock.

2.2. General population versus insured population

When the trend risk is graduated, a discussion is made as to whether this should be done on the values of the exposed portfolio - own experience - or on the contrary, taking as a reference the general population values.

The technical debate has been intense in the past, although in 2017 the consensus among the best experts in trend modeling understands that using data from the **general population in the trend implies an implicit margin of safety** advisable by prudential criteria.

This criterion has the justification that the population insured in annuities has better survival records than the general population, or what is the same, **the insured population has already experienced the improvements in mortality in the past and that in the future it will correspond to experience the general population.**

For instance, among others, the Spanish PERMF-2000 survival tables or the recent ones prepared by the OECD for the supervisory authorities of the Peruvian government in this year 2017, demonstrate that the trend is based on the general population.

In this way we avoid implementing an additional sub-risk that refers to the graduation of trend for insured population, which certainly would also present difficulties in its measure given the insufficient exposure both in number of heads insured and in the time spent to make future projections with the statistical accuracy needed.

3. ACTUARIAL MODELS APPLIED FOR LONGEVITY

Because the life expectancy increases in certain populations in a substantial way, trends in mortality have become the focus of all institutional agents, both private and public.

Traditionally, life insurance and pensions have based their mortality models on life tables, and deterministic projections have been used to take improved mortality into account. The spectacular improvement of longevity throughout the 20th century has shown that this approach is inadequate to handle the mortality risk.

Over the years, different proposals for modeling and forecasting the mortality rates dynamics have been put forward. No model is uniformly superior to the others in every situation and for every population. For this reason and so as to minimize the "model risk", four models have been selected. They match two of the most common approaches used in countries such as the United Kingdom, the United States and Italy: models based on time series techniques, and models based on multidimensional smoothing techniques. There were other models in international actuarial literature to choose from, such as the models that include the cohort variable. However, the use of this variable has caused significant controversy among experts in longevity average; they recommend cautious use, especially when performing future predictions (Currie, 2012).

3.1. Database

3.1.1. Countries selection

To determinate which of the 28 EU countries should be included for calculating the longevity trend risk was not an easy task since they are not all equally representative in the European insurance business.

We could consider it appropriate or acceptable to find the average mortality rate of the countries using their market share and this in turn would have to be determined by the volume of business at risk.

In the document "The underlying assumptions in the standard formula for the Solvency Capital Requirement calculation (EIOPA-14-322 25 July 2014)", it is assumed as hypothesis for the calculation of the life underwriting risk module the diversification in insurance portfolios. Indicating that the reference population that underlies all calibration work is an insured population that is well diversified with respect to:

- Age.
- Gender
- Smoker
- Socio-economic status
- Type of insurance coverage
- Amount of insurance coverage
- Geographical location

However, these variables and those that we thought earlier that could be representative of the mortality rate as the volume of technical provisions per country, are not published.

For these reasons, we decided to use demographic data that cover at least 75% of the global European population and that have representativeness in the volume of the European insurance business for calibrating the longevity trend risk. **The following seven countries represent a population share of almost 75% (1 January 2016) and their market volume was about 77% in 2015.**

MEMBER STATE	POPULATION	PERCENT OF TOTAL EU POPULATION
GERMANY	82.175.684	16,10%
FRANCE	66.759.950	13,08%
UNITED KINGDOM	65.382.556	12,81%
ITALY	60.665.551	11,88%
SPAIN	46.445.828	9,10%
POLAND	37.967.209	7,44%
NETHERLANDS	16.979.120	3,32%
TOTAL	376.375.898	74%

3.1.2. Biometric observation

To analyze the historical evolution of the survival of the seven selected countries, the database contained in **Human Mortality Database (HMD)** was taken as a **point of reference**. We have used these data since they are usually selected for actuarial studies that measure mortality and human survival due to the quality, timeliness and richness of the information. The biometric variables used in the study are those exposed to risk ($E_{x,t}$) and the deceased ($D_{x,t}$) by age and without distinction by gender. The exposed by age used for the weightings of $q(x, t)$ of each country will be seen in the next chapter.

3.1.3. Observation and projection period

Before carrying out the information collection, a descriptive study was made of the time series of exposed and claims available for each of the selected countries.

MEMBER STATE	LAST_DATE	MAX. YEAR	MIN. YEAR	MAX. AGE	MIN. AGE
FRANCE	02/05/2016	2014	1816	110	0
GERMANY (EAST)	29/03/2017	2015	1956	110	0
GERMANY (WEST)	29/03/2017	2015	1956	110	0
GERMANY TOTAL	29/03/2017	2015	1990	110	0
ITALY	28/08/2015	2012	1872	110	0
NETHERLANDS	13/01/2017	2014	1850	110	0
POLAND	26/11/2015	2014	1958	110	0
SPAIN	16/03/2016	2014	1908	110	0
UK	29/06/2016	2013	1922	110	0
TOTAL		2015	1816	110	0

We observe in the previous table that Italy published its latest data in 2012 and Germany, however, in 2015. In the case of the data from Germany, this information is disaggregated between GERMANY EAST and GERMANY WEST (we have verified that this data contains GERMANY TOTAL in their same years). It is understood that there is no problem in using the disaggregated data and joining them, and that it would be like using GERMANY TOTAL and adding 5 years of the other two. Similarly, we used the UK data that contains aggregated population data for Northern Ireland, England and Wales.

It was not considered relevant in terms of improvement and stress factors that the last reference year was different for some of the countries, since the results of the backtesting showed robustness in the analyzes.

Therefore, it was decided to **select the last 30 available years for each country and project the last observed mortality rate until 2024**. The reason to decide not to use more years was to avoid the effect that different political and economic situations could have in the structure of mortality. During the study period, most countries have a stable situation, except Poland, and we will see that the volatility of mortality rates is much higher for this country, but we opted for maintaining this country for prudence principle.

Regarding to ages, **we have chosen the 40 to 100 year interval because it is the most representative for the European insurance industry in annuities**. We can also highlight that it is not necessary to obtain a long-term improvement factor for the issues raised above, and therefore, **it is recommended for a better estimation and calibration to update the model every five years with new observations**.

Finally, the weights used in the calculation of the average $q(x, t)$ of the European sample are attached.

	SPAIN	FRANCE	NETHERLANDS	POLAND	GERMANY	ITALY	UK
EDAD	2012	2012	2012	2012	2012	2012	2012
40	15,4%	16,7%	4,2%	10,8%	18,0%	18,3%	16,6%
41	15,0%	17,1%	4,4%	10,3%	18,0%	18,1%	17,0%
42	14,7%	17,1%	4,6%	9,8%	18,6%	18,1%	17,2%
43	14,2%	16,6%	4,7%	9,3%	20,1%	18,0%	17,0%
44	14,0%	16,2%	4,9%	8,9%	21,3%	17,8%	17,0%
45	13,7%	15,9%	4,8%	8,6%	22,3%	17,8%	17,0%
46	13,5%	15,6%	4,6%	8,4%	23,4%	17,7%	16,9%
47	13,2%	15,5%	4,5%	8,2%	24,0%	17,6%	16,8%
48	13,0%	15,7%	4,6%	8,3%	24,6%	17,1%	16,8%
49	13,0%	15,8%	4,7%	8,4%	25,0%	16,3%	16,7%
50	12,8%	15,9%	4,7%	8,7%	25,4%	16,0%	16,5%
51	12,5%	15,8%	4,7%	8,9%	25,9%	15,8%	16,4%
52	12,4%	15,9%	4,7%	9,3%	25,9%	15,7%	16,2%
53	12,4%	16,1%	4,7%	9,9%	25,6%	15,4%	15,9%
54	12,4%	16,2%	4,7%	10,6%	25,3%	15,2%	15,7%
55	12,3%	16,2%	4,6%	11,2%	24,9%	15,3%	15,5%
56	12,2%	16,3%	4,6%	11,7%	24,4%	15,3%	15,3%
57	12,0%	16,7%	4,7%	12,0%	24,0%	15,5%	15,2%
58	11,6%	16,9%	4,7%	12,3%	23,9%	15,4%	15,2%
59	11,4%	17,2%	4,7%	12,3%	23,7%	15,3%	15,3%
60	11,4%	17,3%	4,7%	12,2%	23,6%	15,4%	15,5%
61	11,4%	17,5%	4,7%	12,1%	23,1%	15,8%	15,4%
62	11,1%	17,5%	4,6%	11,9%	22,9%	16,3%	15,6%
63	10,7%	17,8%	4,6%	11,5%	22,7%	16,8%	15,9%
64	10,7%	17,9%	4,6%	10,9%	22,4%	17,0%	16,4%
65	11,2%	17,7%	4,7%	10,4%	21,9%	16,8%	17,3%
66	11,4%	18,3%	5,0%	10,2%	21,5%	15,6%	18,0%
67	11,5%	19,0%	5,6%	10,1%	21,4%	14,9%	17,4%
68	12,6%	17,8%	5,3%	8,9%	21,2%	16,6%	17,5%
69	13,6%	16,5%	4,9%	8,0%	20,2%	18,2%	18,7%
70	13,2%	16,2%	4,9%	7,7%	22,1%	18,2%	17,7%
71	12,1%	15,3%	4,5%	7,4%	25,3%	19,2%	16,1%
72	11,2%	14,5%	4,4%	7,8%	26,0%	20,6%	15,6%
73	11,7%	13,7%	4,2%	7,9%	27,6%	19,7%	15,2%
74	10,5%	14,1%	4,0%	7,8%	30,3%	18,4%	15,0%
75	9,2%	14,8%	3,9%	8,0%	31,5%	17,6%	15,1%
76	10,5%	14,9%	3,8%	8,0%	30,5%	17,5%	14,8%
77	11,8%	15,1%	3,7%	8,1%	28,9%	17,8%	14,6%
78	12,5%	15,5%	3,7%	8,1%	27,9%	17,8%	14,5%
79	12,6%	16,1%	3,6%	8,1%	27,5%	17,8%	14,3%
80	13,1%	16,5%	3,6%	7,9%	26,5%	18,0%	14,4%
81	13,6%	17,2%	3,7%	8,1%	23,6%	19,0%	14,8%
82	13,7%	18,1%	3,8%	8,4%	21,6%	19,2%	15,2%
83	13,7%	18,4%	3,7%	8,5%	22,0%	18,6%	15,1%
84	13,5%	18,4%	3,7%	8,2%	22,9%	18,6%	14,7%
85	13,4%	18,3%	3,6%	7,9%	23,6%	18,6%	14,5%
86	13,3%	18,7%	3,6%	7,5%	23,6%	18,7%	14,7%
87	13,2%	19,2%	3,5%	7,1%	23,2%	18,8%	14,9%
88	13,2%	19,7%	3,5%	7,1%	22,6%	18,9%	14,9%
89	13,2%	19,8%	3,5%	6,8%	22,5%	19,1%	15,1%
90	13,3%	20,1%	3,5%	6,5%	21,9%	19,1%	15,6%
91	13,2%	20,8%	3,5%	6,2%	21,1%	19,1%	16,1%
92	13,2%	22,1%	3,5%	5,6%	21,7%	16,3%	17,6%
93	13,3%	24,6%	3,7%	5,2%	23,8%	12,2%	17,1%
94	13,8%	23,0%	3,8%	5,2%	26,7%	12,4%	15,1%
95	14,7%	18,7%	3,8%	4,7%	26,0%	16,4%	15,9%
96	15,4%	18,0%	3,8%	3,9%	19,0%	21,9%	17,9%
97	15,4%	17,2%	3,9%	3,9%	14,4%	25,5%	19,7%
98	15,0%	18,3%	3,6%	4,0%	14,0%	25,5%	19,7%
99	13,6%	23,2%	3,2%	4,1%	14,9%	23,0%	18,0%
100	12,4%	26,7%	2,9%	4,2%	17,6%	20,3%	16,1%

3.2. Longevity model

In order to reduce the model risk, and due to the fact that there is no model that gives an overall better fit for all ages, we have used three well known models, and we propose to use the average among them. The models are: Lee-Carter (LeeCarter1992), Cairns-Blake-Dowd (CBD2006) and Psplines (Currie2004). They correspond to models M1, M5 and M4 used in Cairns2009, and they are standard models used in estimating and forecasting mortality.

It is important to note the use of parametric and non-parametric models in the estimation.

3.2.1. Notation

Before we proceed with model specification and estimation we introduce some basic notation

- We define $m_c(t, x)$ to be the crude (i.e., unsmoothed) death rate for age x in calendar year t .

$$m_c(t, x) = \left(\frac{\text{Numbre of deaths during calendar year } t \text{ aged } x \text{ last birthday} = d_{x,t}}{\text{Average populations during calendar year } t \text{ aged } x \text{ last birthday} = e_{x,t}} \right)$$

- The average population is usually approximated by an estimate of the population aged x last birthday in the middle of the calendar year. The underlying death rate is then $m_c(t, x)$, which is equal to the expected deaths divided by the exposure.
- The mortality rate, $q(t, x)$ is the probability that an individual aged exactly x at exact time t will die between t and $t+1$.
- The force of mortality, $\mu(t, x)$ is interpreted as the instantaneous death rate at exact time t for individuals aged exactly x at time t .

In general, it is satisfied that:

1. $m(x,t) \approx \mu(x,t)$
2. $q(x,t) \approx 1 - \exp(-m(x,t))$

3.2.2. Statistical assumptions

In the stochastic mortality models literature, we find that some models attempt to model $\mu(x, t)$ and others $q(t, x)$. This is not a problem, since we saw above the relationship between both quantities. Depending on what the aim is, we will assume that the number of deaths $d_{x,t}$ follows a Poisson or binomial distribution:

- If we treat $d_{x,t}$ as a random variable, and the central exposure, $e_{x,t}^c$, as fixed, then $d_{x,t}$ has a Poisson distribution

$$d_{x,t} \sim P \left(e_{x,t}^c \cdot \mu(x,t) \right)$$

- We have a corresponding distributional result for the mortality rate, $q(x,t)$. We approximate the initial exposed to risk $e_{x,t} \approx e_{x,t}^c + \frac{1}{2}d_{x,t}$. Then $d_{x,t}$ has a binomial distribution

$$d_{x,t} \sim B \left(e_{x,t}^c \cdot q(x,t) \right)$$

We first describe the models and later we will explain issues related to uncertainty in the forecasts

LEE-CARTER:

One of the most commonly used models in mortality projection is the Lee-Carter model (Lee & Carter, 1992). This is the model proposed a single-factor model for the force of mortality defined as follows:

$$\log(\mu_{x,t}) = \alpha_x + \beta_x \kappa_t$$

where α_x is the age-related component of mortality, κ_t is a period effect for year t and β_x is an age-related modulation of κ_t , therefore, represents the pace at which the force of mortality changes at each age over time. The model specified above cannot be fitted on its own, as there are infinitely many parameterizations which yield the same fitted values, therefore, some restrictions need to be imposed to ensure identifiability, in general, it is assumed that $\sum \kappa_t = 0$ and $\sum \beta_x = 0$. This particular set of constraints has the attractive feature that α_x is a measure of average log mortality for age x .

Estimation of the parameters is done via maximum-likelihood as proposed by BDV2002. Although the original Lee-Carter model assumes a Poisson distribution, we have used the Binomial setting, since it has been suggested that it gives a better fit to the data (Currie2013).

Projections are calculated assuming an ARIMA (p,d,q) model for κ_t . Several possibilities were tested and a random walk with drift was chosen since it had the best performance in terms of AIC criteria. Therefore, we assume that:

$$\kappa_{t+1} = \kappa_t + drift + \epsilon_{t+1} \sim N(0, \sigma^2)$$

Since mortality can be regarded as a stochastic process, it makes sense to use a stochastic model to predict future results with regard to mortality trend, and calibrate the trend stress for longevity risk. This stress is calculated from projection confidence bands; these bands are obtained by multiplying the prediction's standard error by

$\Phi^{-1}(p)$, where Φ is the distribution function of a variable $N(0; 1)$; p is the desired probability, i.e. $p = 0,005$ and $\Phi^{-1}(p) = 0,57$ in our case.

CBD:

CBD2006 introduced a two-factor model for the force of mortality defined as follows:

$$\text{logit}(q(x, t)) = \kappa_t^1 + \kappa_t^2 (x - \bar{x})$$

Therefore, again, we are using a Binomial distribution. In this case, the model is completely identifiable, and no further constraints need to be imposed. projections of future mortality are done by means of a bivariate random walk with drift for κ^1 and κ^2 , where both the drift terms and their correlation are estimated from the data.

Important: This models assumes linearity of the age effects. That linearity does not hold for lower and higher ages, and these models are therefore appropriate for the pensioner ages only (60-89).

P-SPLINE 2D (AGE-PERIOD):

Currie et al. (2004) introduced the idea of using bidimensional P-splines (Eilers1996) for the estimation of the force of mortality. Penalized splines or P-splines is now well-established as a method of smoothing in Generalized Linear Models. The main characteristics of the methodology under consideration are the following:

- Using B-splines as the basis for the regression;
- Modifying the log-likelihood by a difference penalty on the regression coefficients.

The aim is to model $\log(\mu_{x,t})$ as a function of age and time, in the context of the GLM model. To this end, the single dimensional focus described above is expanded so as to enable smoothing of the force of mortality based on age as well as time. \mathbf{B}_x is the B-splines matrix built from age, and \mathbf{B}_t is the matrix built from the time variable. The regression matrix for the bidimensional model is the Kronecker product of both:

$$\mathbf{B} = \mathbf{B}_t \otimes \mathbf{B}_x$$

Additionally, a penalty is applied to adjacent coefficients, based on their proximity in the direction of age and time. This penalty is:

$$\mathbf{P} = \lambda_x \sum_i (\Delta^2 \theta_i)^2 + \lambda_t \sum_j (\Delta^2 \theta_j)^2$$

This penalty depends on two parameters that control smoothness in the direction of age and time. With values for λ_x and λ_t , the model is adjusted using the penalized version of the Fisher algorithm proposed by Eilers & Marx (1996). One of the advantages of this method is to be able to obtain the standard errors of adjusted data straightforwardly; this can be used immediately to calculate force of mortality stress.

$$\text{Var}[\log(\hat{\mu}_{x,t})] = \mathbf{B} (\mathbf{B}' \hat{\mathbf{W}} \mathbf{B} + \mathbf{P})^{-1} \mathbf{B}'$$

Where \mathbf{B} and \mathbf{P} come from the equations described above, and $\hat{\mathbf{W}}$ is the weight matrix of the penalized Fisher algorithm once convergence has been achieved.

Figure 2 shows the force of mortality among the one Member State population, and the force of mortality adjusted using the bidimensional P-spline model. As observed, the smoothed surface reflects the data pattern without any noise.

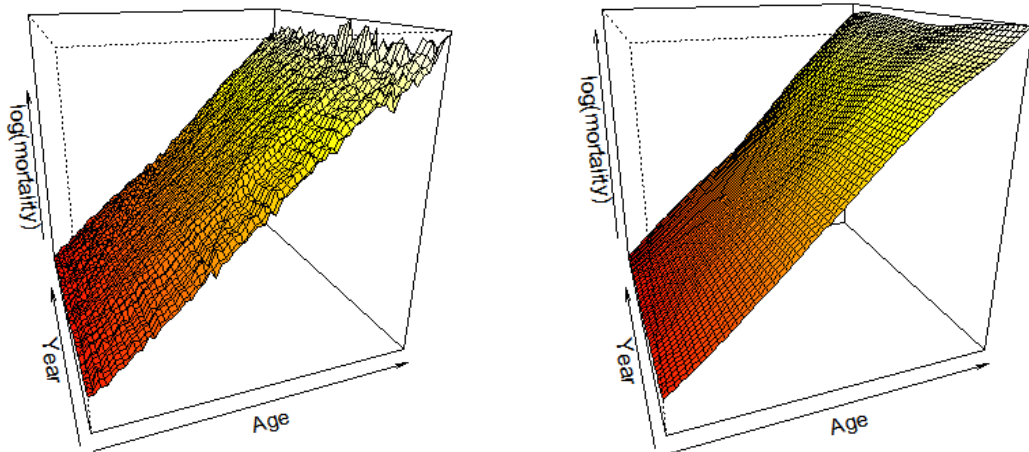


Figure 2 - Force of mortality observed (left) population between the ages of 40 and 100, for 30 years and adjusted force of mortality (right) using the bidimensional P-spline model.

P-splines handle the prediction of future data as missing value problem (Currie et al., 2004). The prediction is completed by extending the B-splines matrix to accommodate the new observations (and the corresponding penalty), and the new coefficients are estimated. These are a linear combination of the two last coefficients used in the adjustment

In this case the model is expressed as

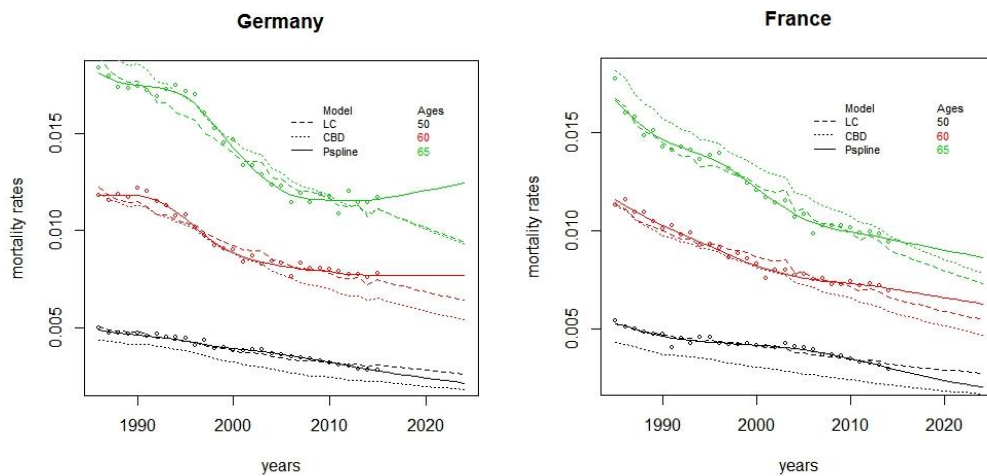
$$\log(\mu_{x,t}) = \sum_j \sum_j \theta_{i,j} B_{ij}(x, t)$$

where $B_{i,j}(x, t)$ is a basis for regression to account simultaneously for the effect of age and time, and is constructed using the Kronecker product of B-splines basis, and $\Theta_{i,j}$ are the parameters to be estimated. The penalty imposed on the coefficients is based of differences of adjacent coefficients. The penalty depends on two smoothing parameters that will determine the smoothness of the fitted values and the shape of the forecast. The values of these two parameters are chosen using BIC criteria. The forecast is obtained by extending the Bsplines basis and penalties to accommodate the new period and re-fit the model to estimate the new parameters.

3.3. Forecast uncertainty

When analyzing the uncertainty in mortality projections in an actuarial context it is important to consider all sources of risk. In the case of the LC and CBD model, there is no analytical expression for the model parameters, therefore, parameter uncertainty is accounted for via bootstrap. We use a semiparametric bootstrap proposed by BDV2005. The procedure is as follows: B samples (we chose B=1000) of the number of deaths $d_{x,t}^b$, $b=1, \dots, B$, are generated by sampling from the Poisson Distribution with mean $d_{x,t}$. Each bootstrapped sample is then used to re-estimate the model to obtain B bootstrapped parameter estimates. Once a stochastic mortality model has been bootstrapped we can simulate it forward to obtain simulated trajectories which account for both the forecast error in the period indexes and the error in the model. In order to simulate the period index, we have used a multivariate adaptation of Algorithm 2 in Haberman2009.

In the case of the Pspline model, analytic expressions for the estimates of the parameters are obtained, and so, it is immediate to account for parameter uncertainty in the forecast. For all models the 99.5 stress of the projections are calculated



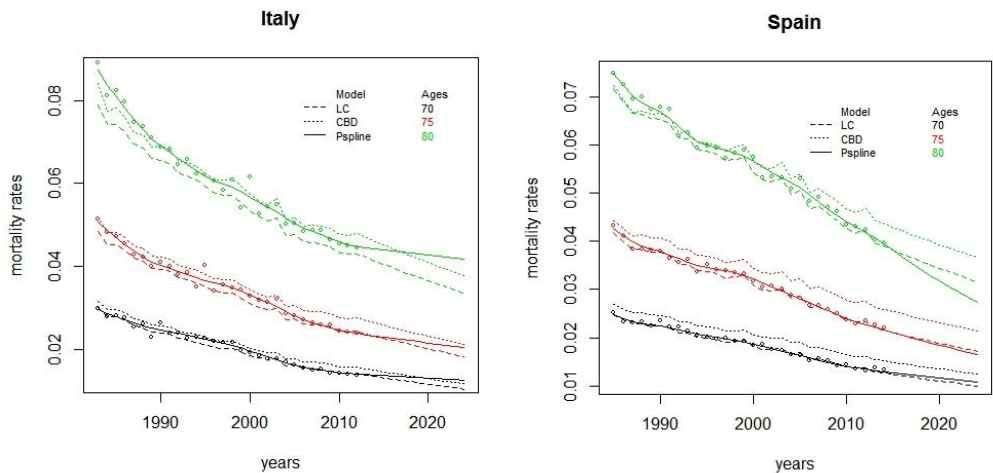


Figure 1: Plots of fitted values and forecast for three models and three different ages

Figure 1 shows the problems of the CBD with younger ages, we can also see that there is no model that fits best for all ages. This is why we choose to average the results obtained.

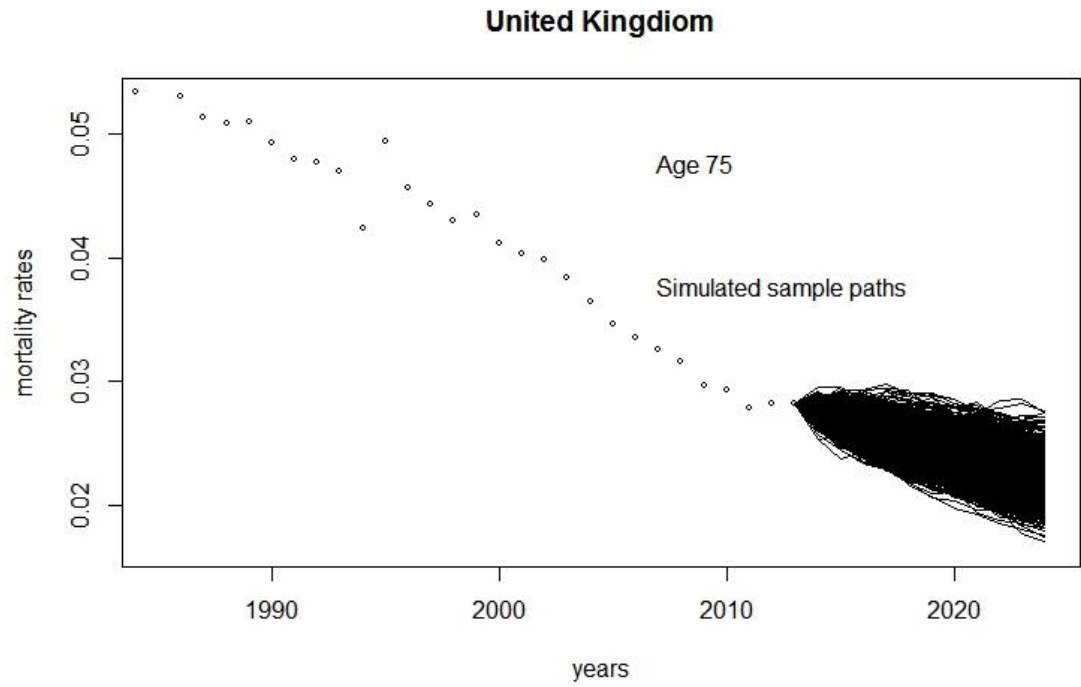


Figure 2: Plot of simulated sample path for the Lee-Carter model

Figure 2 gives an example of the sample paths obtained when accounting for the stochastic and parameter uncertainty

3.4. Improvement factors

Once the models are fitted and projected, the average mortality improvement factors for each age are calculated imposing that:

$$\hat{\mu}(x, f) = \hat{\mu}(x, t)(1 + \lambda_x)^{f-t}$$

where f is the final projected year (in this case 2024) and t is the first projected year (in this case will be 2013, 2014, 2015 or 2016, depending on the country).

Once λ_x is calculated, for each country, we average the results obtained from the three models, and we also calculate the results at 99.5% stress. **Figure 3 show the Best estimates for all countries (averaged over the three models) and the 99.5% obtained by weighted average of the results of each country, with weights corresponding to the proportion of the total population.**

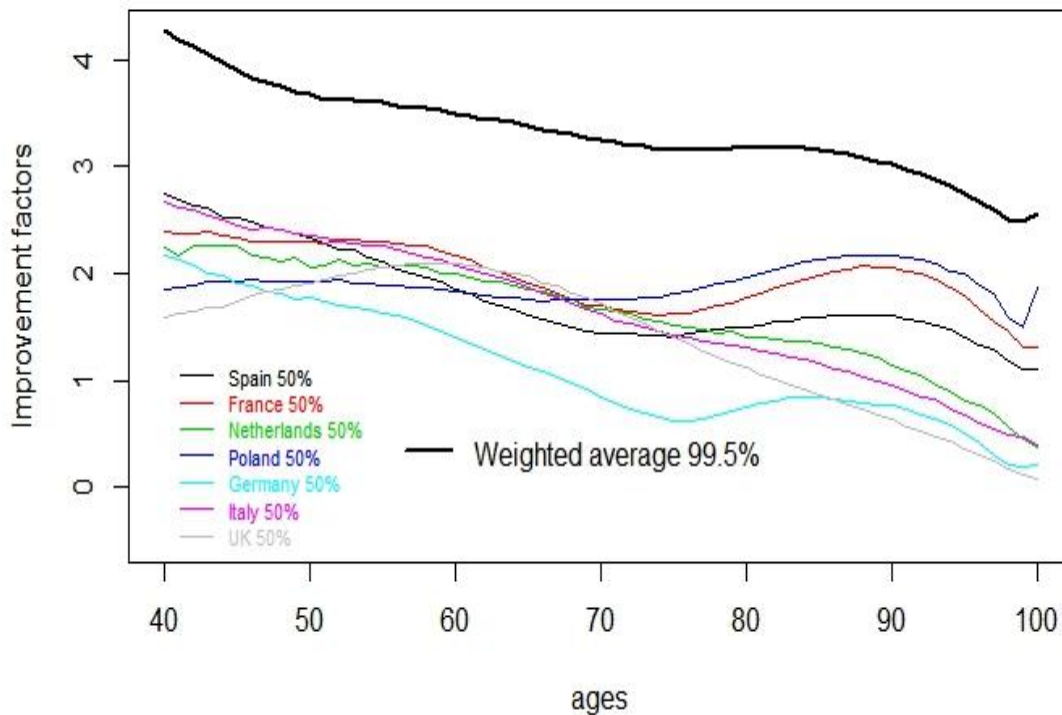


Figure 3: Averaged improvement factors for each country and 99.5% stress obtained as weighted average of the 7 countries

The detail of these figures are represented in the following Figure 4. It represents the **European Longevity Index with a level of confidence of 99,5% usefull for Solvency II streeses (ELI_{99,5})** and the **European Longevity Index with a level of confidence of 50% usefull for Best Estimate and techincal provisions without streeses (ELI₅₀)**.

EUROPEAN LONGEVITY INDEX (ELI)			EUROPEAN LONGEVITY INDEX (ELI)		
AGE	$q_x^{50\%}$	$q_x^{99.50\%}$	AGE	$q_x^{50\%}$	$q_x^{99.50\%}$
40	2,26%	4,31%	71	1,40%	3,18%
41	2,23%	4,22%	72	1,36%	3,16%
42	2,22%	4,15%	73	1,32%	3,13%
43	2,20%	4,06%	74	1,26%	3,10%
44	2,17%	3,98%	75	1,23%	3,09%
45	2,15%	3,91%	76	1,23%	3,08%
46	2,13%	3,83%	77	1,25%	3,10%
47	2,11%	3,78%	78	1,26%	3,12%
48	2,10%	3,74%	79	1,28%	3,14%
49	2,08%	3,69%	80	1,29%	3,15%
50	2,08%	3,66%	81	1,32%	3,16%
51	2,05%	3,62%	82	1,34%	3,18%
52	2,04%	3,61%	83	1,35%	3,19%
53	2,04%	3,62%	84	1,35%	3,18%
54	2,02%	3,59%	85	1,34%	3,16%
55	2,01%	3,60%	86	1,32%	3,13%
56	1,99%	3,57%	87	1,31%	3,11%
57	1,97%	3,56%	88	1,30%	3,09%
58	1,95%	3,56%	89	1,28%	3,05%
59	1,92%	3,54%	90	1,26%	3,02%
60	1,88%	3,50%	91	1,22%	2,96%
61	1,84%	3,48%	92	1,18%	2,92%
62	1,80%	3,45%	93	1,16%	2,90%
63	1,77%	3,44%	94	1,09%	2,84%
64	1,73%	3,41%	95	0,97%	2,70%
65	1,69%	3,38%	96	0,88%	2,60%
66	1,65%	3,34%	97	0,80%	2,52%
67	1,61%	3,33%	98	0,71%	2,44%
68	1,57%	3,29%	99	0,69%	2,46%
69	1,53%	3,26%	100	0,71%	2,55%
70	1,47%	3,22%			

Figure 4: Mortality improvements (ELI₅₀ and ELI_{99,5}).

These three methods used (Lee Carter, Pspline and CBD) are also used by the OECD in their study “*Mortality Assumptions and Longevity Risks: Implications for pension funds and annuity providers*”, and whose results are also in line with those presented in this technical note as we can see in the following table for the case of Spain (this OECD analysis is also implemented for the other six countries chosen for this technical note).

Males	HMD		PERMP		LC		CBD		PS		CMI	
	1990-2000	2000-2009	2010-2020	2020-2030	2010-2020	2020-2030	2010-2020	2020-2030	2010-2020	2020-2030	2010-2020	2020-2030
55-59	1.6%	1.7%	1.5%	1.5%	1.3%	1.3%	1.6%	1.6%	1.6%	1.6%	1.7%	1.7%
60-64	1.6%	2.0%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%	2.0%	2.0%	1.8%	1.7%
65-69	1.7%	2.4%	1.5%	1.5%	1.6%	1.6%	1.5%	1.5%	2.6%	2.5%	1.9%	1.7%
70-74	1.9%	2.7%	1.5%	1.5%	1.6%	1.6%	1.4%	1.4%	2.8%	2.8%	2.1%	1.7%
75-79	1.3%	2.5%	1.5%	1.5%	1.5%	1.5%	1.3%	1.3%	2.6%	2.6%	2.2%	1.8%
80-84	1.5%	1.7%	1.5%	1.5%	1.3%	1.3%	1.2%	1.3%	2.2%	2.3%	2.0%	1.9%
85-89	1.2%	1.3%	1.5%	1.5%	1.0%	1.0%	1.1%	1.2%	1.7%	1.8%	1.8%	1.9%
90-94	0.9%	1.6%	1.3%	1.3%	0.6%	0.6%	1.0%	1.1%	1.5%	1.4%	1.6%	1.7%
95-99	0.6%	1.1%	0.6%	0.6%	0.4%	0.4%	0.9%	0.9%	0.9%	0.9%	1.3%	1.3%
100-104	0.4%	0.9%	0.0%	0.0%	0.2%	0.2%	0.7%	0.8%	0.4%	0.4%	1.0%	1.0%
105-110	0.2%	0.4%	0.6%	0.0%	0.1%	0.1%	0.5%	0.6%	0.0%	0.0%	0.7%	0.7%

Additionally, in the study presented in this document, **the model risk is diluted by using the average of the three models**, and the justification of this is presented in the following section.

3.5. Model risk analysis

As we have pointed out when identifying the longevity sub-risks in chapter 2, we must take into account the model risk.

By model risk we understand the adverse consequences of the use of an incorrect model, or the incorrect use of a model, thus including any errors in the definition, design, processes, or simplifications used in the model.

The proposal that we offer in this Technical Note, is based on making the average of the three proposed models. We must remember that the Pspline model in relation to the other two models, cover in a certain way the whole spectrum of statistical science that deals with graduation of tendency. Thus, the average of models has the main purpose to mitigate model risk.

To measure the model risk, we performed two exercises:

- **To quantify the number of "hits"** in which a specific model for a country and for an age the factor of improvement calculated at 50% exceeds the European factor of improvement calculated at 99.5% of the average of models and countries.

That is, we look for how many times in which the model of European average reinforced to 99.5% is surpassed by particular cases in each country, that is to say 1,200 possible cases (7 countries, three models and all ages):

- Number of global hits are 24 cases (1,87%), that is, the European reinforced model is valid in 98.13% of the possible negative impacts it could have.
- By country, 16 cases in Poland and 8 cases correspond to France.
- By models, all cases correspond to the P-spline model.
- By age, all cases correspond to ages between 85 years and 100 years.

For the purpose we are looking for, the goodness of age granularity, on a European index, we understand that the number of non-compliances over 99.5% of the model average guarantees the robustness of the methodological proposal. And therefore the European longevity index fulfills its purpose.

- **Expert judgment:** As an additional and complementary analysis to the previous one, the model risk is proposed to be analyzed by expert judgment.

Based on the origin of the risks inherent in the model, these can be classified as follows:

Data Quality	Statistical Quality	Management
<ul style="list-style-type: none"> - Definition error - Sources error - Incorrect or insufficient data - No recent data - Not enough frequency - Inadequate historical data - Migration - Absence of relevant variables 	<ul style="list-style-type: none"> - The model does not adequately capture the reality - Wrong assumptions - Overparameterization - Lack of comparability with market - Software Difficulties - Regular update - Volatility estimation - No documentation - Unsupported methodology - Lack of independence in the validation 	<ul style="list-style-type: none"> - The model does not integrate into management - The model lacks of credibility by the user or manager - Error in the interpretation of model results - Management by untrained persons who are not able to understand their limitations and assumptions - Differences between management and supervision practices - The model is not updated or calibrated periodically - Changes are not approved and validated

The next risk matrix of the longevity model by expert judgment helps us to measure this risk (see Rodríguez-Pardo, JM ,Ariza F. Revista Análisis Financiero, número 129, 2015):

RISK MODEL VALUATION MATRIX BASED ON EXPERT JUDGMENT						
RISK CRITERIA		RISK LEVEL				
MODEL RISK	Alert level Evaluation	Level 1 (very low)	Level 2 (low)	Level 3 (moderate)	Level 4 (high)	Level 5 (very high)
		10 points	20 points	30 points	40 points	50 points
	Risk Model Classification (5 levels) Each risk criterion is given a score based on the level of risk					
	Risk of possible limitations using external data	Internal data with some external reference	Internal data with some external reference It does not contain significant errors	Internal and external data Improbable but consistent	External data	Limited external data or without validation
	Risk that data used are not accurate	Accurate, complete and appropriate data	It does not contain significant errors	Approaches	Questionable data	Erroneous and incomplete data
	Risk that the model does not meet the minimum statistical quality demanded	Best actuarial, financial and statistics practices	Reasonable assumptions	Approaches	Questionable simplifications	Inadequate techniques or absence of significant variables
	Risk that the documentation model would not be revised and updated	Complete, updated and reviewed by a third party (internal auditor)	Complete and updated (at least mathematical and empirical bases).	It does not reflect the limitations and uncertainty linked to the model	There is some documentation but there are no updating procedures	There is no documentation
	Risk that the model could not be explained autonomously and independently	The model can be replicated in a separate software to get the same results	The model can be replicated in a separate software to get similar results	The model can be replicated by simplifications	Problems are detected by independents	The model validation is difficult
	Risk that the processes, procedures and responsible for each stage of calculation and use of the model is no detailing	Processes and responsibilities fully documented and updated	Processes and responsibilities documented but updated at the request of Board or Supervisor	There is just some documentation and slightly updated	There is no documentation or hierarchy of responsibilities	No assigned responsibilities
	Risk that the model is not integrated into the risk management and the decision-making	The model is well understood by the Board and regularly supports the decision-making	The model is well understood by the Board but supports only some decision-making	The company can not demonstrate that the model is used for decision making	The model is not well understood by the Board	The model is not well understood by the Board and it doesn't take part of decision-making
	Risk that the results are not consistent as to the nature, volume and complexity of the entity	Accurate results and segmented by LOB's	Accurate results	Consistent results	Some inconsistencies detected	Inadequate results as to the nature, volume and complexity of the entity
	Risk that the model does not explain in whole or in part the causes and sources of profits and losses	The model fully explain the causes of losses and gains	The model approximates the causes of losses and gains	The model approximates the main causes of profits and losses	The model approximates only some of the causes of profits and losses	The model does not explain any of the causes of losses and gains
	Risk that deficiencies in the model are not remediable	Without deficiencies detected	No relevant deficiencies	Some improvement areas	The model needs some additional corrections	The model is wrong
	Risk of trend risk calibration	Using several of the suggested methods for calibrating the trend risk (Medium, BIC, SAINT)	Using at least one of the suggested methods for calibrating the risk of trend	Using other calibrating methods	Using simplifications	Trend risk is not calibrated
	Longevity Risk					
RISK MODEL ASSESSMENT						
	Sum of the scores for each criterion (10 to 50 each)	110 points	From 110 to 220 points	From 220 to 330 points	From 330 to 440 points	From 440 to 550 points
ADDITIONAL CAPITAL CHARGE	Additional capital charge based on expert judgment	0-1%	1-3%	3-5%	5-8%	The model should not be implemented

The recent document of **the Actuarial Function Self-Regulation Guide under the framework of Solvency II** prepared by the Institute of Spanish Actuaries and presented on May 2017, incorporates the model risk measure into the key functions that the actuary must perform to the extent of risk. Therefore, the contrast of models that we have made is aligned with the precepts emanated in the Guide in section 5.6 *Implication in internal models and specific parameters* (page 67 and following)

The analysis of every units of measures-rows of the matrix, allows us to conclude that the average of models mitigates in such a way the risk of model.

4. SHOCK ANALYSIS UNDER SOLVENCY II

The new Solvency II framework Directive introduces a new margin of solvency which, unlike the previous margin, will be dynamic. It rewards companies that show better risk management, and protects policyholders and beneficiaries: poorer solvency margins will be immediately reported to the market.

For a previous focus, we must remember some other goals of the new Directive:

- To integrate the regulations of the European insurance market.
- To improve the competitiveness of the insurance industry.
- To promote a solvency system that is sensitive to the risks assumed.
- To make decisions that fit the company's actual risk.

To this end, appropriate technical provisions must be defined using statistical and actuarial methods. The concept of Best Estimate was created for this purpose. Additionally, this regulation introduces a Solvency Capital Requirement (SCR) in order to handle potential business deviations. Its calculation is based on a standard formula that allows all companies to evaluate their economic capital objectively.

4.1. Standard formula

This paper focuses on analyzing the impact on insurance companies of longevity risk under Solvency II, and the possibility of ensuring that the assets of entities are always enough to cover liabilities generated by this risk. For this, Solvency II introduces a shock equivalent to a permanent, instantaneous and unique reduction of 20 percent in expected mortality rates.

$$SCR_{long} = NAV_0 - (NAV_0 \cdot shock | shock_{longevidad})$$

So that the need for examining the suitability of this shock is understood, we would like to translate this data into more tangible terms.

What does a permanent 20 percent reduction in mortality mean in practice? For example, this means eradicating about 60 percent of male deaths derived from circulatory problems (the same as eliminating ischemic heart disease) permanently and overnight, or eradicating all female deaths from cancer, permanently and overnight.

At a first glance, these scenarios are markedly extreme. In reality, this is even more because eradicating illnesses overlooks the natural process where the disappearance of one illness automatically leads to increased prevalence of other illnesses, since causes of death are not independent.

4.2. Alternative shock to the standard formula

For the reasons mentioned above and since we believe that the standard formula for longevity risk does not fit the actual progress of expected mortality improvements, we suggest that this formula must be recalibrated.

This alternative shock we propose will be linked to the policyholder age in addition to the residual duration of the insurance contract.

Hence, using the index ELI as a reference, the estimated mortality is calculated (q'_{x}):

$$q'_{x} = [q_{x} \cdot (1 - \lambda_{x})]$$

where:

q_{x} is the observed probability (HMD) that an individual aged x dies between x and $x+1$.

q'_{x} is the expected probability that an individual aged x dies between x and $x+1$.

λ_{x} is the mortality improvement factor for each age x (ELI_{99,5}).

For the year $x+1$:

$$q'_{x+1} = [q_{x+1} \cdot (1 - \lambda_{x}) \cdot (1 - \lambda_{x+1})]$$

And for the last projection period, which corresponds to the maturity of the contract:

$$q'_{x+n} = [q_{x+n} \cdot (1 - \lambda_{x}) \cdot (1 - \lambda_{x+1}) \cdot \dots \cdot (1 - \lambda_{x+n})]$$

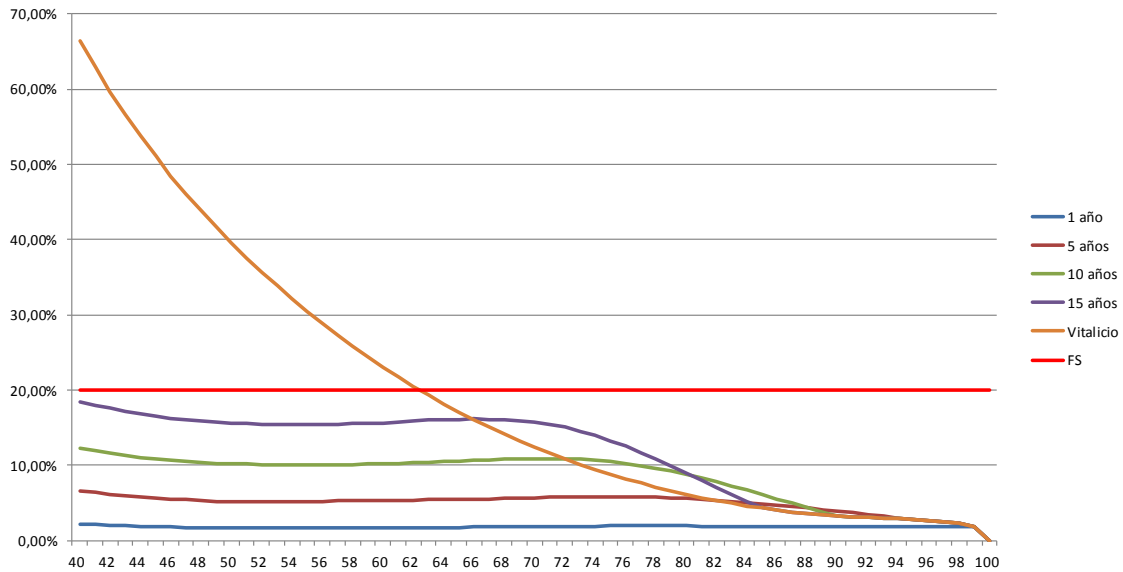
Furthermore, defining longevity shock as the reduction in expected mortality over estimated or base mortality, we have compared the number of deaths among the basis population with mortality improvement (ELI₅₀) at the end of the observed period, over the expected population with mortality improvements stressed (ELI_{99,5}):

$$shock_long = \frac{q_{x+n}}{q'_{x+n}} - 1$$

As a consequence and using the aforementioned methodology and premises, alternative shocks to the standard formula (20 percent) are obtained. These shocks are also unique, instantaneous and permanent but they combine age and residual duration of contract. **These would be the proposed longevity shocks as an alternative to the unique, permanent and immediate 20%:**

AGE	MATURITY				
	w	1 year	5 years	10 years	15 years
40	66,38%	2,14%	6,55%	12,30%	18,49%
41	62,93%	2,07%	6,35%	11,94%	18,01%
42	59,69%	2,01%	6,13%	11,58%	17,57%
43	56,62%	1,94%	5,93%	11,28%	17,16%
44	53,74%	1,88%	5,77%	11,02%	16,80%
45	51,03%	1,83%	5,63%	10,80%	16,50%
46	48,47%	1,77%	5,51%	10,62%	16,25%
47	46,06%	1,73%	5,39%	10,47%	16,04%
48	43,78%	1,70%	5,30%	10,33%	15,86%
49	41,60%	1,67%	5,23%	10,21%	15,70%
50	39,53%	1,65%	5,20%	10,16%	15,58%
51	37,56%	1,62%	5,19%	10,13%	15,50%
52	35,68%	1,62%	5,18%	10,10%	15,46%
53	33,87%	1,63%	5,17%	10,09%	15,42%
54	32,13%	1,63%	5,16%	10,07%	15,40%
55	30,45%	1,65%	5,19%	10,06%	15,38%
56	28,84%	1,64%	5,21%	10,07%	15,41%
57	27,30%	1,64%	5,23%	10,10%	15,45%
58	25,81%	1,67%	5,25%	10,12%	15,50%
59	24,38%	1,68%	5,25%	10,15%	15,56%
60	23,02%	1,68%	5,26%	10,16%	15,65%
61	21,72%	1,69%	5,30%	10,24%	15,77%
62	20,47%	1,70%	5,35%	10,31%	15,88%
63	19,29%	1,72%	5,38%	10,38%	15,97%
64	18,14%	1,74%	5,43%	10,46%	16,04%
65	17,05%	1,74%	5,42%	10,52%	16,08%
66	16,02%	1,75%	5,48%	10,62%	16,13%
67	15,03%	1,78%	5,53%	10,72%	16,12%
68	14,09%	1,78%	5,56%	10,77%	16,04%
69	13,20%	1,79%	5,63%	10,84%	15,92%
70	12,35%	1,81%	5,68%	10,86%	15,70%
71	11,56%	1,84%	5,76%	10,89%	15,42%
72	10,80%	1,85%	5,80%	10,85%	15,03%
73	10,08%	1,87%	5,82%	10,77%	14,53%
74	9,40%	1,89%	5,83%	10,64%	13,95%
75	8,76%	1,91%	5,83%	10,46%	13,27%
76	8,16%	1,91%	5,82%	10,22%	12,52%
77	7,60%	1,92%	5,79%	9,94%	11,69%
78	7,06%	1,92%	5,73%	9,59%	10,80%
79	6,56%	1,92%	5,65%	9,19%	9,86%
80	6,10%	1,91%	5,55%	8,75%	8,90%
81	5,67%	1,90%	5,46%	8,28%	7,93%
82	5,27%	1,90%	5,34%	7,76%	6,97%
83	4,91%	1,90%	5,21%	7,23%	6,03%
84	4,58%	1,89%	5,05%	6,66%	5,13%
85	4,29%	1,88%	4,88%	6,08%	4,29%
86	4,03%	1,87%	4,71%	5,50%	
87	3,80%	1,85%	4,51%	4,92%	
88	3,60%	1,84%	4,31%	4,35%	
89	3,42%	1,82%	4,10%	3,80%	
90	3,28%	1,82%	3,89%	3,28%	
91	3,16%	1,80%	3,68%		
92	3,05%	1,79%	3,46%		
93	2,97%	1,79%	3,25%		
94	2,88%	1,80%	3,01%		
95	2,78%	1,78%	2,78%		
96	2,66%	1,77%			
97	2,49%	1,76%			
98	2,23%	1,77%			
99	1,81%	1,81%			
100	0,00%	0,00%			

The representation of this shocks and the comparison with the current stress given by the standard formula (20%) is as follows:



With this study, we demonstrate that there should be one stress for each age and each year of the projection. In particular younger persons would need to have higher stresses given that they benefit more from future mortality improvements than older persons. It appears that more granular stresses per age would provide for a more risk-sensitive SCR calculation.

It's also obvious the relevance of the maturity of the contract, but **if we look for simplicity, we really think that could be enough for insurance industry to project different stresses by age and year of the projection**, because the higher longevity risk is in annuity contracts in the long term and not in the short term.

In addition, and looking for simplicity in the application of this more granular shock, we can adjust to the logarithm of the shock and then calculate the exponential. To make the adjustment more accurate, the age range has been divided into two: 40-55 and 56-100 and two exponential functions have been adjusted, or what is the same, two straight lines to the logarithms of the shocks.

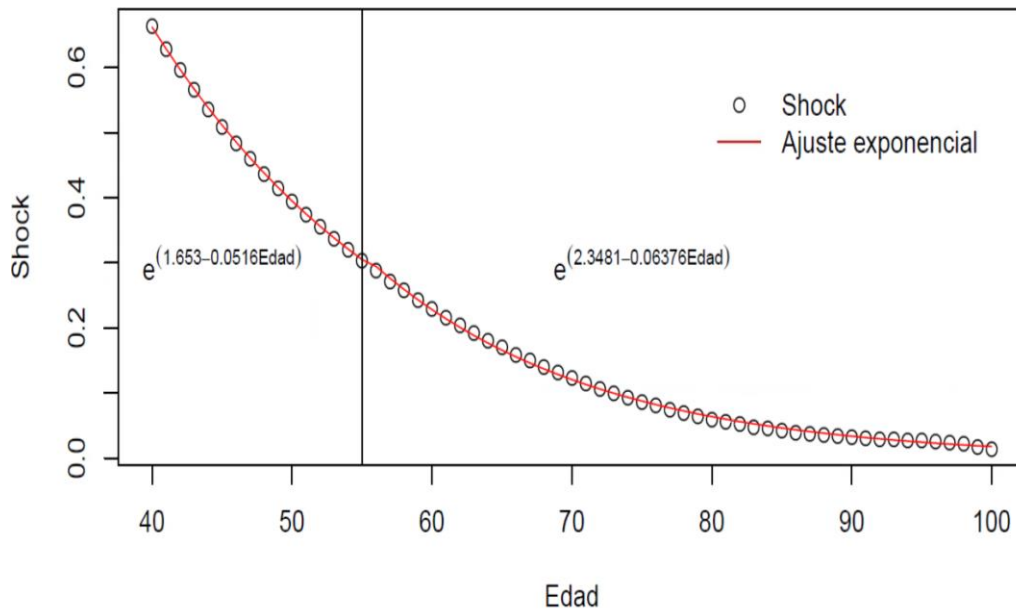
Following this approach, these granular shocks for annuities could be represented for the following functions:

$$\text{Ages 40-55: shock} = e^{1,653 - 0,0516x}$$

$$\text{Ages 56-100: shock} = e^{2,3481 - 0,06376x}$$

Where

X = age of the policyholder



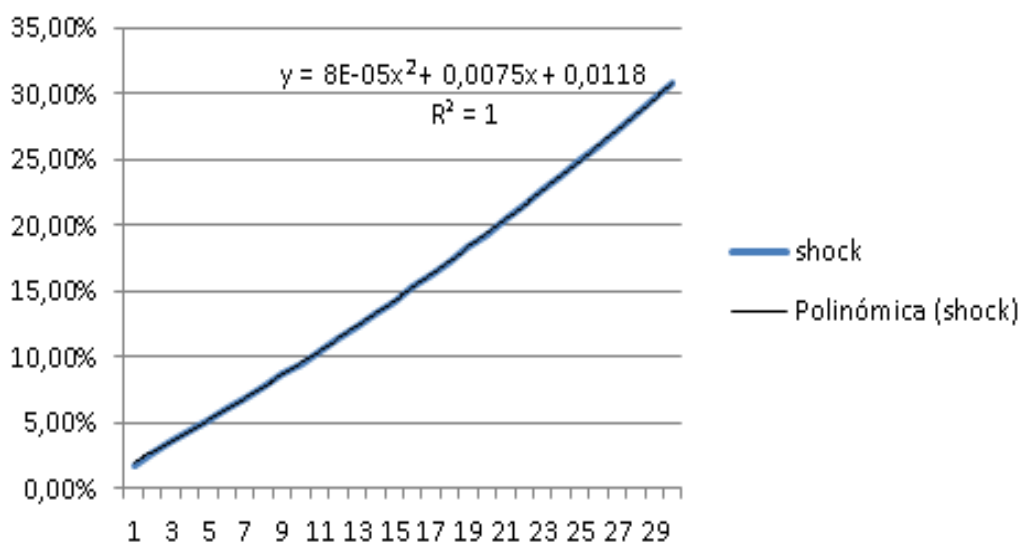
Furthermore, if we wanted to take into account the granularity of the residual maturity of the contracts, it could be represented by the following polynomic function

$$\text{Shock} = 0,00008X^2 + 0,0075X + 0,0118$$

Where

X = residual maturity of the contract

residual maturity / shock



The following can be concluded from these results:

- For longer durations, there is a higher chance that mortality will improve, with life annuities being the most extreme case. Consequently, it is evident that **life annuities should not be handled in the same way as temporary annuities.**
- **The same longevity shock should not apply to all ages:** the younger you are, the more likely it is that mortality will improve.
- Since most longevity insurance products target people over 40, our analysis does not focus on younger ages. However, it is clear that a single shock should not be established for the entire insured portfolio; it should vary based on a combination of age and duration.
- **Looking for more simplicity, it could be an alternative to the standard formula a more granularity shock depending just on the age on every year of the projection.**

4.3. Variable analysis

Following common practice in the insurance market, a multiple regression study has been carried out to analyze the relationship between the age, duration and gender variables and the longevity shock variable. In other words, an analysis was performed on how the dependent variable can be explained by simultaneous treatment of the three independent variables.

The dependent variable “longevity shock” is predicted from the independent variables of age, gender and residual maturity of the contract. The following general equation for multiple linear regression is used:

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

based on the following premises:

- Linear relationship between the variables.
- The distribution of the dependent variable is conditioned by each possible combination of values of the independent variables.
- Variables are independent of each other. As a consequence, residuals will be independent of each other and comprise a random variable.
- Homogeneity of variance (homoscedasticity): The dependent variable variance that is conditioned by the values of the independent variables shows homogeneity.

Since Normality is not present in residuals from the multiple regression analysis, a Generalized Linear Model has been developed. This model does not require normality in errors and explains a variability of 80 percent. Hence, it has been possible to conclude that “maturity” and “age” are significant variables; and even though “gender”

has been deemed not relevant, if necessary it could be regarded as a confounding variable since it is closely associated with the response variable.

4.4. Application to the insurance market

This paper confirms that the shock recommended by the Standard Formula (20 percent) does not adequately reflect the longevity risk faced by life annuity portfolios. In general, this model generates a higher SCR for younger ages with life annuity, while the standard formula requires higher SCR for all other combinations; the difference in relation to older ages is particularly significant. Consequently, depending on the breakdown of the insured portfolio, **the standard model's longevity shock will over or underestimate the current longevity risk in almost all cases.** Insurance companies will have to make payments that do not match the current longevity risk contained in their balance sheets.

4.5. Prudence principle of the model

This model has been developed using methodology premises based on expert considerations. These premises were always the most prudent. Some of these premises were:

1. The benchmark population is from **seven EU countries**. So the projected improvement factor and longevity shocks are higher than the factors and shocks of several other EU countries.
2. The projection models weight the last observed years more than the remaining interval; higher results generate higher shocks.
3. **The selected European Longevity Index (ELI) is derived from the median of the improvement factor from the three projection models and**, as such, gives much more prudence because of the most conservative model.
4. **Improvement factors have been projected by generating multiple scenarios, and choosing the worst 99.5 percent among them all (ELI-99.5).**
5. **General population rather than insured population data has been used.** Even though life expectancy of the general population is lower, its improvement factors are higher since this population shows a higher ability to improve than the insured parties. **Consequently, purely biometric shocks will also be higher.**
6. The maximum age for human life (ω) has been set at 100 years. In this way, prudence is added to the model: from that age onward, mortality factors are very volatile and start decreasing; for some ages, they are even negative. Had another method been chosen, the resulting longevity shock would be much lower.

7. By choosing 1984 as the start year for the observation period, a prudence margin is achieved, because an **abrupt mortality reduction caused by epidemics and wars is excluded.**

4.6. Validation and Testing

Once the qualitative and quantitative analyses were successfully completed, the model was review and tested. Some of these tests and findings are listed below:

1. Results match our a priori expectations.
2. Mortality projection is a very complex and multifaceted process; for this reason, some of the most important aspects have been documented.
3. The model demonstrates usefulness for the purpose of its application to the insurance market.
4. **Backtesting was carried out and showed that this method is robust.** If applied to different population groups in different time periods and age ranges, the conclusions are the same.
5. As it happens with the standard formula's model, **this model is easy to apply.**
6. Variables used are statistically well documented and validated, ensuring the model's consistency.
7. This model **meets the Usability Test requirements**¹ from the supervisory body and, as such, can be used to manage and make business decisions. These requirements are:
 - Traceability
 - Transparency
 - Objectivity
 - Robustness
 - Easy to manage
 - Survivor trend
 - Continuity
 - Consistency
 - Simplicity
 - Universal

5. MORTALITY RISK

If we only pay attention to the methodology followed for the trend of longevity risk, in principle it could be thought that the study of longevity is also useful for mortality risk since we start from the general population and not insured and therefore the qbase that we stress later is the same.

¹ <http://www.llma.org/>

In other words, given that we have already generated the improvement factors that reflect the trend and volatility for longevity, as we are dealing with mortality instead of longevity, we would pick up instead of that increasing trend, and model it in a decreasing way.

That is, instead of taking the 99.5 percentile of the projection, EIOPA took 0.05 of the projection for mortality. This is what EIOPA presented in the last CP in November 2017 and it's not correct as we demonstrate in the following paragraphs.

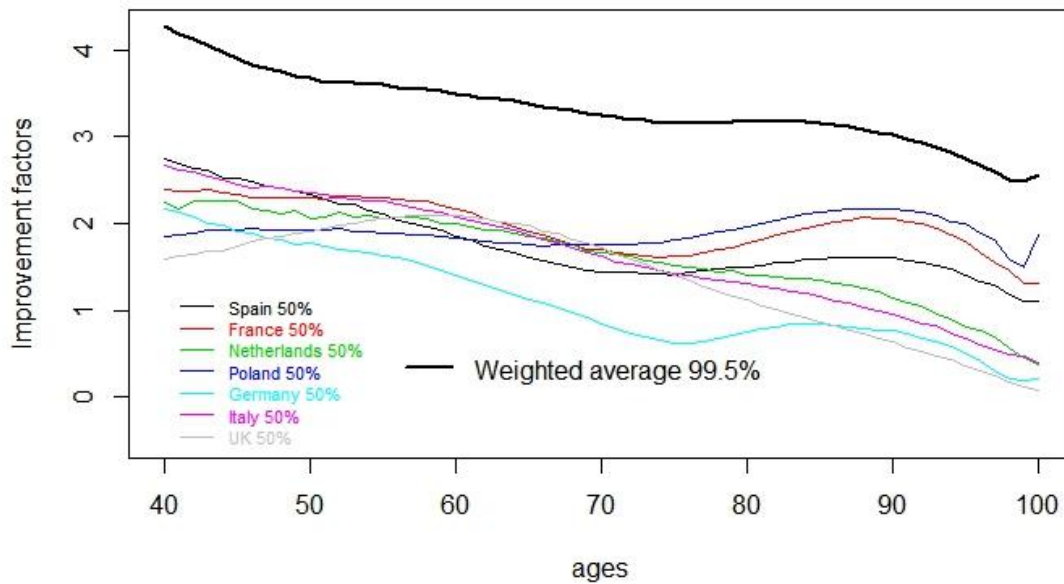
However, in a qualitative analysis and based on the experience analyzed of the seven countries of the study, where what we have done is to measure the trend risk, **given that the historical experience of all the countries analyzed is of increased survival, it is not transferable to mortality, because to measure the deviation of mortality we should not take into account that trend risk is zero (does not exist) based on that experience (there have been no mortality increases that have generated a trend).**

The analysis of the tendency to increase the risk of survival has shown us that in all the scenarios and models studied this is always growing and therefore **there is no experience in any country that any increase in mortality has consolidated a trend.** This conclusion applied to the risk of mortality clears us a first uncertainty, that the time axis is not relevant, so **to calibrate the mortality risk we should not use the trend analysis already done for longevity.**

So it's is an error to think that the mortality stresses for mortality risk are provided with the negative stresses for longevity risk. This conclusion is justified because the historical data taking into account for this studio never demonstrate a trend for mortality, just a trend for longevity because for any of the seven countries studied there is no an increase of mortality rates which generated a trend in the past, just aleatory and catastrophic movements are demonstrated in the past.

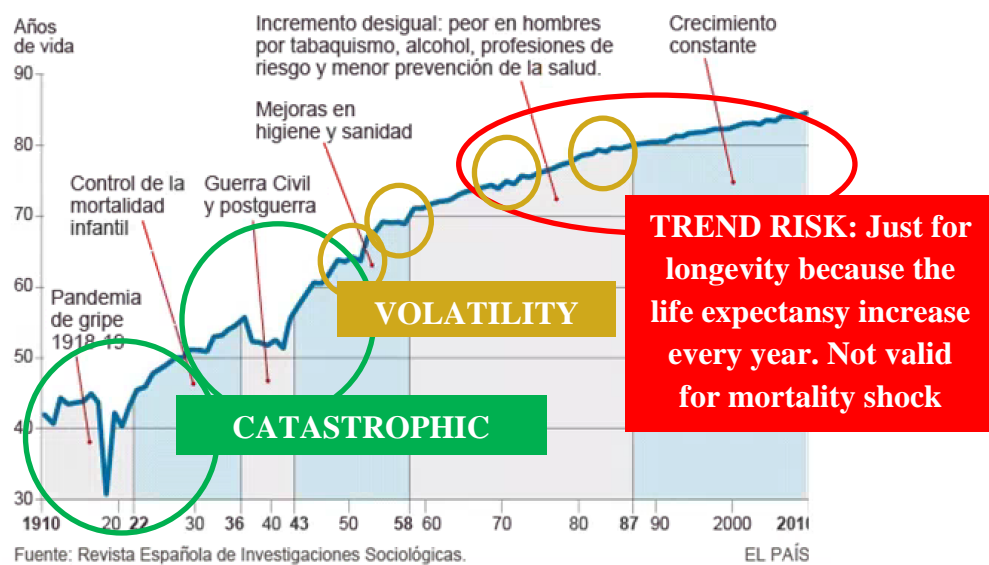
Here below we describe some examples about this conclusion:

In our model results presented in chapter 4, we can see that the mortality improvements are always positive, so the longevity trend is positive and mortality trend is negative. We can't reflect a negative trend for mortality, so we can say that it's equal to zero.



So, mortality risk has not a trend risk, just volatility, level and catastrophic risks as we can also demonstrate in the following graphs representing some of the countries selected in our study.

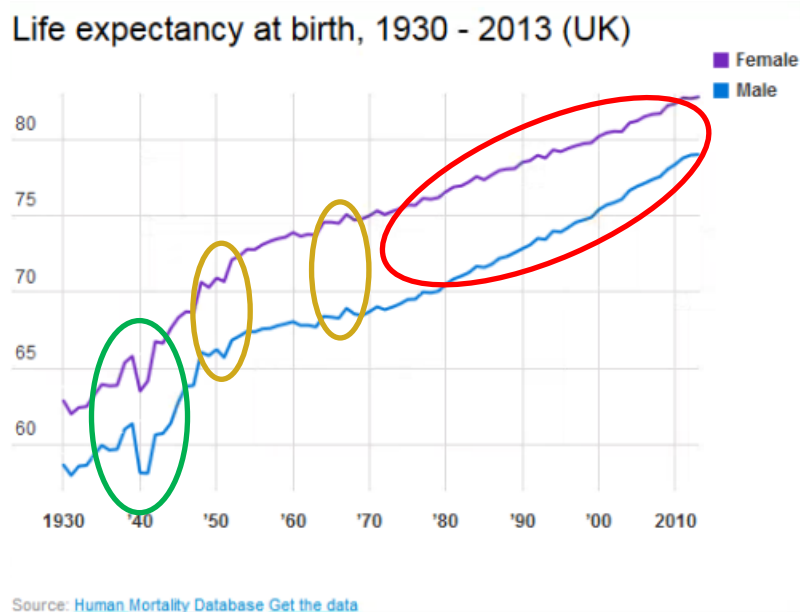
If we chose the evolution of the life expectancy at birth in Spain:



So, according to this qualitative analysis, we can conclude that for mortality risk we have the following subrisks:

- MORTALITY RISKS:
- Volatility
 - Catastrophic
 - ~~Trend~~

The same conclusion is valid for every countries analyzed in this paper, as we can also observe in the next UK life expectancy evolution graph.



Therefore, once this trend sub-risk has been ruled out, for an analysis of mortality shocks we should analyze the risk of volatility and level of mortality (base risk) whose charge should already be implicit in the experience table itself.

In addition, the other sub-risk, **the catastrophic one is already collected by the SCRcat** shock itself, and in case of taking it into account for the SCRdeath we would be doubling the capital charge.

Since it is normal that volatility occurs outside the confidence intervals used in the construction of the mortality tables, we take as a reference the methodology applied in the Spanish mortality tables (PASEM), which in turn take as a methodological reference the German mortality tables (DAV2008T).

In the graduation of the insured experience mortality tables, as is the case of the TABLE PASEM 2010, there are already included charges that mitigate the aforementioned risk of random fluctuation, also known as the volatility charge. The level of confidence applied in this charge will be that which indicates the level of fluctuation that a portfolio could present above the expected one.

In Spain, the PASEM table contemplated a confidence level of 99%, somewhat below the requirement established in Solvency II of 99.5%. However, this small difference

between the different levels of confidence (99% with respect to 99.5% of Solvency II) would be the randomness that complements the regulatory requirements. That is, this difference would be the shock to contemplate for the case of Spain.

Given that the tables already incorporate all the unexpected fluctuation in the risk of mortality, we should consider the risk of catastrophe, for example epidemics, since the Solvency II regulations already contemplate a specific sub-risk for it. It is another added difficulty because we should isolate and exclude that catastrophic charge or catastrophe to perform the mortality shock analysis.

The charge for level or model risk is also contemplated, in order to assess its adequacy, the graduation of the table should be recalibrated with several models and the impact should be assessed. However, the charge contemplated in the PASEM table is the same as the international actuarial literature used for other tables, as is the case of the German table DAV2008T that served as a methodological source for the Spanish table.

Below we analyze in detail the amounts of the different charges contemplated in the table PASEM 2010 of Spain, and whose conclusions are absolutely valid and extrapolated to the analysis that concerns us linked to the mortality shock under Solvency II:

• **Security charges for trend risk:** “As the table does not consider future improvements in mortality, it is proposed not to add an explicit security surcharge for future changes in mortality”.

- The PASEM table incorporates a total charge of 39.5%, where the volatility charge is 11.6% calculated at 99%. This level of confidence is similar to that established in Solvency II, so we are facing a first conclusion, the Spanish mortality risk table could be thought that the requirements of the stress regulation applied to the mortality risk were taken into account when elaborating (99.5% percentile).
- If the 99% charge is 11.6%, 99.5% would be 11.7%. That is, the maximum volatility of the PASEM by mortality would have been approximately **11.7%, still significantly lower than the 15% currently proposed by the Solvency II standard formula.**
- Therefore, maintaining the 15% surcharge that should be applied to the gross rates of the second order, before the surcharges that the qbase introduces in the tables themselves, is aligned with the Spanish experience and also retains a relevant prudential margin.
- Regarding the other charges contemplated, the justification is what is stated in point 10.4 of the technical note of the PASEM:

$$\text{Charge PASEM} = (1+11.6\%)*(1+10\%+15\%)-1=39.5\%$$

$$Qx(1 \text{ order}) = 1,395 * qx (2\text{nd order})$$

If we use the same methodology where we must apply the current 15% of Solvency II, that is, with volatility of 15% instead of 11.6%, the stressed charge would be:

$$\text{charge} = (1 + 15\%) (1 + 10\% + 15\%) - 1 = 43.75\%$$

$$Qx \text{ (1 order)} = 1.4375 * qx \text{ (2nd order)}$$

Compared this stressed charge of Solvency II, with respect to the PASEM surcharge of 39.5%, the final shock would be

$$43.75\% - 39.5\% = 4.2\%$$

It is worth remembering that this same methodology could be replicated on the German table DAV2008T, since the PASEM is a replica of this and that at the time it already envisaged the entry into force of Solvency II.

In our opinion, although the mortality shock for the case of Spain is 11.7%, to safeguard the principle of prudence, could be justified that the regulatory authorities kept 15% of mortality shock, but as we have seen, **there is no justification for this 15% shock to increase to 25%, based on a projected trend risk that does not exist in mortality but in longevity.**

Additionally, **regulators and supervisors should perhaps clarify that this mortality shock does not apply to the already reloaded reference qx (published for example in the PASEM), but rather to the second order rates (before surcharges) and not on the charges of the table or even less on the own qx already recharged.**

As a quantitative conclusion at least for the Spanish case, the current stress charge of 15% exceeds the technical calculation for Spain which is close to 12%. **The Solvency II charge applied to the entire European insurance market, for reasons of prudence, may justify the need for 15%, but in any case it justify an additional increase over the current 15% of mortality shock.**

If this EIOPA proposal were approved, it would have impact not just in the SCR for mortality risk and the total SCR for the life underwriting risk, in addition, technical provisions would increase (due to the increase in the risk margin).

There could be a second impact in the Matching Adjustment business, given that a very important part of the liabilities that apply this measure include mortality as underwriting risk, so, in order to apply it, it is necessary to demonstrate that the best estimate (BEL) of said portfolio does not increase by more than 5% after the application of the mortality shock.

When the mortality shock increases from 15% to 25%, the flows stressed by the mortality shock would be higher and it could happen that in some cases the limit of 5%

above was exceeded, which would result in the non-compliance of one of the required requirements by the regulations to apply the Matching Adjustment.

6. CONTRAST WITH EIOPA-CP-17-006 November 6, 2017

In the CP-17-006 published in November 2017, EIOPA answered about *the standard parameters for mortality and longevity risk in the life and health underwriting module. Specifically, EIOPA presented a more granular approach for longevity risk, with a view to a calibration differentiated by age groups. EIOPA also was asked to assess the costs and benefits of these more granular approaches, in particular in view of their risk sensitivity and complexity.*

6.1. Paragraphs

Below is shown what EIOPA said about these topics (blue) and the opinion of this working group about the main EIOPA's conclusions developed in this CP (black)

188. Most of stakeholders agreed with the methodology used by EIOPA in its discussion paper (EIOPA-CP-16/008) to derive longevity stresses.

Longevity risk: We present an alternative methodology but it is quite close to EIOPA's methodology. The main difference is that we implement the Pspline as a third method for modeling the trend of longevity risk. This third method is also implemented by ODCE and covers the whole methodology internationally accepted in the statistical and actuarial science for modeling the trend for longevity risk. Also, using the median of these we mitigate the risk model.

Mortality risk: As we presented in chapter 5, we are not agree with the methodology implemented by EIOPA. Mortality has not a trend risk as longevity has. We'll also develop this idea in the following points.

189. Most of stakeholders seem to be in favor of more granular stresses, for instance per age group.

Longevity risk: We are also in favor of a more granular stresses, **at least per age** for longevity risk. For simplicity we can forget the more granular stresses for the maturity of the contracts because the main longevity risk is concentrated in the long term annuities.

Mortality risk: We are not agree in a more granular stress because this risk does not present a trend risk as we justified in chapter 5. This mortality risk just presents volatility, level and catastrophic risks and it does not depends on the age or the maturity of the contract as longevity does.

191. EIOPA discusses the costs and benefits of using a more granular approach.

Longevity Risk: The benefits of using a more granular approach are obvious because the capital charge for entities is more appropriated to their portfolios in the sense that for elder ages of policyholders, the capital charge is lower than for young policyholders. In addition, it's important for companies to know that they can release capital charge in the future for the current portfolio and it has a huge impact in its profitability-risk binomial (RORAC).

With this new approach and these two ideas, entities we'll be encouraged to commercialize annuity products to the citizens.

The cost of this new granularity can't be a problem because companies must implement their life technical provisions depending on the age of the policyholders. So, a shock by age just suppose a decrease in their mortality rates of their survival or mortality tables. The implementation costs of a more granular shock must be close to zero.

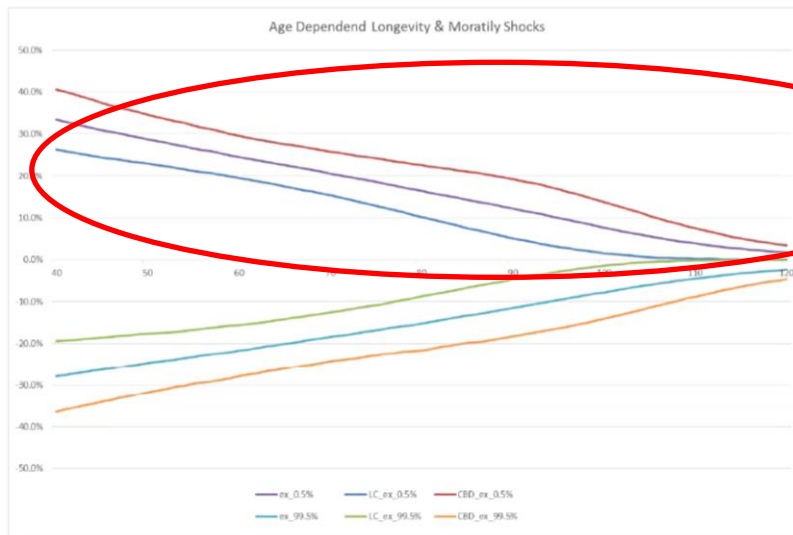
Mortality Risk: It has not a benefit for insurance companies because a more granular mortality shock or an increase of it from 15% to 25% does not reflect the reality of the risk assumed in their portfolio.

204. To demonstrate the proposed methodology two commonly used mortality models will be used. The results of both models will be combined to incorporate to some extent the effects of model risk.

Longevity risk: The main goal of this methodology is achieved because it demonstrated the useful of a more granularity shock. However we also demonstrate the benefits of applying the Pspline model.

Mortality risk: The Lee Carter, Cairns- Blake-Dowd and Pspline models are appropriated for the trend risk assessment, so they are not useful for mortality risk shock assessment.

230. The positives $h(x)$ provide for mortality stresses, while the negatives $h(x)$ provide for the longevity stresses.



Mortality risk: It's an error to think that the mortality stresses for mortality risk are provided with the negative stresses for longevity risk. This conclusion is justified because the historical data taking into account for this studio never demonstrate a trend for mortality, just a trend for longevity because for any of the seven countries studied there is an increase of mortality rates which generated a trend in the past, just aleatory and catastrophic movements are demonstrated in the past.

In other words, given that we have already generated the improvement factors that reflect the trend and volatility for longevity, as we are dealing with mortality instead of longevity, we would pick up instead of that increasing trend, and model it in a decreasing way. That is, instead of taking the 99.5 percentile of the projection, we would take 0.05 of the projection. This is what EIOPA presented in the last CP in November 2017.

In chapter 5 we describe some qualitative examples about this conclusion

231. Both for mortality and longevity stresses, one can observe the results stemming from the Lee-Carter and from the CBD model. The final stresses are provided by the average of both models.

It could be correct for longevity risk but not for mortality risk. Mortality risk are represented just by volatility, level and catastrophic risks included in the 15% mortality shock and catastrophic shock under the underwriting risk of Solvency II (see more details in chapter 5).

235. The stresses defined above are “equivalent stresses”. The mortality and longevity risk that affects the own funds corresponds to changes in the mortality rates used in the best estimate calculation. These mortality rates are defined via a mortality table which gives, for each entry age, the 1-year probability of dying every year of the best estimate projection until the age limit (e.g. 120 years old). In theory, there should be one stress for each age and each year of the projection.

However, given the complexity it would introduce, we define an equivalent stress that is applied to all mortality rates of an insured person over the projection.

Longevity Risk: The proper implementation of this methodology is in this way, one stress for each age and year of the projection. It is not necessary more complex than currently 20% for every ages and years of the projection.

In addition, companies must implement their current mortality rates for the best estimate assessment by age and year of the projection, so this new shock does not incur in any complexity or additional cost for companies and the benefits of this implementation are so huge in terms of capital charge and RORAC.

Furthermore, this kind of methodology is currently used under the framework of Solvency II for the calibration and implementation of Spread risk for corporative bonds in the sense that the capital charge depends on the maturity and rating of these bonds.

Mortality risk: There is not any sense for increasing mortality shock or applying a trend risk for this underwriting risk.

235. The outcome is the same as if we would have defined a mortality stressed table.

Longevity risk: It's not true, the output is just the same for a portfolio with an average age of 60 years old, but the reality of the annuity portfolios all over Europe is that the average age is about 73 years old, so this unique shock as an alternative to a granular shock for longevity risk obliges to the life insurance companies to overcharge their capital sources. On the other hand, for a portfolio with an average age under 60 years old it underestimate the real risk the company assume in it's business and so expose to future solvency problems.

One of the main consequences of applying a unique shock for longevity risk is that companies are definitely discourage to commercialize and offer annuity contracts to the citizens.

Mortality risk: The shock based on volatility and level risk is not expose to the age and year of the policyholders.

236. We observe that for age close to 60 years old, the longevity stress of 20 % is confirmed. Given the uncertainties described above that are not fully taken into account, the 20 % stress appears appropriate.

We demonstrated along this technical note that this approach is not correct

237. For mortality stresses, the results provide for a stress of 25 % for age 60 years old, which is higher than the current stress of 15 %. EIOPA would welcome

further evidence from stakeholders on the appropriateness of the mortality stress factor.

We demonstrated with several evidences in this technical note, specifically in chapter 5 and in precedent paragraphs that this approach is not correct for mortality risk.

239. As one can observe in the results displayed in the graphs that the stresses are different depending on the age of the insured person. In particular younger persons would need to have higher stresses given that they benefit more from future mortality improvements than older persons. It appears that more granular stresses per age group would provide for a more risk-sensitive SCR calculation.

We agree with this idea but just for longevity risk, not for mortality risk.

243. Finally, one of the key objectives of this SCR review is to simplify, where possible, the standard formula. Increasing granularity in an arbitrary manner compared to the *Best Estimate* would cause more complexity and implementation costs than benefits.

Longevity risk: We do not agree with this conclusion. Companies must assess their current best estimate age by age and year by year, so this more granularity for longevity risk do not suppose an increase of costs for life entities.

The benefits derived on this more granularity approach for longevity risk are so important for companies in terms of future benefits and reduction of capital charges, but also for citizens because life companies are going to be more attracted to offer annuity contracts to the citizens.

Maintaining a unique 20% shock for mortality risk could also be a problem for entities with a huge exposure to annuities for young policyholders because these entities are underestimating the real longevity risk they assume in their business.

6.2. Conclusions

Mortality and longevity stresses

244. EIOPA advises to maintain the 20 % stress for longevity risk, which appears appropriately calibrated.

245. EIOPA advises to increase the mortality stress factor for mortality risk to 25 %, so that it is appropriately calibrated.

Granularity

246. EIOPA does not advise improving the granularity of the mortality and longevity stresses: the added complexity due to the interaction with the *Best Estimate* model points, the implementation costs and the fact that it would not be in line with simplifying the standard formula provide for more arguments against than in favour.

Longevity Risk:

In order with the conclusions exposed in this technical note, the longevity shock is expected to be calculated with much more granularity.

There should be one stress for each age and each year of the projection. In particular younger persons would need to have higher stresses given that they benefit more from future mortality improvements than older persons. It appears that more granular stresses per age would provide for a more risk-sensitive SCR calculation.

The possible cost of implementation of this granularity does justify the need of application this granularity. The stakeholders demand this more granularity depending on the age and year as the own EIOPA-CP exposes.

Mortality risk:

In this technical note, we advise that an increase of the mortality stress factor for mortality risk to 25 % is an error of interpretation of the sub-risks around the mortality risk. This conclusion is because mortality risk is not affected by trend risk as we argue in chapter 5 and in precedent paragraphs. So the only sub-risks for mortality are catastrophic, level and volatility which calibration with a 99,5 of level of confidence is about 12% as we demonstrated in chapter 5.

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