CUADERNOS
DE LA FUNDACIÓN

C / 223

Contributions to Risk Analysis: RISK 2018

José María Sarabia
Faustino Prieto
Montserrat Guillén

(editors)

Fundación
MAPFRE

Área de Seguro y Previsión Social

Contributions to Risk Analysis: RISK 2018

José María Sarabia (Universidad de Cantabria)
Faustino Prieto (Universidad de Cantabria)
Montserrat Guillén (Universidad de Barcelona)
Editors

Fundación MAPFRE

Paseo de Recoletos, 23
28004 Madrid (España)

www.fundacionmapfre.org

ISBN: 978-84-9844-683-8
Depósito Legal: M-9011-2018
Maquetación y producción editorial: Arias Montano Comunicación
www.ariasmontano.com

PRESENTATION

Since 1975 FUNDACIÓN MAPFRE has been carrying out general interest activities for society in different professional and cultural areas, in addition to many actions aimed at improving the economic and social conditions of the most disadvantaged people and sectors of society.

We work within the Insurance and Social Protection Area with the aim of promoting and disseminating knowledge on, and the cultures of, Insurance and Social Protection

In terms of the activities aimed at society in general, we create free and universal content on insurance, which we make available via the Seguros y Pensiones para Todos (Insurance and Pensions for Everyone) website. We organize training courses and educational and awareness-raising activities for teachers, as well as work-shops for schoolchildren and free group visits to the Insurance Museum. We also publish informative guides to help people learn the basics of insurance.

In addition to this informative work, we support research by preparing reports on insurance markets and other subjects of interest, awarding research grants related to insurance and social protection, publishing books and surveys on insurance-related matters and organizing conferences and seminars. Our commitment to knowledge is embodied in our specialized Documentation Center that supports all our activities and is open to the general public.

These activities include the publication of this book, which contains articles that were presented at the International conference Contributions to Risk Analysis RISK 2018, held in Santander, from April 25 to 27, 2018.

All information regarding our activities is available online and accessible to users around the world quickly and efficiently via our website: www.fundacionmapfre.org

Fundación MAPFRE

PRESENTACIÓN

Desde 1975 FUNDACIÓN MAPFRE desarrolla actividades de interés general para la sociedad en distintos ámbitos profesionales y culturales, así como acciones destinadas a la mejora de las condiciones económicas y sociales de las personas y de los sectores menos favorecidos de la sociedad.

El Área de Seguro y Previsión Social trabaja con el objetivo de promover y difundir el conocimiento y la cultura del Seguro y la Previsión Social.

En cuanto a las actividades orientadas hacia la sociedad en general, creamos contenidos gratuitos y universales en materia de seguros que divulgamos a través de la página web Seguros y Pensiones para Todos. Organizamos actividades educativas y de sensibilización mediante cursos de formación para el profesorado, talleres para escolares y visitas gratuitas para grupos al Museo del Seguro. Asimismo publicamos guías divulgativas para dar a conocer aspectos básicos del seguro.

Además de esta labor divulgativa, apoyamos la investigación mediante la publicación de informes sobre mercados aseguradores y otros temas de interés, la concesión de ayudas para la investigación en seguros y previsión social, la publicación de libros y cuadernos de temática aseguradora y la organización de jornadas y seminarios. Nuestro compromiso con el conocimiento se materializa en un Centro de Documentación especializado que da soporte a todas nuestras actividades y que está abierto al público en general.

Dentro de estas actividades se encuadra la publicación de este libro, que recoge las ponencias presentadas en el congreso international *Contributions to Risk Analysis: RISK 2018*, celebrada en Santander del 25 al 27 de abril de 2018.

Todas nuestras actividades se encuentran disponibles y accesibles en internet, para usuarios de todo el mundo, de una manera rápida y eficaz, a través de nuestra página web: www.fundacionmapfre.org

Fundación MAPFRE

FOREWORD

The 7th Workshop on Risk Management and Insurance Research (RISK, 2018) held in Santander (Spain) from April 25 to April 27, 2018 is an international forum for disseminating recent advances in the field of Risk Analysis, organized by the Department of Economics of the University of Cantabria, in collaboration with the Research Group on Risk in Insurance and Finance of the University of Barcelona.

In line with previous conferences, RISK 2018 provides a platform to share new ideas, research results and development experiences in actuarial science and finance. In this edition, Prof. Dr. Enkelejd Hashorva (Université de Lausanne) is the keynote speaker of the inaugural session.

Workshops on Risk Management and Insurance (RISK) have been taken place biannually in Spain since 2005. More information on the workshop and also on the previous meetings can be found at: http://www.risk2018.unican.es/

There were around 50 submissions. A selection of short papers is included in this book. All of them went through a double peer-review process.

Risk analysis is a field of research that studies measurement of adverse events, its prevention and mitigation. The fundamental aspects of the quantitative analysis of risks are fundamentally two: the probability of occurrence of rare phenomena and the severity of the losses. The works published in this volume address this subject with many different perspectives, ranging from the analysis of probability distributions to the use of massive databases. They include classic actuarial science models, mathematical methods, predictive modeling, or the study of dependencies.

The number research groups in the world working on issues related to risks has not ceased to increase in recent years. Since 2005, a small group of Spanish researchers have been meeting once every two or three years to discuss topics related to risk analysis with a high potential for expansion.

The members of the organizing committee would like to thank the scientific committee for their valuable help to make this conference a success. We want to acknowledge participants, presenters, chairpersons and all authors in general for their contribution to the development of new advances in the analysis of risk. We also want to thank our invited speaker for accepting to play a central role in this conference.

We are indebted to Fundación Mapfre for sponsoring this publication and to all other sponsors for their generous support. In particular we thank the Universidad de Cantabria for providing us with an excellent location to celebrate the working sessions, we also thank the Santander Financial Institute (SANFI) and the support received from the Spanish Ministry of Economy FEDER grants in projects ECO2016-76203-C2-1-P and ECO2016-76203-C2-2-P (Proyectos de I+D correspondientes al Programa Estatal de Fomento de la Investigación Científica y Técnica de Excelencia).

Santander, April 2018

EDITORS

José María Sarabia (Universidad de Cantabria)
Faustino Prieto (Universidad de Cantabria)
Montserrat Guillén (Universidad de Barcelona)

PRÓLOGO

El 7º Congreso sobre gestión de riesgos e investigación en seguros (RISK 2018) celebrado en Santander (España) del 25 al 27 de abril de 2018 es un foro internacional para difundir los avances recientes en el campo del análisis de riesgos, organizado por el Departamento de Economía de la Universidad de Cantabria, en colaboración con el Grupo de Investigación de Riesgo en Seguros y Finanzas de la Universidad de Barcelona.

En línea con los congresos anteriores, RISK 2018 proporciona una plataforma para compartir nuevas ideas, resultados de investigación y experiencias de desarrollo en ciencias actuariales y financieras. En esta edición, el Prof. Dr. Enkelejd Hashorva (Universidad de Lausana) es el orador principal de la sesión inaugural.

Desde 2005 se celebran congresos de Gestión del Riesgo y Seguros (RISK) bianualmente en España. Toda la información sobre este evento y las ediciones anteriores, se puede encontrar en: http://www.risk2018.unican.es/

En esta ocasión se presentaron 50 ponencias, de las cuales, una selección se incluye en este libro. Todas ellas seleccionadas a través de un proceso doble de revisión por pares.

El análisis de riesgos es un campo de investigación que estudia la medición de eventos adversos, su prevención y mitigación. Los aspectos fundamentales del análisis cuantitativo de riesgos son fundamentalmente dos: la probabilidad de ocurrencia de fenómenos raros y la gravedad de las pérdidas. Los trabajos publicados en este volumen abordan este tema desde muchas perspectivas diferentes, que van desde el análisis de distribución de la probabilidad hasta el uso de bases de datos masivas. Incluyen modelos clásicos de ciencia actuarial, métodos matemáticos, modelos predictivos o el estudio de las dependencias.

El número de grupos de investigadores en el mundo que trabajan las cuestiones relacionadas con el riesgo no ha dejado de aumentar en los últimos años. Desde 2005, un pequeño grupo de investigadores españoles se ha reunido una vez cada dos o tres años para debatir temas relacionados con el análisis de riesgos con un alto potencial de expansión.

El comité organizador quisiera expresar su agradecimiento al comité científico por su valiosa ayuda para que esta conferencia sea un éxito. Queremos reconocer a los participantes, presentadores, presidentes y todos los autores en general por su contribución al desarrollo de nuevos avances en el análisis del riesgo. Especialmente queremos agradecer a nuestro orador invitado por aceptar desempeñar un papel principal en este congreso.

Estamos en deuda con Fundación Mapfre por patrocinar esta publicación y con todos los demás patrocinadores por su generoso apoyo. En particular, agradecemos a la Universidad de Cantabria por brindarnos una excelente ubicación para celebrar las sesiones de trabajo, también agradecemos al Instituto Financiero Santander (SANFI) y al apoyo recibido del Ministerio de Economía español. Becas FEDER en proyectos ECO2016-76203-C2 -1-P y ECO2016-76203-C2-2-P (Proyectos de I+D correspondientes al Programa Estatal de Fomento de la Investigación Científica y Técnica de Excelencia).

Santander, Abril 2018

EDITORES

José María Sarabia (Universidad de Cantabria)
Faustino Prieto (Universidad de Cantabria)
Montserrat Guillén (Universidad de Barcelona)

COMMITTEES

ORGANIZING COMMITTEE CHAIRS

Montserrat Guillén Estany, Universitat de Barcelona, Spain.
José María Sarabia Alegría, Universidad de Cantabria, Spain.

LOCAL ORGANIZING COMMITTEE

Vanesa Jordá Gil, Universidad de Cantabria, Spain.
Faustino Prieto Mendoza, Universidad de Cantabria, Spain.
Lorena Remuzgo Pérez, Universidad de Cantabria, Spain.
Carmen Trueba Salas, Universidad de Cantabria, Spain.

SCIENTIFIC COMMITTEE

Pilar Abad Romero, Universidad Rey Juan Carlos, Spain.
Alejandro Balbás de la Corte, Universidad Carlos III de Madrid, Spain.
Catalina Bolancé Losilla, Universitat de Barcelona, Spain.
Enrique Calderín Ojeda, University of Melbourne, Australia.
Ana Debon Aucejo, Universitat Politecnica de Valencia, Spain.
José M. Feria Domínguez, Universidad Pablo Olavide, Spain.
Emilio Gómez Déniz, Universidad de Las Palmas de Gran Canaria, Spain.
Montserrat Guillén Estany, Universitat de Barcelona, Spain.
Antonio J. Heras Martínez, Universidad Complutense de Madrid, Spain.
Enrique J. Jiménez Rodríguez, Universidad Pablo de Olavide, Spain.
Luís Ortíz Gracia, Universitat de Barcelona, Spain.
Faustino Prieto Mendoza, Universidad de Cantabria, Spain.
Juan Manuel Rodríguez Poo, Universidad de Cantabria, Spain.
José María Sarabia Alegría, Universidad de Cantabria, Spain.
Miguel Angel Sordo Díaz, Universidad de Cádiz, Spain.
Francisco José Vázquez Polo, Universidad de Las Palmas de Gran Canaria, Spain.
José Luis Vilar Zanón, Universidad Complutense de Madrid, Spain.

TABLE OF CONTENTS

ESTIMACIÓN DE LA CONCURRENCIA DE PENSIONES PARA LAS GENERACIONES NACIDAS DURANTE EL BABY-BOOM ESPAÑOL

Estefanía Alaminos
Organización Internacional del Trabajo - OIT, Departamento de Estadística, Suiza
alaminosestefania@gmail.com

Mercedes Ayuso
Universidad de Barcelona, Riskcenter, Departamento de Econometría, España
mayuso@ub.edu

ABSTRACT

El reto que supone la llegada de las abultadas generaciones nacidas durante el *baby-boom* español a la clase pensionista para la sostenibilidad del sistema público de pensiones está desencadenado diversos debates sociales en torno a la financiación de las pensiones contributivas y al diseño de algunas figuras tales como la pensión de viudedad. Uno de los problemas que se deja patente en este trabajo, así como en estudios anteriores realizados por las autoras, es el hecho de que en un futuro próximo habrá un elevado número de individuos concurrentes en pensión de jubilación y viudedad, lo que incrementará el gasto público en pensiones de manera notable. Mediante cálculo actuarial se demuestra como el coste de la concurrencia en ambas pensiones sufrirá un gran incremento con las próximas generaciones de pensionistas, siendo este hecho debido mayoritariamente a las pensionistas femeninas.

1. INTRODUCCIÓN

En los próximos años las generaciones nacidas durante el *baby-boom* español alcanzarán la edad ordinaria de jubilación. Con ellas, el gasto público que la Seguridad Social afrontará por este concepto se verá incrementado, no sólo por el hecho del aumento en el número de pensiones de jubilación a pagar, sino también por el incremento en las pensiones de viudedad devengadas. El número de pensionistas de la Seguridad Social mayores de 65 años se incrementará por varios motivos. Primero, por la llegada a la edad legal de jubilación de abultadas generaciones (Abellán *et al*., 2017; OECD, 2015; Määttänen *et al*., 2014). Segundo, el incremento del número de mujeres con carreras de cotización completas hará que el número de pensionistas de jubilación femeninas se incremente (Alaminos y Ayuso, 2016). Este último punto incrementará a su vez el número de pensionistas de viudedad masculinos, que serán susceptibles de percibir la correspondiente pensión de viudedad en caso de fallecimiento de la esposa. El aumento del número de pensionistas de jubilación de sexo femenino, y el aumento del número de pensionistas masculinos de viudedad, llevará asociado un incremento en el número de pluripensionistas de jubilación y viudedad (Alaminos y Ayuso, 2016).

Actualmente uno de los debates que ha suscitado la sostenibilidad del sistema de pensiones gira en torno a la fuente de financiación de la pensión de viudedad. Una de las propuestas es la de que dicha pensión deje de ser financiada mediante contribuciones –Seguridad Social– y pase a financiarse vía presupuestos generales. Esto supondría un transvase entre fuentes de financiación, pero no solventaría el problema que causa el tener que afrontar un gasto tan elevado por tal partida y que además se pronostica que aumentará de manera creciente en los próximos años. En una sociedad como la actual, donde cada vez existen más matrimonios en los que ambos cónyuges son económicamente independientes, la figura de la de la pensión de viudedad se pone en entredicho, al haberse perdido la razón por la que fue creada (solventar la situación de necesidad que le surgía a la mujer al fallecer el marido del que dependía económicamente).

Este estudio se ha realizado con la finalidad de estimar el monto total que causarán las generaciones que causen alta en jubilación y viudedad nacidas durante el baby-boom español. Dado que el fenómeno demográfico del *baby-boom* se fecha entre 1957 y 1970 (Castro, 2000), si tomamos una edad ordinaria de jubilación de 65 años[1], estas generaciones comenzarán a jubilarse a partir del año 2022, terminándose por incorporar a dicha etapa en el año 2035 la cohorte nacida en 1970. El trabajo se estructura como sigue. En el apartado 2 se presenta la metodología y los datos, en el apartado 3 los principales resultados obtenidos, y en el apartado 4 las conclusiones derivadas del presente estudio.

---

1 Suponemos que los individuos cumplen con los requisitos legales establecidos para poder jubilarse a los 65 años.

2. METODOLOGÍA Y DATOS

El desarrollo metodológico empleado en este trabajo parte de calcular el número de individuos ocupados casados que llegan vivos a la edad de jubilación, para lo que se han utilizado probabilidades temporales de supervivencia ${}_{n}p_{x}^{c}$ [2]. Las probabilidades utilizadas han sido obtenidas de las tablas de mortalidad de la población asegurada PASEM2010, por ser las tablas generalizadas de uso común más recientes de las que se dispone.

Otro de los aspectos metodológicos de nuestro estudio consiste en el cálculo del valor actual actuarial a percibir por pensionista de jubilación o por pluripensionista de jubilación y viudedad. Para ello se ha seguido el modelo actuarial multiestado casado y viudo desarrollado en Alaminos (2017).

En lo referente a la información utilizada respecto a los ocupados, se han recopilado, en primer lugar, datos sobre el número medio[3] anual de ocupados casados por edad y sexo en 2015 de la Encuesta de Población Activa (INEbase, 2016). Las cohortes (edades quinquenales) analizadas aparecen en la Tabla 1 y recogen la generación que se jubilará en 2020 (colectivo de 60-64 años en 2015)[4], la que se jubilará en 2025 (colectivo de 55-59 años), los ocupados que se jubilarán en 2030 (con 50-54 años) y los que terminarán por alcanzar la edad de 65 años en el año 2035 (colectivo con 45-49 años).

Para estimar los importes iniciales medios que se devengarán en los próximos años por clase de pensión y sexo, se han obtenido del Anuario de Estadísticas de la Seguridad Social los importes iniciales medios en pensión de jubilación y viudedad para hombres y

2 Correspondiendo la notación ${}_{n}p_{x}^{c}$ a la probabilidad de que un individuo casado de edad *x* sobreviva a la edad *x*+*n*. Nótese que a diferencia de lo que hacemos en Alaminos (2017) donde estimamos las probabilidades de supervivencia y muerte para la población española mayor de 65 años según estado civil (casado-viudo), ahora al trabajar con población inferior a 65 años no establecemos dicha diferenciación, por las bajas probabilidades de muerte esperadas para ambos colectivos.

3 Al ser la Encuesta de Población Activa de periodicidad trimestral, el dato anual está referido a la media de los cuatro trimestres dentro del año.

4 Téngase en cuenta que se ha tomado el inicio de cada quinquenio.

mujeres. Las series temporales para las que se han obtenido los datos comprenden el periodo 2006-2016, realizándose un análisis de las mismas en Alaminos y Ayuso (2016)[5].

En cuanto al tope máximo a percibir en concepto de pensiones públicas por un mismo individuo, se ha supuesto que se incrementará en el 0,25% de revalorización anual que marca la ley y que se ha aplicado en los últimos años[6].

Tabla 1. Número de ocupados casados por sexo y edad en 2015, y número estimado de supervivientes por género de dichos colectivos

	Ocupados casados 2015		Año de jubilación	Supervivientes	
	Hombres	Mujeres		Hombres	Mujeres
60-64	414.150	244.450	2020	410.094	243.276
55-59	781.225	512.075	2025	768.410	507.715
50-54	949.950	693.375	2030	930.455	685.785
45-49	1.030.000	751.125	2035	1.006.402	741.725

3. RESULTADOS

Los resultados de nuestro estudio se muestran en la Tabla 2 y en la Figura 1, donde aparece el valor actual actuarial por cohorte de pensionistas que alcanzarán la edad ordinaria de jubilación como casados y bajo el supuesto de que cumplen todos los requisitos mínimos exigidos para percibir la pensión contributiva correspondiente[7].

[5] Siendo las rectas de ajuste estimadas y utilizadas en nuestros cálculos las siguientes: Hombres: $y = 40{,}136x + 1109; R^2 = 0{,}9614$. - Mujeres: $y = 59{,}034x + 529{,}35; R^2 = 0{,}9931$. Viudedad.- Hombres: $y = 10{,}456x + 356{,}14; R^2 = 0{,}9341$. - Mujeres: $y = 17{,}895x + 525{,}34; R^2 = 0{,}9244$.

[6] El tope máximo estimado mediante el supuesto de revaloración del 0,25% resulta: 2.593,05€ (2020); 2.625,62€ (2025); 2.658,61€ (2030); 2.692,01€ (2035).

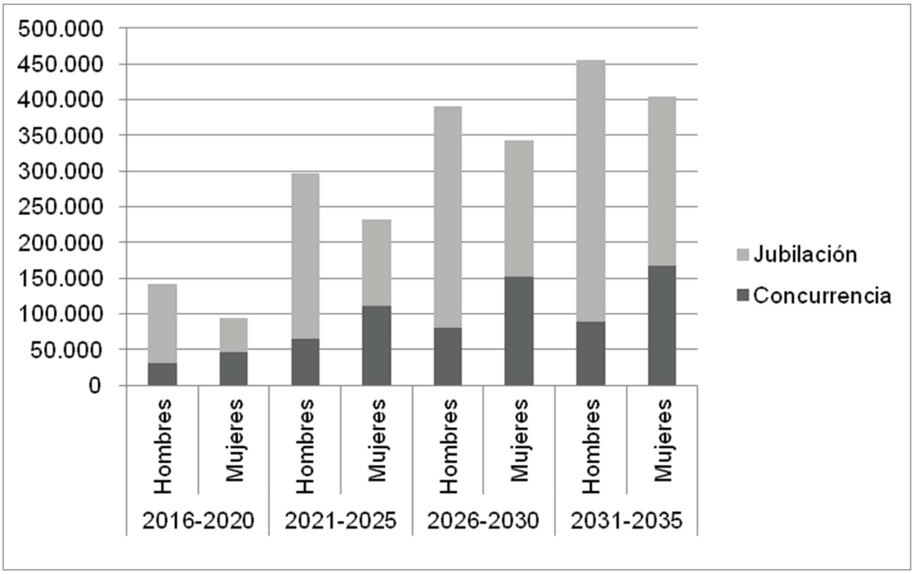
[7] Se ha supuesto que ambos cónyuges son económicamente independientes, por lo que en caso de fallecer uno, el cónyuge superviviente percibirá la correspondiente pensión de viudedad. Respecto a los importes utilizados se han supuesto siempre escenarios medios de pensión.

Tabla 2. Valor actual actuarial por cohorte de pensionistas

Periodo de jubilación \ Coste cohorte (en millones de €)		Coste concurrencia (en millones de euros)	Peso de la concurrencia sobre el coste total	Coste jubilación (en millones de euros)	Peso de jubilación sobre el coste total	Coste total
2016-2020	Hombres	30.844,73	21,78%	110.798,89	78,22%	141.643,61
	Mujeres	46.478,71	49,22%	47.954,15	50,78%	94.432,86
2021-2025	Hombres	64.328,40	21,71%	231.957,76	78,29%	296.286,16
	Mujeres	111.218,00	47,90%	120.958,64	52,10%	232.176,64
2026-2030	Hombres	81.044,67	20,71%	310.358,17	79,29%	391.402,84
	Mujeres	152.207,18	44,27%	191.583,99	55,73%	343.791,18
2031-2035	Hombres	88.761,02	19,45%	367.581,48	80,55%	456.342,50
	Mujeres	166.690,85	41,22%	237.713,57	58,78%	404.404,42

Se demuestra como el monto total por concurrencia de pensiones aumenta notablemente tanto para hombres y mujeres a medida que vamos avanzando en generaciones. El hecho de que, en términos relativos, el peso de la concurrencia sobre el coste total disminuya en el tiempo para ambos géneros (aunque más para las mujeres) se debe al elevado peso que la jubilación tiene sobre el coste total de la cohorte, y a la relación que tiene la evolución de las cuantías medias iniciales por pensión con el porcentaje de revalorización anual de las pensiones máximas.

Figura 1. Valor actual actuarial (en millones de euros) por cohorte de pensionistas según año de jubilación y por sexo.



Si el ritmo de crecimiento de las cuantías iniciales de pensiones de jubilación se mantiene, y sigue siendo notablemente superior al crecimiento anual de las pensiones máximas, serán numerosas las personas que alcancen el máximo de pensión, a lo que contribuirá significativamente la adición de las pensiones de viudedad, calculadas sobre bases cada vez más elevadas, tanto para hombres como para mujeres.

4. CONCLUSIONES

Uno de los principales hechos que se deberían tener en cuenta en una futura reforma del sistema de pensiones contributivas es la concurrencia de pensiones. Gran parte de los individuos que formarán parte de las próximas generaciones de pensionistas contarán con historiales de cotización completos, lo que ocasionará que en el momento de la jubilación éstos perciban la pensión de jubilación por la que han cotizado. Dado que este hecho se da tanto en hombres como en mujeres, en el futuro tal y como se deja patente en Alaminos y Ayuso (2016), el número de individuos concurrentes en pensión de jubilación y viudedad aumentará, pensiones que se devengarán con unos importes mayores a las pensiones que se inician actualmente y que se pagarán durante más años debido al incremento en la longevidad.

El aumento en el pago de pensiones concurrentes de jubilación y viudedad no sólo incrementará el coste que tendrá que hacer frente la Seguridad Social en los próximos años por tal concepto[8], sino que además actuará en contra del efecto redistributivo que se espera de nuestro sistema de protección social. Esta afirmación se apoya en el hecho de que en el futuro se devengarán pensiones de viudedad mayoritariamente a mujeres (u hombres) que ya cuenten con una pensión de jubilación. Sin embargo, de no producirse reformas en este sentido, existirán grupos poblacionales, como el de la población soltera, que únicamente percibirán pensión de jubilación, habiendo contribuido, al igual que los individuos casados económicamente independientes, a financiar todas las prestaciones de la seguridad social durante su vida activa. Cambiar los porcentajes asociados a la determinación de la pensión de viudedad puede cobrar sentido si incluimos las condiciones económicas a las que se accede a la jubilación en función de la vida laboral de la persona.

El trabajo demuestra además como la probabilidad de alcanzar el tope máximo de pensión gracias al efecto derivado de la concurrencia de pensiones aumenta significativamente una vez las generaciones del *baby-boom* alcancen la edad ordinaria de jubilación, especialmente en el caso de las mujeres. Determinar porcentajes de cálculo de la pensión de viudedad que tengan en cuenta las diferencias intrageneracionales por factores sociodemográficos (nivel de renta y estado civil, entre otros, ver Ayuso *et al*. 2017) constituye nuestra línea actual de investigación.

AGRADECIMIENTOS

M. Ayuso agradece la ayuda recibida en el proyecto del Ministerio de Economía y Competitividad y FEDER ECO2015-66314-R

[8] En el caso de que la pensión de viudedad se financie vía impuestos supondrá un cambio en la fuente de financiación, pero el coste asociado a dicha partida no variará.

REFERENCIAS

Abellán García, A., Ayala García, A., y Pujol Rodríguez, R. (2017). “Un perfil de las personas mayores en España, 2017. Indicadores estadísticos básicos”. Madrid, Informes Envejecimiento en red nº 15.

Alaminos (2017) “Heterogeneidad en la mortalidad y su impacto en el Estado de Bienestar: pensiones y dependencia”. Tesis doctoral. Universidad de Barcelona.

Alaminos, E., Ayuso, M. (2016). “Modelo actuarial multiestado para el cálculo de probabilidades de supervivencia y fallecimiento según estado civil”. Anales del Instituto de Actuarios Españoles, 3ª Época, 22, 41-71.

Ayuso, M., Bravo, J. M., y Holzmann, R. (2017). Adressing longevity heterogeneity in pension scheme design. Journal of Finance and Economics, 61, 1, 1-21.

Castro, T. (2000). “Un caso especial: la generación del baby-boom”. En Las personas mayores en España. Informe 2000. Observatorio de Personas Mayores 101-108.

Määttänen, N., Võrk, A., Piirits, M., Gal, R. I., Jarocinska, E., Ruzik, A., y Nijman, T. (2014). “The Impact of Living and Working Longer on Pension Income in Five European Countries: Estonia, Finland, Hungary, the Netherlands and Poland”. NETSPAR Discussion Paper Series.

OECD (2015). “Pensions at a Glance 2015: OECD and G20 indicators”. Paris: OECD Publishing.

TRAFFIC INTENSITY AND ESTIMATION OF THE PREVALENCE OF ALCOHOL-IMPAIRED DRIVERS IN RANDOM ROADSIDE SURVEYS

Manuela Alcañiz Zanón
Universitat de Barcelona, Dept. Econometría, Estadística y Economía Aplicada
Riskcenter-IREA, España, malcaniz@ub.edu

ABSTRACT

Psychoactive substances, alcohol and illicit drugs in drivers are one of the main causes of traffic accidents in the world. Here, an analysis of roadside surveys is presented for a random sample of drivers in Catalonia (Spain). The estimation of the prevalence of alcohol suffers from many methodological problems due to the need to introduce information in the sample design in order to guarantee the representativeness of the observed data. A comparative analysis shows that excluding information on traffic intensity and location may lead to results that are substantially different from those that include the sample design information. The role of sample weights is central for a correct analysis.

1. INTRODUCTION

In 2006 the EU funded the DRUID Project (*Driving Under the Influence of Drugs, Alcohol and Medicines*) to assess the prevalence of driving under the influence of alcohol and other psychoactive substances. A decade later, we have learned that the results of that project have outlined a serious situation, even if for some countries the participation in roadside tests was voluntary. The highest prevalence of drivers with high alcohol levels and positive drug tests was found in Southern Europe and, at least in Spain, all efforts to reduce driving after alcohol or drug use do not seem to have eradicated the problem. Between 2001 and 2010, there was a significant reduction in the number of drivers that were found positive in breath tests for alcohol screening, but an increased presence of drivers after having consumed illicit drugs and in particular, cannabis, was found. The latest estimate by Domingo-Salvany, Herrero, Fernández et al. (2017) referred to the 2015 data, indicates that in a sample of 2744 drivers, 2.6% were positive for at least alcohol, while 11.6% were positive at least for one of the substances tested. A fraction of 0.8% of the tested drivers resulted in a positive result on both alcohol and drugs. This percentages were not weighted by traffic intensity, as this information was not available at the time of the study.

The purpose of this paper is to show that including or ignoring the sample design associated to the data collection mechanism has relevant effects, as it leads to differences in the prevalences of driving under the influence of alcohol or drugs. Here we will focus on alcohol-impaired driving, as our sample size is higher and the results will gain consistency.

2. DATA AND METHODS

Our cross-sectional study forms part of an initiative promoted by the Catalan Traffic Authority addressed at periodically measuring the prevalence of alcohol and drug-impaired driving in Catalonia (2014-2020 Strategic Road Safety Plan). The drivers that circulate on the region's main interurban roads make up the population of interest. Given the impossibility of testing all drivers, a roadside survey, providing a random and representative sample, was designed. The checkpoints were randomly located across the territory divided into eight police operational areas known as Regional Traffic Areas (RTAs). Alcohol and drug tests were performed by traffic officers and non-response was absent, because no driver refused to perform the test.

The roadside tests were conducted in the autumn of 2014 in line with normal police procedures by traffic officers. A standardized form was used in each test to record time and location, the characteristics of the driver according to his/her driving licence (gender, age, nationality, and years holding a driving licence), type of vehicle, number of occupants, as well as the results of the test for the following substances: alcohol, THC, methamphetamines, amphetamines, cocaine, opiates and benzodiazepines.

A two-stage probabilistic sample was designed to gather the data. The first stage involved selecting the road sections that would constitute the primary sample units (PSUs).

These sections comprised the stretch of road between a given access and exit point. Road sections with an average daily traffic (ADT) below 4,000 vehicles per day were excluded. As a result, the number of road sections included in the analysis was 3,469. Random drivers passing through these road sections formed the secondary sample units (SSUs).

PSUs were selected using stratified sampling. The stratification variables were geographical area (RTAs), road type (conventional or motorway), flow direction (according to the road kilometre counter), day of the week, and time-slots (divided into six four-hour intervals beginning at 10pm). The selection of the road sections ensured the proportional representation of RTAs and road types across the territory, as the probability of choosing a particular road section was set proportional to its length. Sampling was performed by replacement, although each road section could only be selected a maximum of three times. Half the road sections were selected in the rising flow direction and half in the decreasing direction. Given a selected road section and flow direction, the traffic agents could then choose the specific kilometre point where performing the tests, as to guarantee a safe location. The selected points were randomly equi-distributed across the days of the week and time-slots.

The number of alcohol test results was 7,133. The number of tests per regions was: 1024 in Girona, 712 in Metropolitan North, 855 in Metropolitan South, 915 in Camp de Tarragona, 720 in Terres de l'Ebre, 1005 in Ponent, 963 in Central, and 939 in Pirineu Oriental. There was also a difference in the number of tests obtained in conventional (77.0%) versus motorway roads (23.0%). The number of tests per day of the week was as follows: Monday: 18.2%; Tuesday to Thursday: 22.3%; Friday: 17.8%; Saturday: 19.9%; and Sunday: 21.8%. The tests were performed during all 24 hours, 9.9% between 2am and 6am, 18.7% between 6am and 10am, 18.0% between 10am and 2pm, 19.2% between 2pm and 6pm, 19.8% between 6pm and 10pm and 14.5% between 10pm and 2am. 79.7% drivers were men and 20.3% were women. 8.8% of drivers were aged under 25, 50.4% between 25 and 44, 34.5% between 45 and 64, and 6.2% were 65+. 13.2% of the tested drivers were not Spanish. 84.6% of the vehicles were regular cars. The number of passengers including the driver were one in 54.0% of the cases; 2 in 30.1%; 3 in 9.1%; 4 in 5.2% and higher in 1.6%.

A logistic regression analysis for alcohol-impaired driving is presented, both including and excluding sample weights. The models are computed using a sample comprising the

6,906 cases that presented no missing values for any of the variables. The regressors were gender; an indicator for the population aged 30-39; the number of occupants in the vehicle, comparing only one with at least two occupants; the time-slot in 3 slots per day; and the period of the week, comparing 24-hour weekdays to weekend days and weekend nights. The computation of weights is described at length in Alcañiz , Guillen, Sánchez-Moscona et al. (2014).

According to Vanlaar (2005), the strong points about roadside survey design procedures to observe actual drivers is that we seek random selection of road sites and drivers; stratification in space and in time; correct use of weights based on traffic counts, and uniformity in the way data are being collected. Moreover, ignoring the clustering will generally cause standard errors of regression coefficients to be underestimated. When these data were gathered Catalonia had 7,4 million inhabitants (49.1% men, 50.9% women). Official figures indicate that 4.2 million of those citizens have a driving licence. The difference between the proportion of men and women that was actually captured in the sample (79.7% and 20.3% for men and women, respectively) and that in the potential population of people with a driving license (57.9% are men and 42.1% are women) is large. As reported by Ayuso, Guillen and Perez-Marin (2016) on the average males drive a larger distance per day than women, which as a consequence makes the later much less likely to be stopped in a roadside random survey.

3. RESULTS

The effect of ignoring sample weights is substantial, but not to a large when looking at the general prevalence of alcohol impaired driving. With no weights, prevalence of alcohol-impaired drivers is 1.00%, while with sample weights, the result is 1.08%.

Table 1 shows the results of a logistic regression. It illustrates that ignoring the sampling weights has a relevant impact, not only on the odds-ratios values, but also on their statistical significance.

Tabla 1. Prevalence and 95% confidence intervals (CI) for alcohol-impaired driving, using and ignoring intensity-based weights

	Weighted		Unweighted	
	OR	**p-value**	**OR**	**p-value**
Gender (ref. male)				
Female	1.152	0.611	0.934	0.808
Age (ref. 30-39)				
<30 or ≥40	2.339	0.013	1.191	0.507
Occupants (ref. >1)				
1 (driving alone)	1.830	0.016	1.394	0.139
Time-slot (ref. 6am – 2pm)				
2pm – 10pm	3.304	0.002	2.274	0.028
10pm – 6am	2.694	0.000	3.695	0.032
Period of the week				
(ref. weekdays, 24 hours)				
Weekend days (6am – 10pm)	2.256	0.008	2.895	0.006
Weekend nights (10pm – 6am)	5.805	0.001	5.340	0.000
χ^2 ***Hosmer-Lemeshow test***	13.352	0.100	5.721	0.573

OR=odds-ratio estimate.

Results following intensity-based weighted data show that the influence of gender (OR=1.152) is not relevant when explaining alcohol impaired-driving. The population aged 30-39 is the one with the lowest probability of showing positive breath tests, because being a driver younger than 30 or older than 40 has a significantly higher risk (OR=2.339). This result should be analysed in more detail, but it may be reflecting that risk aversion is highest during maternity/paternity age, which is in line with Alcañiz, Santolino and Ramon (2016). Drivers alone in the vehicle show a higher probability of impaired-driving (OR=1.830) compared to drivers that have other passengers with them. Afternoons (OR=3.3.04) and nights (OR=2.694) are time-slots with a greater presence of drunk drivers than mornings. This result is in connexion with the ones for the period of the week. Weekend days (OR=2.256) and especially weekend nights (OR=5.805) present a much higher probability of detecting alcohol impaired-drivers than weekdays, all 24 hours long.

When sample weights are ignored, the observations are not representing the population of drivers correctly, because of two reasons. First, the size of the strata that have been used to select the roadside locations is ignored and therefore the results would ignore that some roads are longer or some types of road are much more frequent than others. Secondly, traffic intensity reflects the amount of drivers that travel in that particular section. When traffic intensity is ignored, the extrapolation could be inaccurate because the driver who has been stopped by the police officers is sampled randomly from all those that pass by the point during the time-slot for the chosen date. If weights are ignored, then some of the reported variable effects are not significant and this is in contradiction with what has been obtained when the sample weights are included in the analysis. For instance, the parameter estimates corresponding to the indicator for population aged 30-39 and driving alone have much higher standard errors than those obtained with the weighted sample. Some other parameters also have increased standard errors, though p-values remain smaller than 0.05. This may suggest that significance could be lost if samples were smaller.

4. CONCLUSIONS

This paper concludes that the factors that reflect the sample design cannot be ignored when reporting the likelihood of finding a driver positive for alcohol. Traffic intensity has a direct impact on the number of drivers represented by any sample unit. Indeed, this kind of observational studies relies on a complex sample design where there is a primary sampling unit, namely the roadside control site, and a secondary sampling unit, which is the driver. Locating a breath test control unit on a road with a higher traffic intensity is radically different from locating it in a low traffic road. The probability of selecting one particular driver in a low intensity road is higher than in a high intensity road. Increasing the sample size in roads with high traffic intensity is unlikely to substitute the sample weights, as the differences between road sections are very relevant, and the police officers stop all possible drivers until their control site is full, and then they can only stop a new driver when there is enough space for him/her to stop safely. So using sampling sizes high enough to be proportional to traffic intensity would be inoperative and inefficient in practical terms.

ACKNOWLEDGMENTS

This work has been partially supported by the Spanish Ministry of Economy and Competitiveness (grant ECO2015-66314-R). A special acknowledgement goes to the Traffic authorities.

REFERENCES

Alcañiz, M., Guillen, M., Sánchez-Moscona, D., Santolino, M., Llatje, O., and Ramon, L. (2014). "Prevalence of alcohol-impaired drivers in Catalonia based on random breath tests in a roadside survey". Accident Analysis and Prevention, 65, 131-141.

Alcañiz, M., Santolino, M., and Ramon, Ll. (2016). "Drinking patterns and drunk-driving behaviour by age and gender in Catalonia, Spain: a comparative study". Transportation Research Part F: Traffic Psychology and Behaviour, 42, 522-531.

Ayuso, M., Guillén, M., and Pérez-Marín, A.M. (2016). "Telematics and gender discrimination: some usage-based evidence on whether men's risk of accidents differs from women's". Risks, 4, 2, 10.

Chuliá, H., Guillen, M., and Llatje, O. (2016). "Seasonal and time-trend variation by gender of alcohol-impaired drivers at preventive sobriety checkpoints", Journal of Studies on Alcohol and Drugs. 77, 3, 413-420.

Domingo-Salvany, A., Herrero, M.J., Fernández, B., Pérez, J., del Real, P., González-Duque, J.C., and de la Torre, R. (2017). "Prevalence of psychoactive substances, alcohol and illicit drugs, in Spanish drivers: a roadside study in 2015". Forensic Science International, 278, 253-259.

Vanlaar, W. (2005). "Drink driving in Belgium: results from the third and improved roadside survey". Accident Analysis & Prevention, 37, 3, 391-397.

A NOTE ON SLASH METHODOLOGY TO DEAL WITH SYMMETRICAL AND UNIMODAL DISTRIBUTIONS WITH HEAVY TAILS

Inmaculada Barranco-Chamorro
Universidad de Sevilla, Departamento de Estadística e I.O., España
chamorro@us.es

Jimmy Reyes
Universidad de Antofagasta, Departamento de Matemáticas, Chile
jimmy.reyes@uantof.cl

Diego Gallardo Mateluna
Universidad de Antofagasta, Departamento de Matemáticas, Chile
diego.gallardo@uantof.cl

ABSTRACT

In this paper a new slash model is introduced. The resulting distribution exhibits greater kurtosis than other slash models previously considered in literature. Closed expressions are given for the probability density function, moments and kurtosis coefficient. Inference based on iterative maximum likelihood methods is carried out. Real applications of interest in Economics are included.

1. INTRODUCCIÓN

In real-world data, it is quite common to find symmetrical and unimodal histograms with heavy tails that do not fit well to a normal distribution. Slash models are a good option to deal with this kind of situations, in which departures of Gaussianity are a serious problem for the data analyst. This is one of the reasons why slash distributions have received a great deal of attention during the last decades. In this context, we face the problem of improving slash models by introducing a generalisation able to model more kurtosis than other slash's previously proposed in literature. The emphasis is on kurtosis, because as Moors (1988) pointed out, the presence of heavy tails produces high kurtosis.

First, we briefly describe the most relevant features of these models along with our proposals.

The canonic slash distribution was introduced by Rogers and Tukey (1972). This model was defined as the ratio of a N(0,1) normal and a uniform U(0,1) distribution independent. It was proposed as a model for bell shaped data with heavier tails than the corresponding normal distribution. Their theoretical properties can be seen in Rogers and Tukey (1972), Mosteller and Tukey (1977), and Johnson et al.(1995). Maximum likelihood estimation in this slash model with location and scale parameters can be found in Kafadar (1982). Wang and Genton (2006) described multivariate symmetrical and skew-multivariate extensions of the slash-distribution, while Gómez et al. (2007) and Gómez and Venegas (2008) extended the slash distribution by introducing the slash-elliptical family; an asymmetric version of this family was discussed in the paper by Arslan (2008). Arslan and Genc (2009) discussed a symmetric extension of the multivariate slash distribution, and Genc (2007) discussed a symmetric generalization of the slash distribution. Recently, Reyes et al. (2013) proposed a modified slash distribution and Rojas et al. (2014) extended the slash distribution by considering a beta distribution at the denominator. All these papers illustrate the relevance of slash methodology from a theoretical and practical point of view.

2. MAIN RESULTS

In this paper, we focus on univariate symmetrical slash models. The starting point of our study is the stochastic representation of the (standard) slash distribution defined as the quotient of a standard normal distribution and uniform on (0,1) distribution to the $1/q$ power, with $q>0$. The general model is obtained as a scale-location transformation of the standard distribution. Several extensions of this first slash model are considered. In our study the emphasis will be on the parameter controlling the kurtosis of the distribution. For the new model which we introduce, a closed expression for the probability density function (pdf) is given; the convergence in law to a normal distribution is proven, and moments are obtained. Comparisons are carried out in terms of pdf's and kurtosis coefficients of other slash models. Iterative maximum likelihood estimation methods are applied.

3. SIMULATIONS AND APPLICATIONS

A simulation study has been carried out, by using the software R, in order to illustrate the performance of our results.

Also, we consider one real data set, of interest in Economics, in which our methodology can be applied. Graphic and descriptive criteria are proposed to show that, in this case, the new slash distribution fits better than other slash models previously considered in literature.

ACKNOWLEDGMENTS

The research of J. Reyes was supported by SEMILLERO UA-2017. The research of I. Barranco-Chamorro was partially supported by CMT2015-68276-R MINECO/FEDER, UE.

REFERENCES

Arslan, O. (2008). "An Alternative Multivariate Skew-Slash Distribution".Statistics and Probability Letters, 78(16), 2756-2761.

Arslan, O., Genc, A.I. (2009). "A Generalization of the Multivariate Slash Distribution". Journal of Statistical Planning and Inference, 139(3), 1164-1170.

Gómez, H.W., Quintana, F.A., Torres, F.J. (2007). "New Family of Slash-Distributions with Elliptical Contours". Statistics and Probability Letters, 77(7), 717-725.

Johnson, N.L., Kotz, S., Balakrishnan, N. (1995). "Continuous univariate distributions, Vol 1, 2nd edn. New York: Wiley.

Kafadar, K. (1982). A biweight approach to the one-sample problem. J. Amer. Statist. Assoc. 77, 416-424.

Mosteller, F., Tukey,J.W. (1977). Data analysis and regression, Addison-Wesley, Reading, MA.

R Core Team (2015). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL http://www.R-project.org/.

Reyes, J., Gómez, H.W., Bolfarine, H. (2013). Modified slash distribution. Statistics: A Journal of Theoretical and Applied Statistics, 47(5), 929-941.

Rogers, W.H., Tukey, J.W. (1972). Understanding some long-tailed symmetrical distributions, Statist. Neerlandica, 26, 211-226.

Rojas, M.A., Bolfarine, H., Gómez, H.W. (2014). An extension of the slash-elliptical distribution. Statistics and Operations Research Transactions (SORT), 38(2), 215-230.

Wang, J., Genton, M.G. (2006). The multivariate skew-slash distribution. Journal Statistical Planning and Inference, 136, 209-220.

CONSISTENCIA DE MEDIDAS DE CO-RIESGO Y MEDIDAS DE CONTRIBUCIÓN DE RIESGO CON ORDENACIONES ESTOCÁSTICAS. ALGUNOS EJEMPLOS

Alfonso J. Bello
Universidad de Cádiz, Departamento de Estadística e I.O., España
alfonsojose.bello@uca.es

Miguel A. Sordo
Universidad de Cádiz, Departamento de Estadística e I.O., España
mangel.sordo@uca.es

Alfonso Suárez-Lloréns
Universidad de Cádiz, Departamento de Estadística e I.O., España
alfonso.suarez@uca.es

ABSTRACT

Cuando analizamos el riesgo de una cartera, no debemos ignorar la interacción entre los riesgos individuales de esta. En este contexto, las medidas de riesgo además de evaluar los riesgos marginales y agregados, también sirven para evaluar el riesgo sistémico, que está relacionado con el riesgo de que el fracaso o pérdida de una componente de la cartera se extienda a otra, o incluso a toda la cartera. Para ello, en la literatura se ofrecen, básicamente, dos tipos de medidas: las medidas de co-riesgo y las medidas de contribución de riesgo. Vamos a presentar la consistencia de algunas medidas de co-riesgo y de algunas medidas de contribución de riesgo en términos de ordenaciones estocásticas de las componentes marginales bajo distintas suposiciones de dependencia positiva, que son suposiciones razonables cuando consideramos carteras de seguros. Ilustramos los resultados presentados con diversos ejemplos y, a partir de simulaciones computacionales, mostramos gráficas de casos particulares de los anteriores.

1. INTRODUCCIÓN

Las medidas de riesgo no se utilizan solo para evaluar los riesgos marginales y agregados, sino también, lo que es más importante desde la reciente crisis financiera mundial, para evaluar el riesgo sistémico, el cual está relacionado con el riesgo de que el fracaso o pérdida de una componente de la cartera se extienda a otra componente, o incluso a toda la cartera. Para abordar la cuestión anterior, en la literatura se ofrecen, básicamente, dos tipos de medidas: las medidas de co-riesgo y las medidas de contribución de riesgo.

Sea el vector aleatorio absolutamente continuo (X,Y) de riesgos o pérdidas, con función de distribución conjunta H, función cópula asociada C y funciones de distribución marginales F_X y F_Y. Así, podemos escribir $H(x,y)=C(F_X(x),F_Y(y))$. Ahora vamos a definir algunos órdenes estocásticos.

Definición 1 (Nelsen, 1999) Dadas dos cópulas C y C', decimos que C es menor que C' en orden de concordancia, $C < C'$, si $C(u,v) \leq C'(u,v)$, $\forall u,v \in (0,1)$.

Definición 2 (Shaked y Shanthikumar, 2007) Sean las variables aleatorias X e Y con respectivas funciones de distribución F_X y F_Y, funciones de supervivencia $\bar{F}_X=1-F_X$ y $\bar{F}_Y=1-F_Y$, y funciones de densidad f_X y f_Y. Entonces, X es menor que Y:

(i) en el orden estocástico, $X \leq_{st} Y$, si $F_X(t) \geq F_Y(t)$, $\forall t$,

(ii) en el orden razón de verosimilitud, $X \leq_{lr} Y$, si $f_Y(t)/f_X(t)$ es creciente en t en la unión de los soportes de X e Y.

(iii) en el orden en razón de fallo, $X \leq_{hr} Y$, si $\bar{F}_Y(t)/\bar{F}_X(t)$ es creciente en t,

(iv) en el orden creciente convexo, $X \leq_{icx} Y$, si $\int_t^{\infty} \bar{F}_X(x)dxp \leq \int_t^{\infty} \bar{F}_Y(x)dx$, $\forall t$.

(v) en el orden en dispersión, $X \leq_{disp} Y$, si $F_X^{-1}(p) - F_X^{-1}(q) \leq F_Y^{-1}(p) - F_Y^{-1}(q)$, para todo $0<q<p<1$,

(vi) en el orden excess wealth, $X \leq_{ew} Y$, si $E[(X-F_X^{-1}(p))^+] \leq E[(Y-F_Y^{-1}(p))^{\wedge +}]$,

$\forall t \in (0,1)$, siendo $(x)^+ = \max\{x,0\}$.

A continuación, vamos a definir algunas nociones de dependencia positiva mediante las ordenaciones estocásticas anteriores.

Definición 3 (i) (X,Y) es TP_2, si $\{X \mid Y=y_1\} \leq\_lr \{X \mid Y=y_2\}$, para todo $y_1 \leq y_2$, e $\{Y \mid X=x_1\} \leq_{lr} \{Y \mid X=x_2\}$, para todo $x_1 \leq x_2$.

(ii) $X \uparrow_{SI} Y$, si $\{X \mid Y=y_1\} \leq\_st \{X \mid Y=y_2\}$, para todo $y_1 \leq y_2$.

(iii) (X,Y) es *PDS*, si $X \uparrow_{SI} Y$ e $Y \uparrow_{SI} X$.

Se sabe que $\leq_{lr}$ implica $\leq_{hr}$, que implica $\leq_{st}$, que implica $\leq_{icx}$, y que $\leq_{disp}$ implica $\leq_{ew}$. Por tanto, también TP_2 implica PDS. Se tiene que (X,Y) es TP_2 (PDS) si, y solo si su cópula asociada es TP_2 (PDS) (Müller y Stoyan, 2002, y Cai y Wey, 2012). La conocida cópula Gumbel (Joe, 1997), usada frecuentemente para el modelado de fuerte dependencia en la cola derecha, es un ejemplo de cópula PDS.

Las medidas de co-riesgo pueden interpretarse como versiones ajustadas de las medidas usualmente utilizadas para evaluar los riesgos aislados. Dos medidas escalares de riesgo populares son el valor en riesgo (VaR) y el expected shortfall (ES).

$$VaR_\alpha [X] = F_X^{-1}(\alpha) = \inf\{x : F_X(x) \geq \alpha\},\ \forall \alpha \in (0,1),$$

$$ES_\alpha [X] = \frac{1}{1-\alpha} \int_\alpha^1 VaR_t [X]\, dt,\ \forall \alpha \in (0,1).$$

Adrian y Brunnermeier (2016), Girardi y Ergün (2013) o Mainik y Shaanning (2014), entre otros, ajustaron el VaR y el ES a la dependencia entre X e Y, condicionando la distribución de Y (una componente de riesgo en particular, o el riesgo agregado de toda una cartera) con un escenario de estrés de X (otra componente de riesgo).

Definición 4 (Girardi y Ergün, 2013) Para todo $\alpha,\beta \in (0,1)$, definimos el valor en riesgo condicional como

$$CoVaR_{\alpha,\beta} [Y|X] = VaR_\beta [Y \mid X > VaR_\alpha [X]].$$

Definición 5 (Mainik y Shaanning, 2014) Para todo $\alpha,\beta\in(0,1)$, definimos el expected shortfall condicional como

$$CoES_{\alpha,\beta}[Y/X]=\frac{1}{1-\beta}\int_{\beta}^{1}CoVaR_{\alpha,t}[Y/X]\,dt.$$

Por otro lado, las medidas de contribución de riesgo cuantifican cómo una situación de estrés en la componente X afecta a otra componente Y. Una forma es mediante la comparación del $CoVaR_{\alpha,\beta}[Y/X]$, que es el VaR de la componente Y condicionada a que la componente X se encuentra en dificultades financieras, con el $VaR_{\beta}[Y]$, que representa el riesgo de Y en un estado de normalidad. Otra forma es la de remplazar el término de centralización $VaR_{\beta}[Y]$ por el valor en riesgo condicional de Y condicionado a que X se encuentra en un estado de referencia determinado, usualmente definido en términos de su mediana. Podemos proceder igualmente con el $CoES_{\alpha,\beta}[Y|X]$.

Definición 6 (Mainik y Shaanning, 2014) Para todo $\alpha,\beta\in(0,1)$, definimos

$$\Delta CoVaR_{\alpha,\beta}[Y\mid X]=CoVaR_{\alpha,\beta}[Y\mid X]-VaR_{\beta}[Y],$$

$$\Delta^{med}CoVaR\_(\alpha,\beta)[Y\mid X]=CoVaR_{\alpha,\beta}[Y\mid X]-CoVaR_{1/2\beta}[Y\mid X].$$

Definición 7 (Mainik y Shaanning, 2014) Para todo $\alpha,\beta\in(0,1)$, definimos

$$\Delta CoES_{\alpha,\beta}[Y\mid X]=CoES_{\alpha,\beta}[Y\mid X]-ES_{\beta}[Y],$$

$$\Delta^{med}CoES_{\alpha,\beta}[Y\mid X]=CoES_{\alpha,\beta}[Y\mid X]-CoES_{1/2,\beta}[Y\mid X].$$

En la teoría actuarial, es importante el estudio de la consistencia de las medidas de riesgo con respecto a órdenes estocásticos. Estos resultados, que implican a diferentes órdenes estocásticos y medidas de riesgo, nos permiten evaluar los riesgos por separado, sin tener en cuenta cómo se relacionan entre sí. Vamos a presentar la consistencia de algunas medidas de co-riesgo y de algunas medidas de contribución de riesgo en términos de ordenaciones estocásticas de las componentes marginales bajo distintas suposiciones de dependencia positiva (que son suposiciones razonables al considerar carteras de seguros). Ilustramos los resultados presentados con ejemplos y, a partir de simulaciones computacionales, presentamos gráficas de casos particulares de los ejemplos anteriores.

2. RESULTADOS DE CONSISTENCIA CON MEDIDAS DE CO-RIESGO

Mainik y Shaaning (2014) han estudiado la monotonía del ***CoVaR*** y del ***CoES*** respecto a los parámetros de dependencia de la cópula con marginales estocásticamente ordenadas. Para el ***CoES*** demostraron el siguiente resultado.

Teorema 8 (Mainik y Shaanning, 2014) Sean (X,Y) y (X',Y') dos vectores aleatorios con cópulas Gumbel de respectivos parámetros θ y θ'. Supongamos que F_X y $F_{X'}$ son las respectivas funciones de distribución de X y X', continuas. Entonces, si $Y \leq_{st} Y'$ y $\theta \leq \theta'$, se verifica que

$$CoES_{\alpha,\beta}[Y \mid X] \leq CoES_{\alpha,\beta}[Y' \mid X'],\ \alpha,\beta \in (0,1).$$

Se tiene que el resultado anterior sigue cumpliéndose cuando el orden $\leq_{st}$ se remplaza por el orden $\leq_{icx}$.

Teorema 9 (Sordo, Bello y Suarez-Llorens, 2018) Sean (X,Y) y (X',Y') dos vectores aleatorios con respectivas cópulas C y C'. Supongamos que $X \uparrow_{SI} Y$ o

$X' \uparrow_{SI} Y'$, o ambas se cumplen. Entonces, si $Y \leq_{icx} Y'$ y $C \prec C'$, se verifica que

$$CoES_{\alpha,\beta}[Y \mid X] \leq CoES_{\alpha,\beta}[Y' \mid X'],\ \alpha,\beta\ (0,1).$$

3. RESULTADOS DE CONSISTENCIA CON MEDIDAS DE CONTRIBUCIÓN DE RIESGO.

Ahora, dados (X,Y) y (X',Y'), vamos a proporcionar condiciones para las marginales Y e Y' y las respectivas cópulas asociadas para poder comparar las medidas de contribución de riesgo $\Delta CoVaR_{\alpha,\beta}[Y \mid X]$ y $\Delta^{med} CoVaR_{\alpha,\beta}[Y \mid X]$.

Teorema 10 (Sordo, Bello y Suarez-Llorens, 2018) Sean (X,Y) y (X',Y') dos vectores aleatorios con respectivas cópulas C y C'. Supongamos que $X \uparrow_{SI} Y$ o

$X' \uparrow_{SI} Y'$, o ambas se cumplen. Entonces, si $Y \leq_{disp} Y'$ y $C \prec C'$, se verifica que

$$\Delta CoVaR_{\alpha,\beta}[Y \mid X] \leq \Delta CoVaR_{\alpha,\beta}[Y' \mid X'],\ \alpha,\beta \in (0,1).$$

Teorema 11 (Sordo, Bello y Suarez-Llorens, 2018) Sean $(\mathbf{X,Y})$ y $(\mathbf{X',Y'})$ dos vectores aleatorios con la misma cópula $\mathbf{C}$, siendo $\boldsymbol{TP_2}$. Entonces, si $\mathbf{Y}\leq_{\mathbf{disp}} \boldsymbol{Y'}$ se cumple que

$$\Delta^{med}\,\boldsymbol{CoVaR}_{\alpha,\beta}[Y\,|X]\leq\Delta^{med}\,\boldsymbol{CoVaR}_{\alpha,\beta}[Y'\,|X'],\ \boldsymbol{\alpha}\in(\mathbf{1/2,1}),\ \boldsymbol{\beta}\in(\mathbf{0,1}).$$

También, dados $(\mathbf{X,Y})$ y $(\mathbf{X',Y'})$, proporcionamos condiciones para las marginales $\mathbf{Y}$ e $\mathbf{Y'}$ y las respectivas cópulas asociadas para comparar las medidas de contribución de riesgo $\Delta\boldsymbol{CoES}_{\alpha,\beta}[Y\,|X]$ ***y*** $\Delta^{med}\,\boldsymbol{CoES}_{\alpha,\beta}[Y\,|X]$.

Teorema 12 (Sordo, Bello y Suarez-Llorens, 2018) Sean $(\mathbf{X,Y})$ y $(\mathbf{X',Y'})$ dos vectores aleatorios con respectivas cópulas $\mathbf{C}$ y $\mathbf{C'}$. Supongamos que $\mathbf{X}\uparrow_{\mathbf{SI}}\mathbf{Y}$ o $\mathbf{X'}\uparrow_{\mathbf{SI}}\mathbf{Y'}$, o ambas se cumplen. Entonces, si $\mathbf{Y}\leq_{\mathbf{ew}}\mathbf{Y'}$ **y** $\mathbf{C}\prec\mathbf{C'}$, se verifica que $\Delta\boldsymbol{CoES}_{\alpha,\beta}[Y\,|X]\leq\Delta\boldsymbol{CoES}_{\alpha,\beta}[Y'\,|X'],\ \boldsymbol{\alpha,\beta}\in(\mathbf{0,1})$.

Teorema 13 (Sordo, Bello y Suarez-Llorens, 2018) Sean $(\mathbf{X,Y})$ y $(\mathbf{X',Y'})$ dos vectores aleatorios con la misma cópula C, siendo $\boldsymbol{TP_2}$. Entonces, si $\mathbf{Y}\leq_{\mathbf{ew}}\mathbf{Y'}$ se cumple que

$$\Delta^{med}\,\boldsymbol{CoES}_{\alpha,\beta}[Y\,|X]\leq\Delta^{med}\,\boldsymbol{CoES}_{\alpha,\beta}[Y'\,|X'],\ \boldsymbol{\alpha}\in(\mathbf{1/2,1}),\ \boldsymbol{\beta}\in(\mathbf{0,1}).$$

4. EJEMPLOS

En esta sección vamos a proporcionar ejemplos para ilustrar los resultados de consistencia de las anteriores secciones. Para cada ejemplo, mediante simulación computacional, hemos simulado un caso particular y representado gráficamente.

Ejemplo 14 Para ilustrar el Teorema 9, consideremos $(\mathbf{X,Y})$ y $(\mathbf{X',Y'})$ con cópulas Gumbel $\mathbf{C}_\theta$ y $\mathbf{C}_{\theta'}$, $\boldsymbol{\theta<\theta'}$ así son $\mathbf{PDS}$ y $\mathbf{C}_\theta\prec\mathbf{C}_{\theta'}$), e $\mathbf{Y\sim N(\mu,\sigma)}$ e $\mathbf{Y'\sim N(\mu',\sigma')}$ con $\boldsymbol{\mu=\mu'}$ y $\boldsymbol{\sigma<\sigma'}$ (así $\mathbf{Y}\leq_{\mathbf{icx}}$ $\mathbf{Y'}$). Entonces $\mathbf{CoES}_{\alpha,\beta}[\mathbf{Y}\,|\mathbf{X}]\leq\mathbf{CoES}_{\alpha,\beta}[\mathbf{Y'}\,|\mathbf{X'}]$, para todo $\boldsymbol{\alpha,\beta}\in(\mathbf{0,1})$**.** En la Figura 1 podemos ver un caso particular.

Ejemplo 15 Para ilustrar el Teorema 10, consideremos $(\mathbf{X,Y})$ y $(\mathbf{X',Y'})$ con cópulas Gaussianas $\mathbf{C}_\rho$ y $\mathbf{C}_{\rho'}$, $\mathbf{0\leq\rho<\rho'}$ (así son $\mathbf{TP}_2$ y $\mathbf{C}_\rho\prec\mathbf{C}_{\rho'}$), e $\mathbf{Y\sim W(a,b)}$ e $\mathbf{Y'\sim W(a',b')}$ dos distribuciones Weibull con $\mathbf{a\leq a'}$ y $\mathbf{b=b'}$ (así $\mathbf{Y}\leq_{\mathbf{disp}}\mathbf{Y'}$). Entonces $\Delta\mathbf{CoVaR}_{\alpha,\beta}[\mathbf{Y}\,|\mathbf{X}]\leq\Delta\mathbf{CoVaR}_{\alpha,\beta}[\mathbf{Y'}\,|\mathbf{X'}]$, para todo $\boldsymbol{\alpha,\beta}\in(\mathbf{0,1})$. En la izquierda de la Figura 2 podemos ver un caso particular.

Ejemplo 16 Para ilustrar el Teorema 11, consideremos $(\mathbf{X,Y})$ y $(\mathbf{X',Y'})$ con la misma cópula Gaussiana $\mathbf{C}_{\rho}$, $\mathbf{0\leq\rho}$ (así es $\mathbf{TP}_2$), e $\mathbf{Y\sim W(a,b)}$ e $\mathbf{Y'\sim W(a',b')}$ dos distribuciones Weibull con $\mathbf{a\leq a'}$ y $\mathbf{b=b'}$ (así $\mathbf{Y\leq_{disp} Y'}$). Entonces $\boldsymbol{\Delta^{med}\, CoVaR_{\alpha,\beta}}\,[Y\,|X] \leq \Delta^{med}\, \boldsymbol{CoVaR_{\alpha,\beta}}\,[Y'|X']$, para todo $\boldsymbol{\alpha}\in(\mathbf{1/2,1})$ y $\boldsymbol{\beta}\in(\mathbf{0,1})$. En la derecha de la Figura 2 podemos ver un caso particular.

Ejemplo 17 Para ilustrar el Teorema 12, consideremos $(\mathbf{X,Y})$ y $(\mathbf{X',Y'})$ con cópulas Gaussianas $\mathbf{C}_{\rho}$ y $\mathbf{C}_{\rho'}$, $\mathbf{0\leq\rho<\rho'}$ (así son $\mathbf{TP}_2$ y $\mathbf{C}_{\rho}\prec\mathbf{C}_{\rho'}$), e $\mathbf{Y\sim W(a,b)}$ e $\mathbf{Y'\sim W(a',b')}$ dos distribuciones Weibull con $\mathbf{b>b'}$ y $\mathbf{E}[Y]\leq\mathbf{E}[Y']$ (así $\mathbf{Y\leq_{ew} Y'}$ e $\mathbf{Y\nleq_{disp} Y'}$, Belzunce et al., 2015). Entonces $\Delta\boldsymbol{CoES_{\alpha,\beta}}\,[Y\,|X]\leq\Delta\boldsymbol{CoES_{\alpha,\beta}}\,[Y'\,|X']$, para todo $\boldsymbol{\alpha,\beta}\in(\mathbf{0,1})$. En la izquierda de la Figura 3 podemos ver un caso particular.

Ejemplo 18 Para ilustrar el Teorema 13, consideremos $(\mathbf{X,Y})$ y $(\mathbf{X',Y'})$ con la misma cópula Ali-Mikhail-Haq $\mathbf{C}_{\theta}\theta$ $\mathbf{0\leq\theta\leq1}$ (así es $\mathbf{TP}_2$), e $\mathbf{Y\sim W(a,b)}$ e $\mathbf{Y'\sim W(a',b')}$ dos distribuciones Weibull con $\mathbf{b>b'}$ y $\mathbf{E}[Y]\leq\mathbf{E}[Y']$ (así $\mathbf{Y\leq_{ew} Y'}$ e $\mathbf{Y\nleq_{disp} Y'}$). Entonces $\Delta^{med}\,\mathbf{CoES_{\alpha,\beta}}\,[Y|X]\leq\Delta^{med}\,\mathbf{CoES_{\alpha,\beta}}\,[Y'|X']$, para todo $\boldsymbol{\alpha}\in(\mathbf{1/2,1})$ y $\boldsymbol{\beta}\in(\mathbf{0,1})$. En la derecha de la Figura 3 podemos ver un caso particular.

Figure 1. En el gráfico podemos ver las superficies de los $\mathrm{CoES}_{\alpha,\beta}\,[Y|X]$ en función de α y β. La roja corresponde a una cópula Gumbel con $\theta=3$ y marginales $N(1,2)$ y $N(0,1)$, y la amarilla a una cópula Gumbel con $\theta=4$ y marginales $N(0,1)$ y $N(0,2)$.

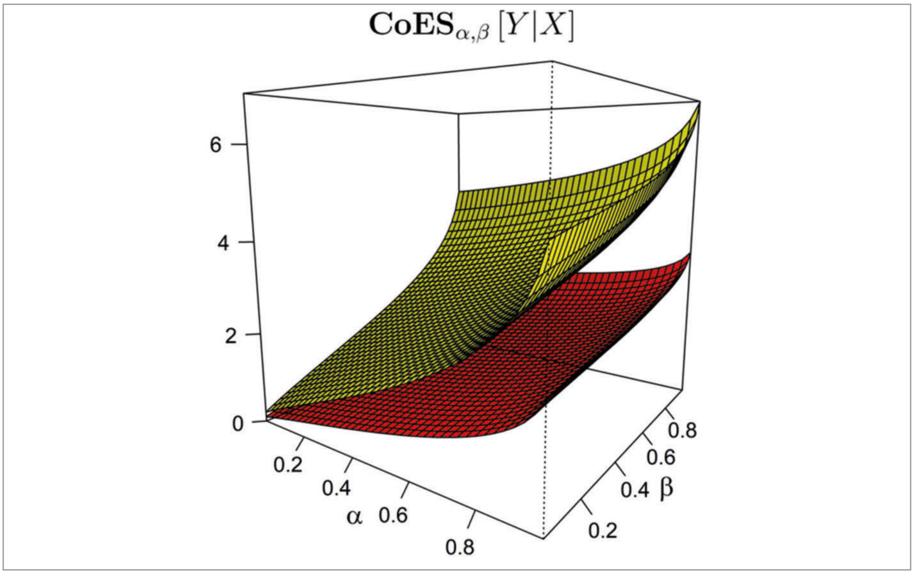


Figure 2. En el gráfico de la izquierda podemos ver las superficies de los $\Delta\mathrm{CoVaR}_{\alpha,\beta}$ [Y|X] en función de α y β. La roja corresponde a una cópula Gaussiana con ρ=0.8 y marginales N(1,2) y W(1,1), y la amarilla a una cópula Gaussiana con ρ=0.9 y marginales N(1,2) y W(1.5,1). En el gráfico de la derecha podemos ver las superficies de los $\Delta^{med}\,\mathrm{CoVaR}_{\alpha,\beta}$ [Y|X] en función de α y β. La roja corresponde a una cópula Gaussiana con ρ=0.8 y marginales N(1,2) y W(1,1), y la amarilla a marginales N(1,2) y W(1.5,1) ligadas con idéntica cópula.

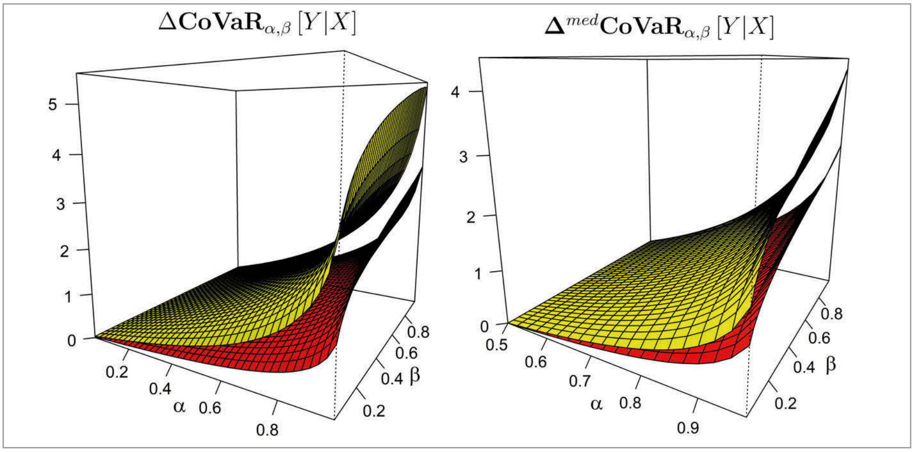
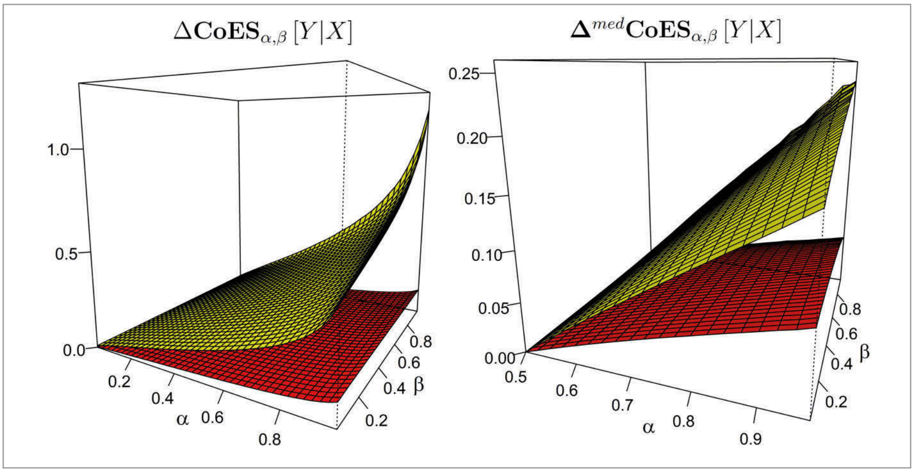


Figure 3. En el gráfico de la izquierda podemos ver las superficies de los $\Delta\mathrm{CoES}_{\alpha,\beta}$ [Y|X] en función de α y β. La roja corresponde a una cópula Gaussiana con ρ=0.1 y marginales N(0,1) y W(1,2), y la amarilla a una cópula Gaussiana con ρ=0.2 y marginales N(0,1) y W(1,1). En el gráfico de la derecha podemos ver las superficies de los $\Delta^{med}\,\mathrm{CoES}_{\alpha,\beta}$ [Y|X] en función de α y β. La roja corresponde a una cópula Ali-Mikhail-Haq con θ=0.8 y marginales N(0,1) y W(1,2), y la amarilla a marginales N(0,1) y W(1,1) ligadas con idéntica cópula.



5. CONCLUSIONES

Cuando analizamos el riesgo de una cartera, no debemos ignorar la interacción existente entre los riesgos individuales que la componen. En este contexto, las medidas de riesgo no se utilizan solo para evaluar los riesgos marginales y agregados, sino también, lo que es más importante desde la reciente crisis financiera mundial, para evaluar el riesgo sistémico, el cual está relacionado con el riesgo de que el fracaso o pérdida de una componente se extienda a otra componente, o incluso a toda la cartera. Para ello, en la literatura se ofrecen dos tipos de medidas: las medidas de co-riesgo y las medidas de contribución de riesgo. Hemos presentado resultados sobre la consistencia de algunas medidas de co-riesgo y de algunas medidas de contribución de riesgo en términos de ordenaciones estocásticas de las componentes marginales bajo distintas suposiciones de dependencia positiva. Hemos presentado ejemplos de cada uno de los resultados e ilustrado mediante un caso particular que hemos simulado computacionalmente.

AGRADECIMIENTOS

Los autores agradecen la financiación del Ministerio de Economía, Industria y Competitividad (proyecto MTM2017-89577-P).

BIBLIOGRAFÍA

Adrian, T., Brunnermeier, M.K. (2016). "CoVaR". American Economic Review, 106, 1705-1741.

Cai, J., Wei, W. (2012). On the invariant properties of notions of positive dependence and copulas under increasing transformations. Insurance: Mathematics and Economics, 50,43-49.

Girardi, G., Ergün, A.T. (2013). Systemic risk measurement: multivariate GARCH estimation of CoVaR. Journal of Banking and Finance, 37, 3169-3180.

Joe, H. (1997). Multivariate Models and Dependence Concepts. Chapman and Hall, London.

Mainik, G., Schaanning, E. (2014). On dependence consistency of CoVaR and some other systemic risk measures. Statistics Risk Modeling, 31, 49-77.

Müller, A., Stoyan, D. (2002). Comparison Methods for Stochastic Models and Risks. Wiley, New York.

Nelsen, R.B. (1999). An introduction to Copulas. In: Lectures Notes in Statistics, vol. 139, Springer-Verlag, New York.

Sordo, M.A., Bello, A.J., Suárez-Llorens, A. (2018). Stochastic orders and co-risk measures under positive dependence. Insurance: Mathematics and Economics, 78, 105-113.

ROLE OF CHOICE OF THRESHOLD ON THE ESTIMATION OF MARKET RISK UNDER THE POT METHOD (EVT)

Sonia Benito Muela
National Distance Education University (UNED). Department of Economic Analysis
Faculty of Economics and Business Administration. Spain
soniabm@cee.uned.es

Carmen López-Martín
National Distance Education University (UNED). Department of Business and Accounting. Faculty of Economics and Business Administration. Spain
carmen.lopez@cee.uned.es

Mª Ángeles Navarro
National Distance Education University (UNED). PhD Student of the Faculty of Economics and Business Administration. Spain
mnavarro900@alumno.uned.es

ABSTRACT

The conditional extreme value theory has been proven to be one of the most successful in estimating market risk. The implementation of this method in the framework of the Peaks Over a Threshold (POT) model requires one to choose a threshold for fitting the generalized Pareto distribution (GPD). In this paper, we investigate whether the selection of the threshold is important for the quantification of market risk. For measuring risk, we use the value at risk (VaR) measure. The study has been done for the S&P 500 index. When we analyse the validity of VaR estimates, the results show than all the thresholds considered provide correct estimations of VaR, with the exception of certain thresholds corresponding to the 99th percentile. The results obtained show that the quantification of the market risk through the VaR does not depend on the threshold selected.

1. INTRODUCCIÓN

One of the most important tasks financial institutions encounter is evaluating their market risk exposure. Although traditionally, the market risk of a portfolio was measured through the variance, one of the possible measures to quantify this risk is the evaluation of losses likely to be incurred when the price of the portfolio assets falls. This is what Value at Risk (VaR) does. (J.Morgan, 1996). The VaR of a portfolio is defined as the worst expected loss over a given horizon under normal market conditions at a given level of confidence. In particular, the $VaR(\alpha)$ of a portfolio at $(1\text{-}\alpha)$% confidence level is the percentile α % of the return portfolio distribution. To estimate this measure, several methodologies have been developed. Among all the methodologies, extreme value theory (EVT) has been proven to be one of the most successful in VaR estimation (see Abad et al., 2014). The extreme value theory approach focuses on limiting the distribution of extreme returns observed over a long time period, which is essentially independent of the distribution of the returns. The two main models for extreme value theory are the block maxima model (McNeil, 1998) and the peaks-over-threshold (POT) model. In the context of the POT model, extreme values above a high threshold are analysed using a generalized Pareto distribution (GPD). The difficulty of this method lies in finding the optimal threshold for GPD fitting. Although many proposals have been made to determine the optimal threshold in the framework of the POT method, the results of the study indicate that according to the literature, the choice of the threshold affects the parameter estimates of the GPD, however, the VaR measures obtained from these parameters do not depend on the choice threshold.

In next section, we present the methodology we use for the study. In section 3, we present the data and the results obtained for the S&P 500 index. The main conclusions are presented in section 4.

2. METHODOLOGY

2.1 Extreme Value Theory

The extreme value theory (EVT) approach focuses on the limiting distribution of extreme returns observed over a long time period. The two main models for EVT are the block maxima model (BM) (McNeil, 1998) and the peaks-over-threshold model (POT). The second model is generally considered to be the most useful for practical applications due to the more efficient use of the data at the extreme values. In the framework of the POT model, there are two types of analysis: the semi-parametric models built around the Hill estimator and its relatives (Beirlant et al., 1996; Danielson et al., 1998) and the full parametric models based on the GPD (Embrechts et al., 1999). In this paper, we focus on the full parametric model. Given a set of random variables $(r_1,r_2,\ldots,r_n)$, i.i.d. ~ F, we choose a low threshold u and examine all values (y) exceeding u: $(y_1,y_2,\ldots,y_{Nu})$, where $y_i=r_i-u$, and N_u is the number of sample data points greater than u. The distribution of excess losses over the threshold u is defined as:

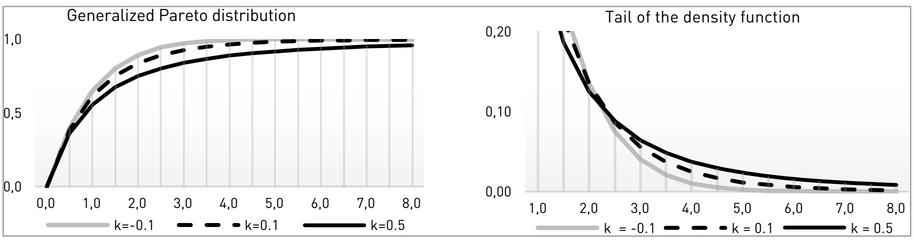
$$F_u(y)=P((r-u)<y \mid r>u)=\frac{F(r+u)-F(u)}{1-F(u)} \qquad (1)$$

According to the theorem of Pickans (1975) and Balkema and de Haan (1974), for a large class of underlying distribution functions F, the conditional excess distribution function F_u (y), for a large u, is well approximated by $F_u(y)\approx G_{k.\xi}(y)$ with $u \rightarrow \infty$, where

$$G_{k.\xi}(\mathrm{y})=\begin{cases} 1-(1+\quad \mathrm{y})^{-1/k} & \text{if } \xi\neq 0 \\ 1-\exp\left(-\frac{\mathrm{k}}{\xi}\right) & \text{if } \xi=0 \end{cases} \qquad (2)$$

$\boldsymbol{G_{k.\xi}}$(y) is the so-called generalized Pareto distribution (GPD) being $\boldsymbol{k}$ and ξ the shape parameter and the scale parameter respectively. The shape parameter can take any value, positive or negative and the scale parameter is always positive. Figure 1 illustrates the shape of the generalized Pareto distribution and the corresponding density function when the shape parameter or tail index take negative and positive values.

Figure 1. Shape of the GPD and the corresponding density function for ξ=1



The expression for the corresponding quantile α, which is denoted by $q(\alpha)$,is calculated by inverting the tail estimation formula to obtain:

$$q(\alpha)=u-\frac{\xi}{k}\left(\left(\frac{n}{N_u}\alpha\right)^{-k}-1\right) \quad (3)$$

2.2 Risk Measure

According to Jorion (2001), the "VaR measure is defined as the worst expected loss over a given horizon under normal market conditions at a given level of confidence". Thus, the VaR is a conditional quantile of the asset return loss distribution. Let r_1, r_2, r_n be identically distributed independent random variables representing the financial returns. Using $F(r)$ to denote the cumulative distribution function, $F(r)= Pr(r_t < r|\Omega_{t-1})$ conditionally on the information set Ω_{t-1} that is available at time $t-1$. Assume that $\{r_t\}$ follows the stochastic process $r_t=\mu_t+\sigma_t z_t$, $z_t \sim iid\ (0,1)$ where $\sigma_t^2 = E(z_t^2 \mid \Omega_{t-1})$ and z_t has the conditional distribution function $G(z)$, $G(z) = P(z_t < z|\Omega_{t-1})$. The VaR with a given probability $\alpha \in (0, 1)$, denoted by $VaR(\alpha)$, is defined as the α quantile of the probability distribution of financial returns: $F(VaR(\alpha)) = Pr(r_t<VaR(\alpha)) = \alpha$.This quantile can be estimated as follows:

$$VaR_t(\alpha)=F^{-1}(\alpha)=\mu_t+\sigma_t G^{-1})(\alpha) \quad (4)$$

where μ_t and σ_t represent the conditional mean and the conditional standard deviation (volatility) of the returns. For estimating the volatility of the return, we use an APARCH model, which is given by the next expression:

$$\sigma_t^{\delta}=\alpha_0+\alpha_1(|\varepsilon_{t-1}|-\gamma\varepsilon_{t-1})^{\delta}+\beta\sigma^{\delta}_{t-1} \qquad \alpha_0,\beta,\delta>0,\alpha_1\geq 0,-1<\gamma<1 \quad (5)$$

In this model, the γ parameter captures the leverage effect (Black, 1976), which means that volatility tends to be higher after negative returns.

To check the accuracy of the VaR estimates, we have used five standard tests: the unconditional coverage test (LRuc) (Kupiec, 1995), backtesting criterion (BTC), independent and conditional coverage tests (LRind and LRcc) (Christoffersen, 1998) and dynamic quantile (DQ) test proposed by Engle and Manganelli (2004).

3. CASE STUDY

3.1 Dataset overview

The data consist of the S&P 500 stock index extracted from the Thomson-Reuters-Etkon database. The index is transformed into returns by taking the logarithmic differences of the closing daily price. We use daily data for the period January 3, 2000, through December 31, 2015. Table 1 shows the descriptive statistics of the daily returns, in percentage, of S&P 500.

Table 1. Descriptive Statistics (S&P500)

Mean(%)	Median(%)	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Jarque Bera
0.0084	0.0535	10.957	-9.469	1.267	-0.1859* (0.039)	11.01* (0.077)	10781 0.808

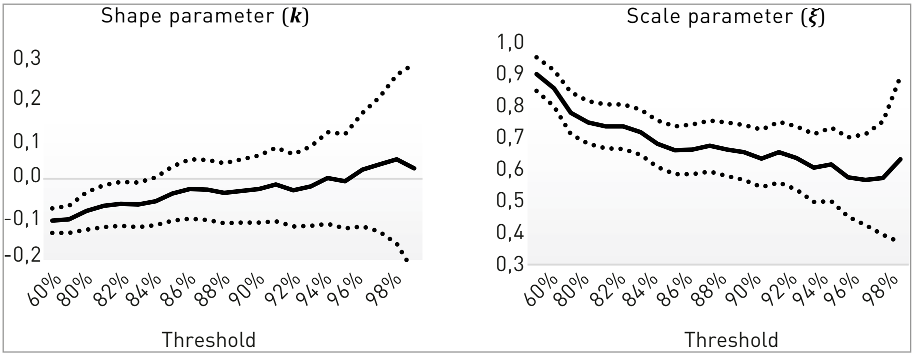
Note: Standard errors of the skewness and excess kurtosis are calculated as $\sqrt{6/n}$ and $\sqrt{24/n}$ respectively.
(*) denotes significance at the 5% level

The unconditional mean daily return is very close to zero (0.008%). The skewness statistic is negative, implying that the distribution of daily returns is skewed to the left. The kurtosis coefficient shows that the distribution has much thicker tails than the normal distribution. Similarly, the Jarque-Bera statistic is statistically significant, rejecting the assumption of normality. All this evidence shows that the empirical distribution of daily returns cannot be fit by a normal distribution, as it exhibits a significant excess of kurtosis and asymmetry (fat tails and peakness).

3.2 Parameter estimation by the maximum likelihood method

In this section, we analyse both the sensitivity of the parameters and the quantiles of the GPD to changes in threshold. For this study, we have selected a set of 22 thresholds that correspond with the β percentiles of the S&P 500 return, for β equal to 60%, 70%, 80%, 81%, 82%, 83%, 84%, 85%, 86%, 87%, 88%, 89%, 90%, 91%, 92%, 93%, 94%, 95%, 96%, 97%, 98% and 99%. Let $u_1, u_2, \ldots, u_n$ be the set of thresholds selected (n=22). For j=1,…,n, let $\hat{k}_{uj}$ and $\hat{\xi}_{uj}$ be the estimators of the shape and scale parameters based on the exceedances over the threshold u_j. The parameters have been estimated by maximum likelihood. In Figure 2 displays the estimation of k and ξ, respectively, as a function of the threshold u. We observe that as the threshold increases, the value of k increases. In the case of the scale parameter, the opposite occurs; as the threshold increases, the value of ξ is reduced. As we expected, in both cases, the accuracy of the estimations decreases as the threshold increases. Thus, in accordance with the literature, we find that the parameter estimations are very sensitive to the threshold we selected for estimating GPD.

Figure 2. Estimation of k and ξ as a function of the threshold u



Note: The dot lines represent the confident interval at 95% confidence level

3.3 Sensitivity of the risk measures to changes in the threshold

To quantify the risk, we use VaR. The expression for this measure is given by:

$$VaR_t\,(\alpha)=F^{-1}\,(\alpha)=\mu_t+\sigma_t\,G^{-1}(\alpha)$$

where σ_t represents the conditional standard deviation of the return, $G^{-1}\,(\alpha)$ is the percentile α of the GPD, and μ_t is the conditional mean return that is assumed constant ($\mu_t=\mu$). For the estimation of the conditional standard deviation of the yields, we use an APARCH model. The sample period is divided into a learning sample from January 3, 2000 to December 31, 2010 and a forecast sample from January 3, 2011 to the end of December 2015. For each day of the forecast period, we will generate estimations of the value at risk measure one day ahead at the 95% and 99% confidence levels. To evaluate the accuracy of the VaR estimates, we have used five standard tests: unconditional (LR_{uc}), backtesting criterion (BTC), independent (LR_{ind}), conditional coverage (LR_{cc}) and dynamic quantile (DQ) tests. Table 2 counts the number of rejections for the 22 thresholds considered.

Table 2. Backtesting VaR

	95% confidence level					99% confidence level				
	LR_{uc}	BTC	LR_{ind}	LR_{cc}	DQ	LR_{uc}	BTC	LR_{ind}	LR_{cc}	DQ
S&P500	0	$1_{(1)}$	0	0	0	0	0	0	0	0

Note: (1) Reject for threshold corresponding to percentile 99% (u=99

We cannot reject the null hypothesis "that the VaR estimates are accurate" for any of the thresholds selected. Only for the threshold corresponding to the 99th percentile, the backtesting criterium test (BTC) rejects this hypothesis at the 95% confidence level. In fact, for a large set of thresholds (from the 82th percentile to the 93th percentile), the number of exceptions is exactly equal to the expected one[1]. Thus, the results presented in this section indicate that the choice of threshold in the framework of the POT method may not be relevant in quantifying market risk when we use the VaR measures for this task.

4. CONCLUSIONS

The conditional extreme value theory has been proven to be one of the most successful in estimating market risk. The implementation of this method in the framework of the POT model requires choosing a threshold return for fitting the generalized Pareto distribution. Although many proposals have been made to determine the optimal threshold in the framework of the POT method, in this paper, we ask whether, in the financial field and specifically in measuring market risk, it is important to choose the threshold. To measure market risk, we have used the value at risk (VaR) and the study has been done for the S&P 500 index. The results obtained are as follows. First, we find that in accordance with the literature, the parameter estimations are very sensible to the selected threshold for estimating GPD. However, the quantiles of the GPD do not change much when the

[1] The results are available for any interested reader upon request to the authors.

threshold changes, particularly for high quantiles (95%, 96%, 97%, 98% and 99%), which are relevant in risk estimation. Third, for a large set of thresholds (from the 80th percentile to the 96th percentile), the VaR estimations at the 95% and 99% confidence levels are practically equivalent. This last result shows that in the framework of the POT method, the choice of the threshold is not relevant in the estimation of risk.

REFERENCES

Abad P., Benito S., and López-Martín C. (2014). "A Comprehensive Review of Value at Risk Methodologies". The Spanish Review of Financial Economic, 12, 15-32.

Balkema A., and De Haan L. (1974). "Residual Life Time at Great Age". The Annals of Probability, 2(5), 792-804.

Beirlant J., Vynckie P. and Teugels J.L. (1996). "Tail Index Estimation, Pareto Quantile Plots and Regression Diagnostics". Journal of the American Statistical Association, 91, 1659-1667.

Black F. (1976). "Studies in Stock Price Volatility Changes". Proceedings of the 1976 Business Meeting of the Business and Economics Statistics Section, American Association, 177-181.

Christoffersen P. (1998). "Evaluating Interval forecasting". International Economic Review, 39, 841-862.

Danielsson J., Hartmann P. and de Vries C. (1998). "The Cost of Conservatism". Risk, 11 (1), 101-103.

Embrechts P. Resnick S. and Samorodnitsky G. (1999). Extreme Value Theory as a Risk Management Tool. North American Actuarial Journal, 26, 30-41

Engle R. and Manganelli S. (2004). "CAViaR: Conditional autoregressive Value at Risk by regression quantiles". Journal of Business and Economic Statistics, 22(4), 367-381.

Jorion P. (2001). "Value at Risk: The new benchmark for managing financial risk". McGraw-Hill.

Kupiec P. (1995). “Techniques for Verifying the Accuracy of Risk Measurement Models”. Journal of Derivatives, 2, 73-84.

McNeil A.J. (1998). “Calculating Quantile Risk Measures for Financial Time Series Using Extreme Value Theory”. Available at http://e-collection.ethbib.etchz.ch/

Morgan J.P. (1996). “Riskmetrics”. Technical Document, 4th Ed. New York.

Pickands J. (1975). “Statistical inference using extreme order statistics”. Annals of Statistics, 3, 119–131.

MODELLING OF CLAIM COUNTS USING FINITE MIXTURE MODELS

Lluís Bermúdez
University of Barcelona, Riskcenter-IREA, Spain
lbermudez@ub.edu

Dimitris Karlis
Athens University of Economics and Business, Greece
karlis@aueb.gr

ABSTRACT

When modelling insurance claim counts data, the actuary often observes overdispersion and an excess of zeros that may be caused by unobserved heterogeneity. A common approach for accounting for overdispersion is to consider models with some overdispersed distribution as opposed to Poisson models. Zero-inflated, hurdle, and compound frequency models are usually applied to insurance data to account for such characteristics of the data. However, a natural way to deal with the unobserved heterogeneity is to consider mixtures of a simpler model. In this paper, we consider K-finite mixtures of some usual regression models. This approach has interesting features: first, it allows for overdispersion and the zero-inflated model represents a special case; and second, it allows for an elegant interpretation based on the typical clustering application of finite mixture models. These models are applied to an automobile insurance claims data set in order to analyse the consequences for risk selection or underwriting and risk classification or ratemaking.

1. INTRODUCTION

In a competitive market, insurance companies need to use a pricing structure that ensures that the exact weight of each risk is fairly distributed within the portfolio. If an insurance company does not achieve at least the same result on this goal than its competitors, the policyholders with lower risk will be tempted to move to another company offering better rates for them. Because of this adverse selection process, the former company will see its financial equilibrium broken, with insufficient premium income to pay for the claims reported by the policyholders with higher risk.

To avoid the above mentioned adverse selection process, a particularly complex pricing structure is designed by actuaries. A thorough review of ratemaking systems for automobile insurance can be found in Denuit (2007). Simultaneously, the actuary must determine a risk selection process (or underwriting system) by which its insurance company chooses applicants to be accepted as policyholders. Adverse selection may be also defined as the tendency of some applicants to look for insurance coverage specifically because they have a greater likelihood of filing claims. In this case, insurance companies try to reduce this effect by either raising premiums or screening out such applicants.

In short, the selection and classification or segmentation of risks involves establishing different classes of risk according to their nature and probability of occurrence. For this purpose, factors are determined in order to classify each risk, and its influence on the observed number of claims is estimated. To achieve this, risk analysis based on generalized linear models (GLM) is widely accepted. Focusing on claim frequency, a regression component is included in the claim count distribution to take the individual characteristics into account.

A very common GLM model used for these purposes has been the Poisson regression model and its generalizations (Dionne and Vanasse, 1989). However, in insurance data sets, for claim count modelling purposes, the Poisson regression model is usually rejected because of the presence of overdispersion (variance greater than mean) and an excess of zeros which cannot be fully remedied by Poisson regression models. This rejection may be interpreted as a sign that the portfolio is still heterogeneous: not all factors influencing risk can be identified, measured and introduced in the model.

The problem of unobserved heterogeneity has been addressed in the actuarial literature in many different ways: zero-inflated, hurdle, and compound frequency models. In this paper, to account for overdispersion and excess of zeros, we consider a k-finite mixture of Poisson, Negative Binomial and Generalized Poisson regressions. As Park and Lord (2009) show for vehicle crash data analysis, a finite mixture of Poisson or Negative Binomial regression models is especially useful where count data are drawn from heterogeneous populations.

The models proposed in this paper account for unobserved heterogeneity by choosing a finite number of subpopulations. The idea behind this is that the data consist of subpopulations, ``revealed" by the unobserved heterogeneity, for which the regression structure, used to account for the observed heterogeneity, is different.

In the next section, the models used here are defined. In Section 3 the database obtained from a Spanish insurance company and the results from fitting the models are summarized. Finally, some concluding remarks are given in Section 4.

2. FINITE MIXTURE OF REGRESSION MODELS

The central idea for a finite mixture of regression models is that we assume that the entire population can be split into k sub-populations (also called clusters, components or segments). Assuming a discrete valued response for the i-th individual we assume that

$$P(y_i)=P(Y_i=y_i)=\sum_{j=1}^{k}\pi_j P(y_i|\Theta_{ij}) \quad \Theta_{ij}>0,\ y_i=0,1,\ldots,$$

where $0<\pi_j<1$ with $\sum\pi_j=1$ are the mixing proportions indicating the probability that a randomly selected observation belongs to the *j*-th sub-population and $P(y|\theta)$ is some discrete distribution indexed by some parameter vector θ. In our case later $P(y|\cdot)$ will be assumed to belong to one of the Poisson, negative binomial and generalized Poisson families. Note that we assume that for each individual we have a set of parameters θ_{ij} that depend on each component and they can possibly depend on some covariate information for the i-th individual.

We further assume that the mean of the *j*-th component can be modelled by a vector of covariates containing information about the *i*-th individual, denoted by $\mathbf{x_i}$. Assuming, without loss of generality that $\theta=(\mu,\phi)$ where μ is the mean of the distribution (this can be easily obtained with a reparameterization) and ϕ some parameter related to overdispersion (set equal to 1 for the Poisson distribution), we further assume that $\log\mu_{ij}=\mathbf{x}_i'\beta_j$ where now β_j is a component specific vector of coefficients.

This modelling has some interesting features: first of all, the zero-inflated model is a special case; secondly, it allows for overdispersion; and thirdly, it allows for a neat interpretation based on the typical clustering usage of finite mixture models.

2.1 Finite mixture of Poisson regressions

The case of finite mixture of Poisson regressions is by far the more well-known and applied in practice. It dates back to Wang et al. (1996) and assumes that

$$P(y_i|\mu_{ij}) = \sum_{j=1}^{k} \pi_j \frac{\exp(-\mu_{ij})\mu_{ij}^{y_i}}{y_i\,!}, \quad y_i = 0,1,...$$

with $\mu_{ij} = \exp(x_i' \beta_j)$. Zero inflated Poisson regression is a special case. The model allows for overdispersion with respect to the simple Poisson regression model.

2.2 Finite mixture of negative binomial regressions

For the negative binomial (NB) model, we assume

$$P(y_i|\mu_{ij}, \phi_j) = \frac{\Gamma(\phi + y_i)}{\Gamma(\phi) y_i\,!} \left(\frac{\mu_{ij}}{\phi_j + \mu_{ij}}\right)^{y_i} \left(\frac{\phi_j}{\phi_j + \mu_{ij}}\right)^{\phi_j}, \quad \phi_j > 0, y_i = 0,1,...$$

and $\mu_{ij}=\exp(\mathbf{x}_i,\beta_i)$ i.e. the probability function of a NB with mean μ_{ij} and variance $\mu_{ij} + \mu_{ij}^2/\phi_j$.

Note that we assume a separate overdispersion parameter ϕ for each component. Such a model has been fitted by Zou et al. (2013). With respect to the finite mixture of Poisson regressions the model has an extra overdispersion parameter and hence allows for more flexible structure.

2.3 Finite mixture of generalized Poisson regressions

The distribution is described in detail in Consul (1989). In this paper we use the mean-parametrization, and introduce covariates to the mean, so we define

$$P(y_i|\mu_{ij}, \phi_j) = \frac{\mu_{ij}\left(\mu_{ij}+(\phi_j-1)y_i\right)^{y_i} \phi_j^{-y_i} \exp\left(-\frac{(\mu_{ij} + (\phi_j-1)y_i)}{\phi_j}\right)}{y_i\,¡}$$

with $\mu_{ij}=\exp(\mathbf{x}_i'\beta_j)$. This parametrization is used in Consul and Famoye (1992). As a model for overdispersion it has been shown that it can have different shapes than the NB model (see Nikoloulopoulos and Karlis, 2008) and hence can fit different data with different properties not satisfied by the NB.

In our application we will use all three models. It is evident that the NB and GP models also contain the simple Poisson model as special case.

3. DATA AND RESULTS

The original database is a random sample of the automobile portfolio of a major insurance company operating in Spain in 1996. The data contains information from 80,994 policyholders. Eleven exogenous variables plus the annual number of accidents recorded were considered. The description of the explanatory variables is presented in Table 1.

Table 1. Explanatory variables used in the models

Variable	Definition
V1	equals 1 for women and 0 for men
V2	equals 1 when driving in urban area, 0 otherwise
V3	equals 1 when zone is medium risk (Madrid and Catalonia)
V4	equals 1 when zone is high risk (Northern Spain)
V5	equals 1 if the driving license is between 4 and 14 years old
V6	equals 1 if the driving license is 15 or more years old
V7	equals 1 if the client is in the company for more than 5 years
V9	equals 1 of the insured is 30 years old or younger
V10	equals 1 if includes comprehensive coverage (except fire)
V11	equals 1 if includes comprehensive and collision coverage
V12	equals 1 if horsepower is greater than or equal to 5500cc

We have fitted models of increasing complexity to this data set, starting from a simple Poisson regression model. We have used AIC to select the best among a series of candidate models. All models were run in R. In Table 2 we have compared the fitted models, resulting that the best fit was obtained with the 2-Finite GP mixture model followed closely by the 2-Finite NB mixture model. Finite mixture models with were also fitted, but no improvement in terms of AIC were achieved.

Table 2. Information criteria for selecting the best model for the data

Model	Log-lik	Parameters	AIC
Poisson	-24172.5	12	48369.00
Negative Binomial	-22442.8	13	44911.60
Generalized Poisson	-22435.0	13	44895.94
Poisson-IG	-22464.0	13	44954.00
Poisson-LN	-22509.7	13	45045.46
ZIP	-22515.4	13	45056.86
ZINB	-22442.8	14	44913.60
ZIGP	-22432.4	14	44892.80
ZIPIG	-22464.0	14	44956.00
Hurdle Poisson	-22554.2	13	45134.38
Hurdle NB	-22489.8	14	45007.60
2-Finite Poisson mixture	-22493.2	25	45036.46
2-Finite NB mixture	-22419.0	27	44892.06
2-Finite GP mixture	-22415.5	27	44884.92

As expected, a large improvement is obtained by moving from a simple Poisson model to a compound frequency model with some overdispersed distributions. Zero inflated and hurdle features while providing an improvement to the basic Poisson model, allowing for overdispersion in this case, are not helpful for NB and GP models. It seems that the problem is not extra zeros but the existence of another group of policyholders. Assuming that we have two distinct subpopulations, it is necessary to move towards a finite mixture model.

In this case, using a 2-finite mixture of regression models, a large improvement is obtained by moving from one component Poisson to a 2-finite mixture of Poisson models. Note that this improvement is slightly better than that obtained with the zero inflated Poisson model. However, since the better fit is obtained by the 2-finite mixture of GP and NB regressions, it seems that there is some extra overdispersion which needs to be modelled appropriately assuming within each component an overdispersed distribution like NB and/or GP.

In Table 3 the results for the 2-Finite NB and GP mixture models are respectively summarized. We report the estimated regression coefficients for each component and p-values for testing the hypothesis that the variable is statistically significant using a Likelihood Ratio Test (LRT) statistic. Note that covariates that perhaps were not significant for a simpler model (no mixture) can be significant in the mixture model, since the two components can allow for separate effects, which are lost when combining to one model. The reason is that they have opposite signs in the mixture and hence when we estimate one coefficient for the simple case, the effect is cancelled out.

Table 3. The fitted models for both the NB and GP 2-finite mixture models. The p-value refers to that of LRT when the variables is removed from both components

	2-FM Negative Binomial			2-FM Generalized Poisson		
	1st Comp	2nd Comp	p-value	1st Comp	2nd Comp	p-value
Intercept	-3.0017	-1.8786	<0.0001	-2.9058	-1.9833	<0.0001
V1	-0.2562	0.1363	0.0031	-0.3085	0.0944	0.0483
V2	-0.0002	-0.0698	0.0006	-0.0537	-0.0556	0.1870
V3	0.1091	-0.0267	0.1888	0.2988	-0.0266	0.0822
V4	0.1230	0.1972	0.3903	0.0662	0.2177	<0.0001
V5	1.2946	-2.0342	<0.0001	0.2416	-0.6373	0.0473
V6	-5.5294	-0.2416	0.0047	-6.8859	-0.3137	<0.0001
V7	0.0076	0.2290	0.0002	-0.1480	0.2320	<0.0001
V9	0.1558	-0.0523	0.1201	0.8305	-0.5768	0.0050
V10	-0.0690	0.0997	0.1594	-0.2345	0.0804	0.2058
V11	0.0918	0.0504	0.1841	0.1427	0.0410	0.1639
V12	-0.2065	0.1533	0.0007	-0.4019	0.1403	0.0006
ϕ	0.3040	0.3270		0.2122	0.1731	
π	0.4788	0.5212		0.5195	0.4805	

Returning to the 2-finite mixture models, the group separation is characterized by low mean for the first component ("good" drivers) and high mean with higher variance for the second one ("bad" drivers).

4. CONCLUSIONS

This paper aims to exploit whether the problem of unobserved heterogeneity requires richer structure by applying finite mixture of regression models, and also to examine the extend that compound frequency models and their zero-inflated or hurdle versions fail to account for this extra heterogeneity. In order to achieve this goal, the proposed models are fitted to an automobile insurance claims data set in order to compare their goodness of fit with the traditional claim frequency models.

The 2-finite mixture models take into account the unobserved heterogeneity more effectively, providing a better classification. Assuming that the data set have been generated from 2 distinct subpopulations, the models allow for a net interpretation of each cluster separately. Note that different regression coefficients can be used to account for the ``observed'' heterogeneity within each population. In our case, the group separation is characterized by low mean for the first component ("good" drivers) and high mean with higher variance for the second one ("bad" drivers). Therefore, the 2-finite mixture models get a wider picture of the portfolio and hence better chances for risk analysis, underwriting and ratemaking purposes.

ACKNOWLEDGMENTS

Research for this paper was initiated while the second author was visiting the Riskcenter Research Group at the University of Barcelona. The authors wish to acknowledge the support of the Spanish Ministry for grant ECO2015-66314-R and ECO2016-76203-C2-2-P.

REFERENCES

Consul, P.C. (1989). Generalized Poisson distributions: properties and applications. M. Dekker.

Consul, P.C. and F. Famoye (1992). "Generalized Poisson regression model". Communications in Statistics-Theory and Methods, 21(1): 89-109.

Denuit, M., Marechal, X., Pitrebois, S. and Walhin, J.-F. (2007). Actuarial Modelling of Claim Counts: Risk Classification, Credibility and Bonus-Malus Systems. Wiley, New York.

Dionne, G. and Vanasse, C. (1989). "A generalization of actuarial automobile insurance rating models: the negative binomial distribution with a regression component". ASTIN Bulletin, 19(2):199-212.

Nikoloulopoulos, A.K. and Karlis, D. (2008). "On modeling count data: a comparison of some well-known discrete distributions". Journal of Statistical Computation and Simulation, 78(3):437-457.

Park, B.-J. and Lord, D. (2009). "Application of finite mixture models for vehicle crash data analysis". Accident Analysis and Prevention, 41(4): 683-691.

Wang, P., Puterman, M.L., Cockburn, I. and Le, N. (1996). "Mixed Poisson regression models with covariate dependent rates". Biometrics, 52(2): 381-400.

Zou, Y., Zhang, Y. and Lord, D. (2013). "Application of finite mixture of negative binomial regression models with varying weight parameters for vehicle crash data analysis". Accident Analysis & Prevention, 50(0): 1042-1051.

MODELLING MOTOR TEMPORARY DISABILITY DATA WITH PERIODICS PEAKS

Lluís Bermúdez
University of Barcelona, Riskcenter, Spain
lbermudez@ub.edu

Dimitris Karlis
Athens University of Economics and Business, Department of Statistics, Greece
karlis@aueb.gr

Miguel Santolino
University of Barcelona, Riskcenter, Spain
msantolino@ub.edu

ABSTRACT

Studies analyzing temporary consequences after a motor crash are scarcer than those analyzing permanent injuries or mortality. A regression model to evaluate risk factors affecting the duration of temporary disability after motor injury occurs is constructed using a motor insurance dataset. The length, measured in days, of medical leave after a motor accident is used here as a measure of temporary disability severity. The probability function of the number of days with medical sick leave shows spikes in multiplicities of five (working weeks), seven (calendar weeks), thirty (months), etc. To account for such a characteristic of the data, a regression model based on finite mixtures of multiple discrete distributions is proposed to fit the data properly. The model provides a very good fit when working week, week, fortnight and month multiplicities are taken into account. Characteristics of the accident such as gender, age, victim's position and severity of the permanent injuries were found significant to explain the length of temporary disability.

1. INTRODUCTION

Road traffic accidents are a major worldwide health problem being the eighth leading cause of death (WHO, 2013). Risk factors associated to mortality and permanent injuries involved in motor crashes have been widely investigated in the literature (Boucher and Santolino, 2010; Savolainen et al., 2011; Alemany et al., 2013; Mannering and Bhat, 2014). Studies analyzing temporary consequences after a motor crash are scarcer. In this paper, a regression model to evaluate risk factors affecting the duration of temporary disability after motor injury occurs is proposed.

The aim of this study is to describe the distribution and determinants of temporary disability duration outcomes for any type of motor injuries. A motor insurance claim dataset is used to evaluate the number of days of medical leave (without including days of hospitalization) taken by motor victims. The empirical frequency distribution of the length of motor temporary disability victims, measured in days, exhibits regular spikes at certain multiples (see Figure 1). The periodic peaks observed in the frequency distribution could imply the different time scales used by doctors when deciding on the number of days of sick leave before the medical reexamination. For example, it is more likely that a doctor programs a revaluation of the medical evolution of injuries in two weeks than in thirteen days. This decision can be motivated because doctors may think on a daily or weekly or monthly scale when evaluating, based on the severity of injuries, the number of days of absence for a patient. Agenda's constraints can also be a reason (i.e. the doctor only visits one day of the week). Indeed, regularly spaced spikes in the frequency distribution are observed at multiples of 5, 7, 15 and 30.

Data with periodic peaks is observed in different applications. Some examples are age misreporting (Camarda et al., 2008), number of smoked cigarettes (Wang et al., 2012) or unemployment duration (Wolff and Augustin, 2003). The phenomenon of rounding at certain multiples is known as digit preference or heaping. The literature in digit preference or heaping assumes that data can be interpreted as indirect (or rounded) observations of a latent distribution. The goal usually pursued is to model the unobserved latent variable using smoothing methods.

A different modelling approach is proposed in this paper. We model directly the random variable with peaks rather than an unobserved smoothed variable. The methodology to fit frequency data with regular spikes is based on finite mixtures of multiple discrete distribution of different multiplicities proposed by Bermudez et al. (2017). The methodology is extended to the regression modelling analysis in this article. A discrete mixture regression model is developed to fit data with regular spikes conditioned on a set of covariates. The duration of motor temporary disability is modelled including as explanatory variables characteristics of the victim (gender and age) and the accident (victim's position, type of vehicle and severity of permanent injuries).

The article is organized as follows. The regression model is presented in next section. Section 3 describes the data. Results are shown in section 4. Concluding remarks are given in section 5.

2. DISCRETE MIXTURE REGRESSION MODEL

Let $X \in N$ be a discrete random variable that takes non-negative integer values including zero. In statistics, the most frequently used parametric distributions to model discrete random variables are the Poisson distribution and the negative binomial (NB) distribution (Boucher and Santolino, 2010). Often, the variable of interest consists of the sum of lower-level units and we are interested on the analysis of the random variable measured in the lower level units. To deal with data measured in a different scale to the scale of interest, multiple discrete distributions are used. These distributions are generalizations of the discrete distributions that allow for different multiplicities (m). This generalization gives positive probability to points 0, m, 2m, … and 0 elsewhere.

When the random variable of interest can be interpreted as resulting from different subpopulations, the finite mixture distribution can be derived from distributions of the individual subpopulations. Let consider the random variable of interest $Y \in N$ is constructed as a mixture of pairwise independent discrete random variables $X_1, X_2, \ldots, X_k$ where X_j takes no negative integer values $\forall j$.

If we assume that X_j is a Poisson distributed random variable with parameter λ_j for j=1,… ,K, then the pf of Y is defined as,

$$P(Y_i = y) = \sum_{j=1}^{k} \pi_j P^{P}_{mj}(y \mid \lambda), \; y = 0,1,\ldots$$

where π_j=P(Zi=j) is the probability that the i-th observation belongs to group j with 0< π_j <1 for j=1,...,K with $\sum\pi_j$=1. This is also applicable if X_j is a NB distributed random variable. The model can be extended if we allow the mean of each component to depend on some covariates related to the i-th individual. We relate the λ's with some covariates. Estimation of the discrete mixture regression model is easy based on an EM algorithm. In Bermudez et al. (2017) an EM type algorithm was described.

3. APPLICATION

The data refer to the duration of outpatient sick leave because of a motor accident. We make use of 20,257 observations from non-fatal victims involved in traffic collisions in Spain. The data are depicted in Figure 1.

A set of regressors is considered in the modelling of the duration of the temporary disability. A description of variables is provided in Table 1.

Figure 1. Empirical distribution of the number of days of temporary disability (limited to 300 days)

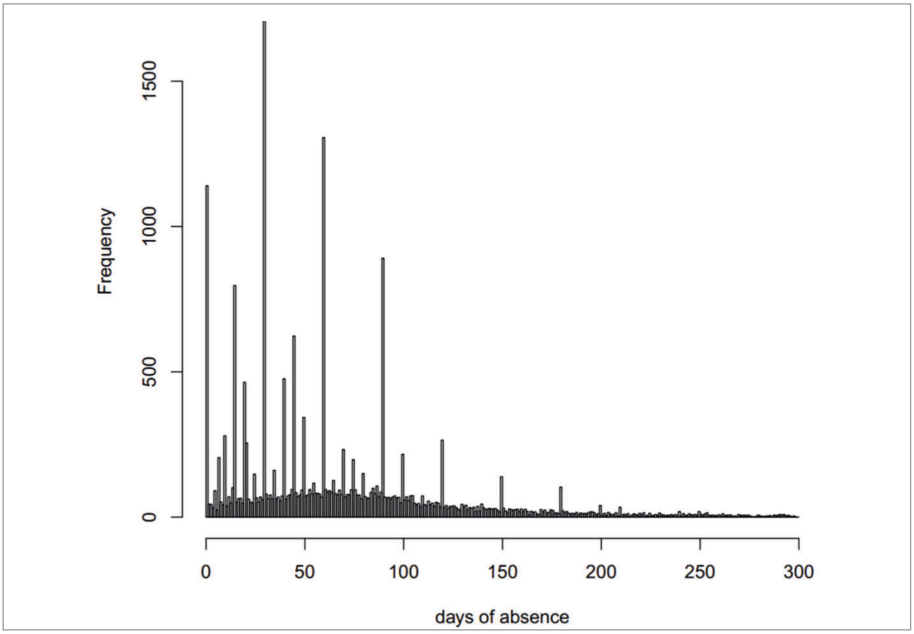


Table 1. Description of variables

Gender	1 if the injured victim is male; 0 otherwise.
Age	Age of the victim in years (divided by 10).
Driver	1 if the injured victim was the driver; 0 otherwise.
Vehicle	1 if the vehicle was a heavy vehicle; 0 otherwise.
Severity	1 if the victim has serious permanent injuries; 0 otherwise.

Based on the BIC criterion, 1-5-7-NB/15-30-Poisson regression model (negative binomial distribution for multiplicities equal to 1, 5 and 7 and Poisson distribution for 15 and 30) was selected. Figure 2 provides the empirical cumulative distribution function of the data, together with that from the fitted 1-5-7-NB/15-30-Poisson regression model and the simple 1-NB regression model. The 1-5-7-NB/15-30-Poisson regression model capture very accurately the spikes of the data.

Parameter estimates are shown in Table 2. Standard errors were obtained on 1,000 non-parametric bootstrap replications. Last two columns show the p-value of the likelihood ratio test (LRT) statistic when the variable is included or excluded from the model and the marginal effect of the binary covariates (ME).

Results show that all covariates were statistically significant except vehicle. The marginal effect of a dichotomous covariate was computed as the difference between the expected value of the period of sick leave when the covariate took value 1 (presence) and when the covariate took value 0 (absence). Sample mean values were considered for the rest of covariates. One can see that the existence of serious permanent injures produces the largest effect on the expected sick leave duration. According to our results, males, drivers and elders have longer periods of temporary sick leave recovering from injuries after a motor accident. These results are in line with other previous studies.

Figure 2. Empirical distribution function and the fitted cdfs from different models

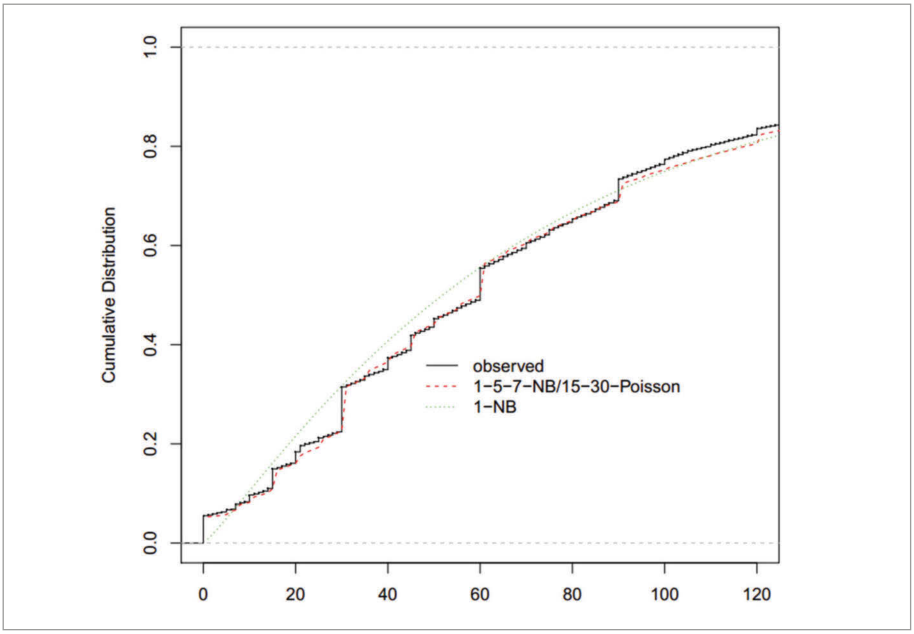


Table 2. Results from fitting the 1-5-7-NB/15-30-Poisson regression model

	Coefficients (St.error) for each component					Significance	
	$m=1$	$m=5$	$m=7$	$m=15$	$m=30$	LRT	ME
Constant	4.313 (0.026)	1.842 (0.052)	0.862 (0.259)	0.598 (0.086)	0.543 (0.043)		
Gender	0.083 (0.017)	0.001 (0.042)	−0.046 (0.228)	−0.060 (0.097)	0.101 (0.037)	< 0.001	6.37
Age	0.040 (0.005)	0.071 (0.011)	−0.209 (0.111)	0.025 (0.022)	0.008 (0.009)	< 0.001	
Driver	−0.079 (0.018)	0.121 (0.039)	0.070 (0.200)	−0.037 (0.091)	−0.027 (0.035)	< 0.001	9.45
Vehicle	0.028 (0.035)	0.001 (0.117)	−0.013 (0.566)	−0.358 (0.280)	0.212 (0.074)	0.143	4.99
Severity	1.072 (0.024)	1.711 (0.079)	3.658 (0.380)	1.939 (1.343)	0.736 (0.162)	< 0.001	164.40
Mixing prop.	0.535 (0.005)	0.161 (0.004)	0.030 (0.003)	0.090 (0.005)	0.184 (0.005)		
NB size	1.876 (0.033)	3.119 (0.181)	9.581 (9178.520)				

4. CONCLUSIONS

This study proposes a regression model suitable for random variables with regular peaks. To account for digit preference/heaping, the regression model is based on finite mixtures of multiple discrete distributions. We found that the negative binomial-Poisson mixture regression model provided the best performance when working week, week, fortnight and month multiplicities were considered (1-5-7-NB/15-30-Poisson). This regression model specification captured very accurately the spikes of the data and the resemblance of its cumulative distribution function to the empirical one was very close compared with standard alternatives, like the usual negative binomial regression model.

The analysis of the risk factors influencing in the length of medical leave showed that characteristics of the accident such as gender, age, victim's position and severity of the permanent injuries were statistically significant to explain the expected sick leave duration. The factor indicating serious permanent injuries was found with the largest impact on the expected temporary disability. The gender, age and victim's position had more moderated impacts. Male and drivers were associated with higher temporary disabilities. Finally, young victims had shorter periods of temporary disabilities. The expected temporary disability duration was relatively stable until early thirties and increasing from this age on.

ACKNOWLEDGMENTS

Research for this paper was initiated while the second author was visiting the Riskcenter Research Group at the University of Barcelona. The authors wish to acknowledge the support of the Spanish Ministry for grant ECO2015-66314-R and ECO2016-76203-C2-2-P.

REFERENCES

Alemany, R., Ayuso, M. and Guillen, M. (2013). "Impact of road traffic injuries on disability rates and long-term care costs in Spain". Accident Analysis and Prevention, 60: 95-102.

Bermudez L., Karlis D. and Santolino, M. (2017). "A finite mixture of multiple discrete distributions for modelling heaped count data. Computational Statistics and Data Analysis, 112(Supplement C): 14- 23.

Boucher, J.P. and Santolino, M. (2010). "Discrete distributions when modeling the disability severity score of motor victims". Accident Analysis & Prevention, 42(6): 2041-2049.

Camarda, C.G., Eilers, P. and Gampe, J. (2008). "Modelling general patterns of digit preference. Statistical Modelling, 8(4): 385-401.

Mannering, F. and Bhat, C. (2014). "Analytic methods in accident research: Methodological frontier and future directions". Analytic Methods in Accident Research, 1: 1-22.

Savolainen, P.T., Mannering, F.L., Lord, D. and Quddus, M.A. (2011). "The statistical analysis of highway crash-injury severities: a review and assessment of methodological alternatives". Accident Analysis and Prevention, 43:1666-1676.

WHO (2013). Global status report on road safety 2013. World Health Organization.

Wang, H., Shiman, S., Griffith, S.D. and Heitjan, D.F. (2012). "Truth and memory: Linking instantaneous and retrospective self-reported cigarette consumption". The Annals of Applied Statistics, 6(4):1689-1706.

Wolff, J. and Augustin, T. (2003). "Heaping and its consequences for duration analysis. Allgemeines Statistisches Archiv, 87: 1-28.

BIASED TRANSFORMED KERNEL ESTIMATOR OF EXTREME QUANTILES

Catalina Bolancé
University of Barcelona, Department of Econometrics, Riskcenter-IREA
bolance@ub.edu

Ramon Alemany
University of Barcelona, Department of Econometrics, Riskcenter-IREA
ralemany@ub.edu

ABSTRACT

When the data are right skewed, the classical kernel estimator (CKE) does not smooth the right tail of the probability and cumulative distribution functions. For estimating probabilities and conditional probabilities associated with the extreme quantiles of a random variable, we propose an estimator that combines a parametric distribution assumption with a kernel estimator. To compare it to the CKE, we carry out a simulation study that shows how our proposal is better when the distribution is heavy-tailed. Finally, we apply the methodology to the estimation of extreme quantiles in the age-at-death distribution of the Spanish population.

1. INTRODUCCIÓN

We propose an estimator for the cumulative distribution and the quantile functions that involves combining nonparametric with parametric fits. We show that this estimator works when the aim is to estimate an extreme pth quantile and the distribution is heavy-tailed.

The concept of extreme pth quantiles of a statistical distribution is related to the need to evaluate a quantity that, in practice, is of very small frequency and which, in theory, is of very low probability. In this work, we refer to an extreme quantile when $p \geq 0.95$. The extreme pth quantile is widely used in risk quantification, where it is known as the value-at-risk (VaR). Both financial and insurance companies need to estimate this VaR to calculate their solvency capital requirement. However, the need to quantify risk, and evaluate the loss associated with a very "rare event", is an increasingly common need in all economic activities. This need gives rise to problems of efficiency when employing either parametric or nonparametric estimators.

Nonparametric methods are inefficient for estimating the probability of a very "rare event". Likewise, in many instances, when the distribution has a heavy tail, parametric methods do not provide a good fit for extreme values, for example, those in the right tail of the distribution.

Azzalini (1981) analyses the properties of the kernel estimator of the cdf and the quantile. Alternatively, the kernel quantile estimators (Harrell and Davis, 1982; Sheather and Marron, 1990) improve the finite-sample properties provided by the estimator proposed by Azzalini (1981), which is based on calculating the inverse of the classical kernel estimator of the cdf. However, the kernel quantile estimator fails when the aim is to estimate an extreme quantile of a heavy-tailed distribution. For estimating these extreme quantiles we define a biased but more efficient transformed kernel estimator of the cdf and the quantile, and we analyse its properties.

In our application of the estimator, we analyse the age-at-death variable for the population aged 65 and over in Spain. Such an analysis is fundamental for evaluating the costs associated with the longevity of a population, for example, costs related to public or private health insurance, long-term care systems and pensions.

2. BIASED TRANSFORMED KERNEL ESTIMATOR

The biased transformed kernel estimator (BTKE) involves selecting a cdf as the first transformation $\boldsymbol{T}$ of the original variable and then applying an optimal second transformation.

Let $\boldsymbol{T}(\cdot)$ be a first transformation coinciding with a cdf and $\boldsymbol{M}$ be a second cdf transformation, then the BTKE of the original cdf $(\boldsymbol{F_X(x)})$ is defined as:

$$\widehat{F}_X^{BT}(x) = \widehat{F}_{M^{-1}[T(X)]}\{M^{-1}[T(X)]\} = \frac{1}{n}\sum K\left(\frac{M^{-1}[T(x)] - M^{-1}[T(X_i)]}{b}\right)$$

where b is the smoothing parameter.

If $M(\cdot)$ is the cdf associated to the $Beta(3,3)$:

$$M(y) = \frac{3}{16}y^5 - \frac{5}{8}y^3 + \frac{15}{16}y + \frac{1}{2}, \quad -1 \leq y \leq 1$$

then the BTKE has optimality properties (see Alemany et al., 2013).

For convenience, we assume that both, F_X and T, are type I extreme value distributions (Gumbel) or type II extreme value distributions (Fréchet) (see, for example, Jenkinson, 1955).

We can analyse the bias and variance of the BTKE and obtain that for a given smoothing parameter b the bias is:

$$E\left[\widehat{F}_X^{BT}(x)\right] - F(x) \approx \left[\frac{m'\{M^{-1}[T(x)]\}}{m'\{M^{-1}[T(x)]\}} f_X(x) + \frac{f_X'(x)}{T'(x)} m\{M^{-1})[T(x)]\}\right] \frac{b^2}{2} \int t^2 k(t)dt$$

and the variance is:

$$V[\widehat{F}_X^{BT}(x)] \approx \frac{F_X(x)[1-F_X(x)]}{n} - \frac{f_X(x)}{T'(x)} m\{M^{-1}[T(x)]\}\frac{b}{n}(1-\int K^2(t)dt)$$

From the expressions of the variance and bias of BTKE we can deduce that, if the true distribution is heavier-tailed than the transformation distribution, for an extreme pth quantiles $x_p = Q_p(X)$, the mean squared error (MSE) of BTKE is lower than the MSE of CKE.

2.1. Smoothing parameter

If we assume $F_X(\cdot) = T(\cdot)$ and $M(\cdot)$ is the $Beta(3,3)$ cdf, the double transformed data follow the $Beta(3,3)$ distribution, and then an asymptotically optimal bandwidth based on minimising the asymptotic MSE of $F_X\ (x_p)$ for estimating $x_p = Q_p\ (X)$ can be used. The resulting smoothing parameter is:

$$b = \left(\frac{m\,\{M^{-1}[T(x_p)]\}\left(1-\int_{-1}^{1} K^2\,(t)dt\right)}{4\left(\frac{1}{2}m'\,\{M^{-1}\left[T(x_p)\right]\}\int_{-1}^{1} t^2\,k(t)dt\right)^2} \right)^{\frac{1}{3}} n^{-\frac{1}{3}}$$

where $m(\cdot)$ is the $Beta(3,3)$ pdf.

The smoothing parameter b is asymptotically optimal if the true cdf fulfils $F_X\ (x_p) = T(x_p)$. However, if $F_X\ (x_p) \neq T(x_p)$ then the bandwidth could be lower or higher than the optimal.

2.2. Selecting the transformation

We assume that $T = T_\theta$ is a parametric cdf and $\widehat{\theta}$ is an estimator of θ, then the asymptotic properties of the BTKE using $T_{\widehat{\theta}}\ (x)$ are equal to those of the Section 2. We compare the ML estimator with more robust alternatives based on distances.

The minimum distance estimators compare true cdf F_X with empirical cdf $\widehat{F}_X$. The three classical criteria are based on the Kolmogorov distance, the Cramér-von Mises statistic and the Anderson-Darling statistic (see DAgostino and Stephens, 1986). Luceño (2006) modified the Anderson-Darling (AD) statistic, thereby, making it possible to give more weight to the left or right tail. Here, therefore, considering that our focus is on the right tail, we use the AD statistic and its right tail modifications of first (ADR) and second order (AD2R) to estimate the parameters θ on the first transformation $T_{\widehat{\theta}}$.

3. SIMULATION STUDY

We analyse the finite sample properties of the CKE, HD (Harrell and Davis, 1982) and BTKE. We generate $2{,}000$ samples of size $\mathrm{n} = 500$ and $\mathrm{n} = 5{,}000$ from different distributions (Weibull with shape parameter , lognormal and two lognormal and Pareto mixtures -70% -30% and 30% -70%, respectively). In all cases, we consider positive skewness and different tail shapes, but our estimator can also be used for alternative distributions with different shapes.

We select three combinations of parameters for the four distributions under analysis. The Weibull (We) and lognormal (Ln) are Gumbel extreme value distributions and the lognormal-Pareto mixtures (Ln-Pa) are Fréchet extreme value distributions. The Weibull, followed by the lognormal, are the least heavy tailed. Conversely, the lognormal-Pareto mixtures are the distributions with the heaviest tails; indeed, when the Pareto shape parameter is $\rho \leq 1$ they do not have a first-order moment.

The results in Table 1 compare the $\widehat{MSE}$ of the CKE with the $\widehat{MSE}$ obtained when using the HD and BTKE for $n = 5{,}000$, respectively. For this comparison, we calculate the quotient between the MSE estimated and that estimated for the CKE, so that when this quotient is less than 1 the corresponding estimator improves the CKE. In some cases BTKE

Table 1. Ratio between the MSE of estimated quantiles using HD and BTKE and the MSE of CKE. Sample size used $n = 5{,}000$

Method		p	Weibull		Lognormal		70%Ln-30%Pa		30%Ln-70%Pa	
HD		0.95	0.999		1.019		0.995		1.000	
		0.99	0.973		1.018		1.043		1.092	
		0.995	0.966		1.008		1.191		1.143	
	T_θ		ML	ADR	ML	ADR	ML	ADR	ML	ADR
BTKE	Ch	0.95	1.018	1.024	1.000	1.004	0.977	0.976	0.955	0.955
		0.99	1.087	1.091	1.037	1.024	0.901	0.902	0.990	0.991
		0.995	1.108	1.107	1.060	1.038	0.922	0.923	0.951	0.950
	We	0.95	0.991	0.992	0.991	0.990	0.978	0.978	0.970	0.970
		0.99	0.943	0.944	0.954	0.942	0.904	0.904	0.925	0.925
		0.995	0.921	0.922	0.890	0.849	0.647	0.647	0.602	0.602
	Ln	0.95	1.144	1.043	0.992	0.993	0.971	0.969	0.961	0.958
		0.99	1.175	1.066	0.975	0.976	0.885	0.856	0.946	0.926
		0.995	1.137	1.051	0.971	0.972	0.792	0.674	0.837	0.777
	T_θ		AD	AD2R	AD	AD2R	AD	AD2R	AD	AD2R
BTKE	Ch	0.95	0.944	0.001	0.930	0.984	0.965	0.991	0.948	0.949
		0.99	0.953	0.003	0.946	0.972	1.042	0.950	1.007	0.957
		0.995	0.936	0.006	1.005	0.988	1.085	0.921	1.088	0.953
	We	0.95	0.928	0.001	0.917	0.998	0.957	0.977	0.950	29.871
		0.99	0.872	0.003	0.827	0.945	1.022	0.932	0.956	1.018
		0.995	0.835	0.005	0.764	0.932	0.774	0.659	0.750	0.666
	Ln	0.95	1.025	0.001	0.920	0.990	0.951	-	0.941	-
		0.99	1.023	0.003	0.888	0.953	1.001	-	0.960	-
		0.995	0.971	0.006	0.923	0.953	0.891	-	0.954	-

4. DATA ANALYSIS

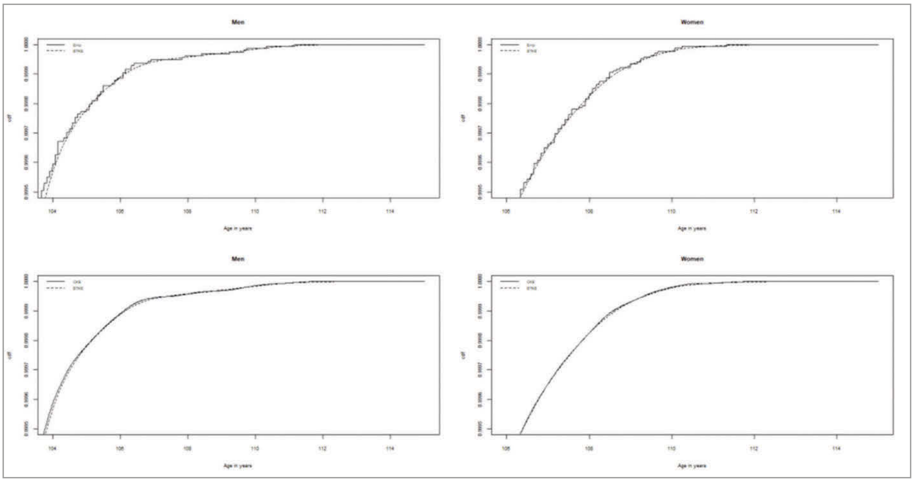
We analyse the longevity of the Spanish population over 65 years in 2011. Accurate estimation of the age-at-death distribution has significant economic consequences for planning the sustainability of public and private pensions, and other health and dependency systems. For the Spanish population over 65, we fitted the distribution of the age-at-death in months and we estimated extreme quantiles and conditional extreme quantiles with p = 0.995 and p = 0.999.

On average, the age-at-death for women is higher than that for men, but that the maxima are similar. Analysing the histograms for men and women we can see that the distributions of the variable age-at-death for the population aged 65 and over is not easy to fit using simple parametric models. We could use mixtures of distributions, adapted that is to the different behaviours of the age-at-death variable (Jasiulewicz, 1997). However, in those ranges of variable values where the sample information is scarce the results remain inefficient. In these cases, our proposed BTKE becomes a good alternative (see Alho and Spencer, 2005, for a review on demography models).

In order to estimate the quantiles and conditional quantiles of the age-at-death variable, the first step for obtaining the BTKE described in Section 2 involves estimating the parametric cdf to be used as the first transformation T_θ. We obtained the results for the three distributions used in the simulation study in Section 3, that is the Weibull, lognormal and Champernowne, and for two methods for estimating parameters also used in the simulation: namely ML, AD, ADR and AD2R. The results shown here are obtained using the AD estimation method and the Weibull cdf for the first transformation.

In Figure 1, we plot the BTKE of the cdf using as the first transformation $T_{\hat{\theta}}$, the Weibull previously estimated using the AD method together with the empirical cdf (top) and CKE (bottom). We can see that BTKE smooths and extrapolates the empirical cdf.

Figure 1. Empirical (Emp) cdf (top) and CKE (bottom) with BTKE using Weibull distributions for men (left) and women (right)



In Table 2 we show the some extreme quantiles of the age-at-death variable for men and women obtained from the BTKE of the cdf plotted in Figure 1.

Table 2. Estimated quantiles of age-at-death variable obtained with BTKE

p	0.95	0.99	0.995	0.999	0.9995	0.9999
Men	93.89	98.49	99.98	102.88	103.82	106.12
Women	97.40	101.23	102.58	105.29	106.39	108.62

5. CONCLUSIONS

Extreme situations occur for which data are scarce, then parametric and nonparametric methods are inefficient. We propose the biased transformed kernel estimator (BTKE), which is easily applicable and guarantees that these extreme quantiles will be consistently estimated. Furthermore, we propose a straightforward formal method to obtain the bandwidth at the desired quantile.

AGRADECIMIENTOS

Este trabajo ha sido financiado por el Ministerio de Educación (beca ECO2016-76203-C2-2).

REFERENCES

Alemany, R., Bolancé, C., Guillén, M. (2013). A nonparametric approach to calculating value-at-risk. Insurance: Mathematics and Economics 52, 255–262.

Alho, J., Spencer, B. (2005). Statistical demography and forecasting. Springer, New York.

Azzalini, A. (1981). A note on the estimation of a distribution function and quantiles by a kernel method. Biometrika 68, 326–328.

Harrell, F., Davis, C. (1982). A new distribution-free quantile estimator. Biometrika 69, 635–640.

Jasiulewicz, H. (1997). Risk measures, distortion parameters, and their empirical estimation. Insurance: Mathematics and Economics 19 (3), 237–241.

Jenkinson, A. F. (1955). The frequency distribution of the annual maximum (or minimum) values of meteorological events. Quarterly Journal of the Royal Meteorological 81, 158–172.

Sheather, S., Marron, J. (1990). Kernel quantile estimators. Journal of the American Statistical Association 85, 410–416.

SCALE-FREE AND LOCATION-FREE RISK MEASURES

Catalina Bolancé
Universidad de Barcelona

Alejandra Cabaña
Universidad Autónoma de Barcelona

Albert Pitarque
Universidad Politécnica de Barcelona

Isabel Serra
Centre de Recerca Matemàtica
Barcelona Supercomputing Center

ABSTRACT

A discussion around characterization risk profile is propose, bearing in mind that a risk profiles should be scale-free and location-free with respect to the stochastic characterization of losses. The Extreme Value Index (EVI) is introduced as a measure of risk profile. Statistical learning methodologies are proposed and an illustrative example with empirical data is shown.

1. INTRODUCTION

The concept of coherent risk measures introduces a fundamental behavior around the scale and location characterization of the stochastic properties of losses. However, the concept of risk can be alternatively characterized along scale-free and location-free properties in order to get a more objective description of the risk behavior of an asset. In this proposed approach, the risk is associated to the fact of evaluating to what extent much greater loss than those most frequent is possible, regardless of the specific values of the losses, given that these can be considered afterwards. The scale-free and location-free properties of a risk measure could be useful if we want to define risk profiles through to use data-mining techniques in supervised learning methodologies.

In classical risk analysis several techniques are produced for using classical statistical ones with some modifications. For instance, value-at-risk analysis with covariates can be modelled with quantile linear regression obtained from linear regression. Accordingly, the objective is not to get a conditional mean expectation

for the target, the objective is to obtain a quantile of the conditional law of the target. In this case, it is known that more accurate modifications are required for high-quantile estimation. Usually those come from extreme value theory [3,7] or non-parametric methods [2].

Risk analysis has an important challenge in order to identify the capabilities of the information obtained in amount of the complex and high-dimensional data, for instance, in credit risk or in fraud detection. The techniques for obtaining analysis in this framework can be identified with the machine learning or data-mining or classical multivariate statistics. In any case, the main examples are trees, random forest, neural networks. Each one of them can be modified in the same way as the above example on value-at-risk [6,8,4], however the same problem of modelling tails appears. The main point in our approach for improving those techniques is to use scale-free and location-free risk measures; the main example is The Extreme Value Index (EVI). The risk characterization through EVI estimation [5] can be used for risk profile characterization. The key point for introducing EVI in data mining techniques is to get a way for comparing EVI estimation, and a solution is proposed. Finally, a nice example on the utility of EVI for risk profile in portfolio composition is shown.

2. METHODOLOGICAL APPROACH

Given a risk measure ϱ, the properties of the scale and location of the stochastic behavior of losses, X, are considered from coherent risk measures approach [1] and are described by

$\varrho(\alpha X) = \alpha \varrho(X)$

$\varrho(\psi + X) = \psi + \varrho(X)$

where α and ψ are scale and location parameters of the given random variable X. However, the invariance of risk measures can be described alternatively, through a scale-free and location-free risk measure defined by

$\varrho(\alpha X) = \varrho(X)$

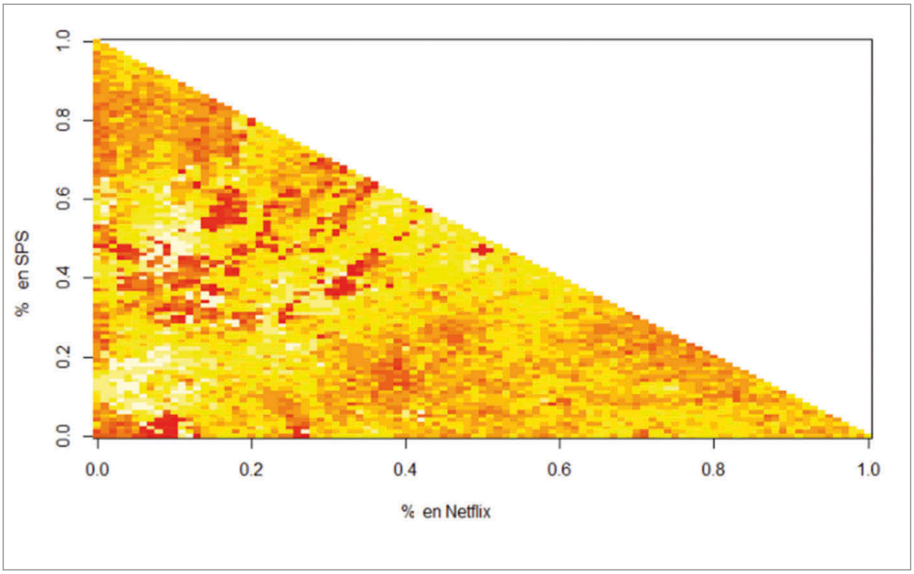
$\varrho(\psi + X) = \varrho(X)$

The EVI as a risk measure has this properties. Exact hypothesis testing for comparing two populations in terms of its EVIs is developed. The exact hypothesis test uses as statistic the ratio between its EVI. With assumptions on the method for estimating EVI, the statistic of the test has a known distribution, a beta prime, allowing us to make inference about the behavior of the tails of the underlying distribution.

3. APPLICATION FOR PORTFOLIO COMPOSITION

We have considered the returns of McDonald's, SPS Commerce and Netflix, from January 3rd, 2011 to December 31st, 2016. The business sectors to which the companies belog are fast-food, software provider and entertainment respectively. Our main purpose is to get a portfolio composition such that the losses risk profile are less or equal than the risk profile for the earnings. Hence, we need to compare the left and right tails of the distribution of the returns of a portfolio. Furthermore, a desirable property is that both tails have a similar EVI, because that would mean hat the unexpected earnings and loses could be considered similar.

Figure 1. Map of p-values for testing comparative on EVI for right and left tails of historical returns varying portfolio composition



Summary of result is shown on Figure 1, through colors, the p-value obtained for testing the null hypothesis: "EVI for both tails of profits of the portfolio are equal". Each p-value is obtained for the analysis of the corresponding law of profits for the portfolio composition: x-axis percentage of Netflix, y-axis percentage of SPS and the rest percentage for McDonald's. The white points correspond to composition with small p-value, i.e. with clearly smaller risk profile for losses. In general, we found that for some portfolios we could consider that tails were different and we were able to distinguish between some portfolios that should be avoided because unexpected losses were higher than unexpected earnings. We also found some cases of portfolios where unexpected earnings were higher than unexpected loses.

4. CONCLUSIONS

This paper has presented an alternative method to characterize risk profile. This measures can be introduced in order to perform data mining techniques as a future work.

ACKNOWLEDGMENTS

This work has been partially supported by the Spanish Ministerio de Economía Industria y Competitividad MTM2015-69493-R, FIS2015-71851-P, "La Caixa" Foundation and 2014SGR-1307 from AGAUR.

REFERENCES

Artzner, P., Delbaen, F., Eber, J-M and Heath, D. "Coherent risk measures", Mathematical Finance, (1999).

Bolancé C, Ayuso M and Guillén M. A nonparametric approach to analyzing operational risk with an application to insurance fraud. The Journal of Operational Risk, (2012).

Coles, S. An Introduction to Statistical Modeling of Extreme Values". Springer, (2010).

Cannon, A.J. Quantile regression neural networks: Implementation in R and application to precipitation downscaling. Computers & Geosciences, (2011).

Castillo, J. del and Serra, I. Likelihood inference for generalized Pareto distribution. Computational Statistics & Data Analysis, (2015).

Chaudhuri, P. and Loh, W. Nonparametric estimation of conditional quantiles using quantile regression trees. Bernoulli, (2002).

Gomes, M.I. and Pestana, D. A Sturdy Reduced-Bias Extreme Quantile (VaR) Estimator Journal of the American Statistical Association, (2007).

Meinshausen, N. Quantile Regression Forests. Journal of Machine Learning Research, (2006).

ESTIMACIÓN MAXIMO VEROSÍMIL CONDICIONADA DEL MODELO LINEAL GENERALIZADO CON FUNCIÓN DE LIGADURA NO PARAMÉTRICA

Catalina Bolancé
Universidad de Barcelona
Departamento de Econometría Estadística y Economía Aplicada
Riskcenter-IREA
bolance@ub.edu

Ricardo Cao
Universidad de A Coruña, Departamento de Matemáticas
Research Group Modes, CITIC, ITMATI
ricardo.cao@udc.es

Montserrat Guillen
Universidad de Barcelona
Departamento de Econometría Estadística y Economía Aplicada
Riskcenter-IREA
guillen@ub.edu

RESUMEN

Concretamente, cuando la variable dependiente mide una cuantía económica, sea ésta, por ejemplo, un coste, un beneficio o una duración, su distribución suele ser de cola pesada y en muchas ocasiones no tiene una forma fácil de modelizar. El modelo lineal generalizado con función de ligadura no paramétrica, ampliamente conocido como modelo *single-index* (índice único), no establece restricciones en la distribución de la variable dependiente ni en la forma de la función de ligadura con el predictor lineal. Sin embargo, la estimación de este modelo no paramétrico plantea algunos retos. Este trabajo propone un método basado en la maximización de la verosimilitud condicionada que aborda algunas de las dificultades relacionadas con los modelos de pérdida económica.

1. INTRODUCCIÓN

Los modelos lineales generalizados son una herramienta muy común en la estimación de gran diversidad de modelos económicos. Particularmente, el modelo lineal generalizado basado en la distribución log-normal es ampliamente utilizado en la modelización de variables de naturaliza cuantitativa que miden ingresos, gastos o costes. Sea Y la variable dependiente y $X = (X_1, \dots, X_d)'$ un vector de variables explicativas o covariables, asumiendo que el logaritmo neperiano $\ln(Y)$ tiene distribución normal y, por tanto, la función de ligadura es logarítmica, se especifica el modelo:

$$\ln(Y) = \theta' X + \varepsilon \qquad (1)$$

donde $\theta = (\theta_1, \dots, \theta_d)'$ es un vector con $d \geq 2$ parámetros que necesitamos estimar y ε es una variable aleatoria con $E(\varepsilon) = 0$ y $E(X\varepsilon) = 0$. El vector de parámetros θ en (1) puede estimarse de forma óptima por el método de mínimos cuadrados ordinarios o por mínimos cuadrados generalizados. Incluso, si no se cumple la propiedad $E(X\varepsilon) = 0$, el uso de variables instrumentales puede resolver el problema de inconsistencia de los estimadores obtenidos por mínimos cuadrados.

El modelo especificado en (1) se basa en que la distribución de la variable aleatoria condicionada $ln(Y)| X$ es normal y, por tanto, $f(\ln(y) \mid x) = f_\theta(\ln(y) \mid \theta' x)$ y $F(\ln(y) \mid x) = F_\theta(\ln(y) \mid \theta' x)$ son, respectivamente, la función de densidad y la función de distribución de la distribución normal con esperanza $E[\ln(Y)] = \theta' x$.

En el contexto de la cuantificación de riesgos, cuando el objetivo es modelizar una variable que mide una pérdida en función de un conjunto de variables explicativas, el modelo lineal especificado en (1) basado en la distribución normal no proporciona un buen ajuste ni de los parámetros asociados a las explicativas ni de la predicción; incluso, suele concluirse que el efecto de las covariables no es estadísticamente significativo, debido a la inconsistencia de la estimación. La regresión no-paramétrica es una alternativa al modelo especificado en (1). En general, el modelo de regresión no-paramétrica plantea la estimación directa de la esperanza condicionada $E(Y \mid X)$ (ver Härdle,1990). En la práctica, la regresión no-paramétrica presenta dos dificultades importantes. La primera es la creciente dificultad de la estimación cuando el número de covariables aumenta (problema de dimensionalidad). La segunda dificultad de la estimación no-paramétrica radica en que la interpretación de los efectos de las variables explicativas no puede realizarse de forma directa. Otra alternativa al modelo especificado en (1) es el modelo aditivo generalizado (ver Hastie y Tibshirani, 1990), sin embargo, este comparte las dificultades descritas para la regresión no paramétrica.

Ante todo lo dicho anteriormente, en este trabajo proponemos el uso del modelo *single-index* para la estimación de los parámetros del modelo lineal generalizado, basado en la maximización de la verosimilitud condicionada (Strzalkowska-Kominiak y Cao, 2013, proponen la maximización de la verosimilitud condicionada para el caso que incluye censura en la variable dependiente). El modelo single-index plantea que la relación entre la variable dependiente y las covariables se establece a partir del predictor lineal $\theta'X$, siendo la ligadura entre ambos una función desconocida que se estima de forma no paramétrica.

La estimación del modelo lineal generalizado con función de ligadura no paramétrica, ampliamente conocido como modelo *single-index*, ha sido abordada en la literatura de distintas formas. En todas ellas, el objetivo es doble, por un lado éste consiste en obtener un estimador para los coeficientes asociados a las variables explicativas en el predictor lineal y, por otro, es imprescindible determinar el grado de alisamiento óptimo relacionado con la estimación de la distribución condicionada o con aquellas funciones específicas que se aproximan no paramétricamente asociadas a cada método (ver, por ejemplo, Härdle et al., 1993; Hristache et al., 2001 y Delecroix et al., 2003).

Con un ejercicio de simulación, en este trabajo mostramos cómo si la distribución de la variable condicionada $Y \mid X$ es de cola pesada, los métodos citados en el párrafo anterior no proporcionan un ajuste eficiente de los parámetros en el predictor lineal. Alternativamente, también mostramos como nuestro método basado en la maximización de la verosimilitud condicionada proporciona estimaciones de los parámetros más eficientes y con menor sesgo para distintas formas de la distribución. En definitiva, esta puede ser una herramienta adecuada en la modelización económica de las variables relacionadas con la cuantificación del riesgo.

2. EL MODELO *SINGLE-INDEX* Y SU ESTIMACIÓN

El modelo *single-index* representa un modo de generalizar el modelo de regresión lineal y supone que la dependencia entre una variable aleatoria Y y un vector de covariables

$X=(X_1,\dots, X_d)'$, que también podría ser aleatorio, puede representarse de forma general como:

$$Y = g(\theta' X) + \varepsilon \,, \qquad (2)$$

donde $\theta=(\theta_1,\dots,\theta_d)'$ es un vector con $d \geq 2$ parámetros que necesitamos estimar y ε es una variable aleatoria con $E(\varepsilon \mid X)=0$.

Sea $\theta=(0_1,\dots,0_d)'$ el vector de parámetros que cumple la propiedad:

$$f_{\theta_0}(y \mid \theta_0' \mathrm{x}) = f(y \mid x) \,, \qquad (3)$$

donde $f_{\theta_0}(y \mid \theta_0' x)$ es la función de densidad de Y condicionada a $\theta_0' X = \theta_0' \mathbf{x}$. Se sabe que si $\theta_0 \in R^d$ cumple (3), para algún valor real λ, el vector $\lambda\theta_0$ también cumple (3), Por tanto, existen infinitos valores para los parámetros en (2). Para resolver el problema de identificación de los parámetros en este trabajo introducimos la restricción $\theta_{0,1}=1$.

Si suponemos que la distribución de la variable dependiente es conocida, la estimación del vector de parámetros en (2) puede realizarse maximizando la función de verosimilitud condicional que, para una muestra de observaciones de la variable dependiente Y_i y el vector de covariables $X_i= (X_{i,1},\dots, X_{i,d})$, $i=1,\dots,n$, equivale a $L_n(\theta) = \prod_{i=1}^{n} f_\theta(Y_i \mid \theta' X_i)$. Sabemos que maximizar $L_n(\theta)$ equivale a maximizar:

$$l_n(\theta) = \frac{1}{n} \ln(L_n(\theta)) = \frac{1}{n} \sum_{i=1}^{n} \ln\left(f_\theta(Y_i \mid \theta' X_i)\right). \qquad (4)$$

Se obtiene el estimador del vector de parámetros como:

$$\hat{\theta}_n = \arg\max_\theta l_n(\theta). \qquad (5)$$

Sin embargo, en la práctica la distribución es desconocida, por tanto, para calcular (4) tenemos que estimar la función de densidad $f_\theta(y \mid \theta' \mathrm{x})$ y para ello utilizamos el estimador núcleo de la densidad condicionada (ver Bashtannyk y Hyndman, 2001) que expresamos como:

$$\hat{f}_\theta(y \mid \theta' \mathrm{x}) = \hat{f}_{\theta,h_1,h_2}(y \mid \theta' \mathrm{x}) = \frac{\hat{r}(\theta' \mathrm{x}, \mathrm{y})}{\hat{s}(\theta' \mathrm{x})}, \qquad (6)$$

donde

$$\hat{r}(\theta'\mathrm{x},\mathrm{y})=\frac{1}{nh_1h_2}\sum_{i=1}^{n}K\left(\frac{\theta'\mathrm{x}-\theta'X_i}{h_1}\right)K\left(\frac{y-Y_i}{h_2}\right)$$

y

$$\hat{s}(\theta'\mathrm{x})=\frac{1}{nh_1}\sum_{i=1}^{n}K\left(\frac{\theta'\mathrm{x}-\theta'X_i}{h_1}\right).$$

donde h_1 y h_2 son parámetros de alisamiento y K es la función núcleo. En el estimador expresado en (6) tiene un papel crucial la elección de los parámetros de alisamiento y no tanto la elección de K. Nosotros proponemos introducir la elección de ambos parámetros de alisamiento en la maximización de la función de verosimilitud obtenida a partir del estimador núcleo de la función de densidad condicionada.

Para obtener un estimador del logaritmo de la función de verosimilitud expresado en (4) y evitar que los parámetros de alisamiento sean artificialmente pequeños es necesario utilizar el procedimiento “dejando uno fuera” (*leave-one-out*) que se obtiene del siguiente modo:

$$\hat{f}^{-i}_{\theta,h_1,h_2}(Y_i\mid\theta'X_i)=\frac{\hat{r}^{-i}(\theta'X_i,Y_i)}{\hat{s}^{-i}(\theta'X_i)} \tag{7}$$

donde

$$\hat{r}^{-i}(\theta'\mathrm{x},\mathrm{y})=\frac{1}{nh_1h_2}\sum_{j=1,j\neq i}^{n}K\left(\frac{\theta'X_i-\theta'X_j}{h_1}\right)K\left(\frac{Y_i-Y_j}{h_2}\right)$$

y

$$\hat{s}^{-i}(\theta'X_i)=\frac{1}{nh_1}\sum_{j=1,j\neq i}^{n}K\left(\frac{\theta'X_i-\theta'X_j}{h_1}\right).$$

Siendo el estimador *leave-one-out* de logaritmo de la verosimilitud condicionada:

$$\hat{l}_n(\theta)=\frac{1}{n}\sum_{i=1}^{n}\ln\left(\hat{f}^{-i}_{\theta,h_1,h_2}(Y_i\mid\theta'X_i)\right). \tag{8}$$

En definitiva, nuestro método consiste en maximizar la expresión (8) respecto al vector de parámetros θ y los parámetros de alisamiento h_1 y h_2.

3. ESTUDIO DE SIMULACIÓN

Presentamos los resultados de nuestro estudio de simulación. El objetivo es evaluar las propiedades de nuestro estimador cuando la variable dependiente toma valores positivos y tiene una distribución con asimetría a la derecha o positiva. En la práctica, esto implica que una gran parte de valores se sitúan cerca del mínimo de la variable y una frecuencia reducida se sitúa en los valores mayores del dominio de la función de distribución. En estos casos donde la distribución posee una larga cola a la derecha es necesario disponer de suficiente tamaño muestral para tener mínimamente representados los valores en dicha cola. De hecho, cuando se trabaja con datos microeconómicos en el ámbito de la cuantificación de riesgos, las bases de datos suelen están formadas por muestras de un tamaño considerable. En nuestro ejercicio de simulación trabajamos con muestras de tamaño $n = 500$ y $n = 200$. Comparamos los resultados de nuestra propuesta basada en la maximización de la estimación núcleo de la máxima verosimilitud condicionada estimada (MVC) con la propuesta de Härdle et al. (1993) basada en la minimización del estimador utilizando validación cruzada (VC) de la distancia entre el modelo teórico $g(\theta' X)$ y su estimación núcleo.

Simulamos 500 muestras de tamaño n donde los valores de la variable dependiente proceden de tres distribuciones condicionadas que se muestran en la Tabla 1. La diferencia entre la distribución lognormal y la Champernowne es que la segunda tiene cola pesada, además, cuanto menor sea el parámetro α más pesada es la cola.

Tabla 2. Modelos utilizados en la simulación

Distribución	Parámetros	Densidad condicionada
Lognormal	$\mu = \theta'\mathrm{x},\ \sigma = \lvert\theta'\mathrm{x}\rvert$	$f(\theta y \mid \theta'\mathrm{x}) = \frac{1}{y\sqrt{2\pi\lvert\theta'\mathrm{x}\rvert^2}} \exp\left(-\frac{1}{2}\frac{(\ln(y) - \theta'\mathrm{x})^2}{\lvert\theta'\mathrm{x}\rvert^2}\right)$
Champernowne	$\alpha = 1,\ M = \lvert\theta'\mathrm{x}\rvert$	$f_\theta(y \mid \theta'\mathrm{x}) = \frac{\lvert\theta'\mathrm{x}\rvert}{(y + \lvert\theta'\mathrm{x}\rvert)^2}$
	$\alpha = 2,\ M = \lvert\theta'\mathrm{x}\rvert$	$f_\theta(y \mid \theta'\mathrm{x}) = \frac{2\lvert\theta'\mathrm{x}\rvert^2 y}{(y^2 + \lvert\theta'\mathrm{x}\rvert^2)^2}$

También, en nuestro ejercicio de simulación, tenemos en cuenta dos formas para el vector de covariables $X = (X_1,\ldots, X_d)'$ y el vector de parámetros $\theta = (\theta_1,\ldots,\theta_d)'$. En primero lugar suponemos, , con las covariables generadas por una distribución normal trivariante con medias 0 y matriz de varianzas y covarianzas igual a la identidad y vector de parámetros θ = $(1,1.3,0.5)'$. En segundo lugar, suponemos $d = 4$, añadiendo una cuarta variable, independiente de las otras tres, con distribución de Bernoulli con probabilidad 0.4 y vector de parámetros $\theta = (1,1.3,0.5,0.8)'$, con ello consideramos el hecho de que pueden existir variables cualitativas representadas en forma de variable binaria (ausencia o presencia de una característica).

En la Tabla 2 mostramos los errores cuadráticos medios de los parámetros estimados con nuestro método MVC y el VC de Härdle et al. (1993). Para evitar los problemas de identificación, obtenemos los resultados fijando el valor $\theta_1 = 1$ y estimando el resto de parámetros del predictor lineal. Los resultados muestran la superioridad del método que proponemos en este trabajo, que es más evidente a medida que la cola de la distribución es más pesada.

4. CONCLUSIONES

En este trabajo proponemos un método para estimar el modelo *single-index* con el que se obtienen resultados excelentes cuando la distribución de la variable dependiente es de cola pesada. Además, también funciona cuando en el predictor lineal se incluyen variables categóricas.

Tabla 2. Error cuadrático medio de las estimaciones

		$n = 500$			$n = 2000$		
Distribución	**Método**	θ_2	θ_3	θ_4	θ_2	θ_3	θ_4
Lognormal	MVC	0.0385	0.0150		0.0047	0.0022	
		0.0292	0.0056	0.0179	0.0018	0.0007	0.0025
	VC	0.0468	0.0672		0.0051	0.0093	
		0.0346	0.0447	0.0495	0.0075	0.0273	0.0076
Champernowne ($\alpha = 2$)	MVC	0.1230	0.0334		0.0047	0.0065	
		0.0354	0.0138	0.1748	0.0043	0.0022	0.0070
	VC	1.8803	0.3626		1.8255	0.3244	
		1.3878	0.4465	0.7689	1.3794	0.2307	0.4978
Champernowne ($\alpha = 1$)	MVC	3.7990	3.9959		0.2428	0.1719	
		1.0092	1.0588	3.1898	0.7567	0.2564	2.8792
	VC	9343	1595		5546	970	
		26469	54956	221162	4540	4275	15880

AGRADECIMIENTOS

Este trabajo ha sido financiado por el Ministerio de Economía y Competitividad (proyectos ECO2016-76203-C2-2, MTM2014-52876-R, MTM2017-82724-R)

REFERENCIAS

Bashtannyk, D.M. y Hyndman, R.J. (2001) "Bandwidth selection for kernel conditional density estimation". Computational Statistics & Data Analysis, 36, 279-298

Delecroix, M., Härdle, W. y Hristache, M. (2003). "Efficient estimation in single-index regression". Journal of Multivariate Analysis, 86, 213-226.

Härdle, W.(1990). Applied Nonparametric Regression. Cambridge University Press, UK.

Härdle, W., Hall, P. e Ichimura, H. (1993). "Optimal smoothing in single-index models". The Annals of Statistics, 21, 157-178.

Hastie, T.J. y Tibshirani, R.J. (1990). Generalized Additive Models. Chapman and Hall/CRD. London.

Hristache, M., Juditsky, A. y Spokoiny, V. (2001). "Direct estimation of the index coefficient in a single-index model". The Annals of Statistics, 29, 595-623.

Strzalkowska-Kominiak, E. y Cao, R. (2011). "Maximum likelihood estimation for conditional distribution single-index models under censoring". Journal of Multivariate Analysis, 114, 74-98.

ESTUDIO DE LA INFORMACIÓN DEL MICROBIOMA EN MODELOS DE ANÁLISIS DE RIESGO

Enrique Calderin Ojeda
The University of Melbourne, Centre for Actuarial Studies, Australia
enrique.calderin@unimelb.edu.au

Guillermo H. Lopez Campos
Queen´s University of Belfast, Wellcome-Wolfson Institute for Experimental Medicine
Reino Unido
g.lopezcampos@qub.ac.ukl

ABSTRACT

A pesar de su potencial interés, la utilización de datos genómicos por parte de las compañías aseguradoras está limitada por motivos éticos y legales en numerosos países. la información, el microbioma surge como una nueva fuente de información con repercusiones en la salud con un potencial valor predictivo. En este trabajo analizamos las posibilidades de aplicación del microbioma, como en elemento relevante para definir el riesgo de un individuo para sufrir ciertas patologías y/o asociados con su longevidad. Posibles métodos estadísticos para el análisis de datos de supervivencia son también analizados.

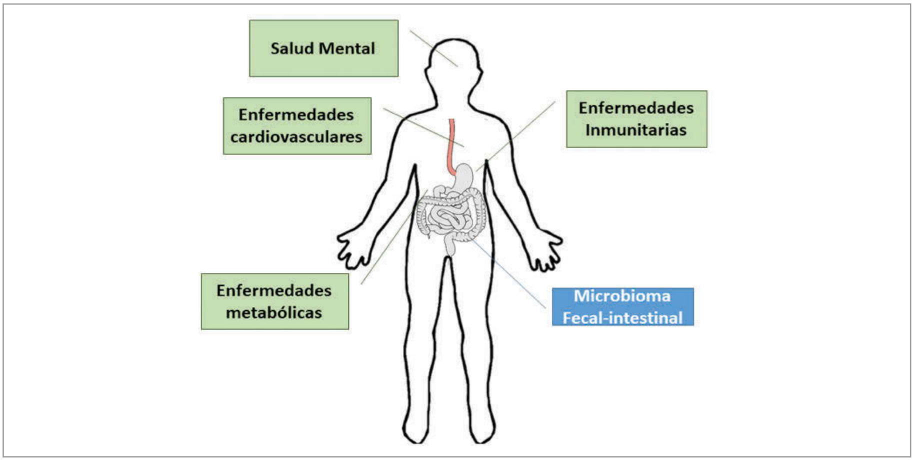
1. INTRODUCCIÓN

En las últimas décadas se han producido grandes avances en las ciencias biomédicas y particularmente en la biotecnología y la biología molecular que han producido toda una revolución en estas áreas generando nuevos conocimientos y técnicas que han podido ser incorporadas en la práctica médica y la salud de los individuos en general. Muchos de estos avances científico-técnicos se han agrupado en nuevos enfoques y paradigmas de la práctica médica tales como la “medicina personalizada” y más recientemente la “medicina de precisión”. En concreto una de las áreas que ha ofrecido unos resultados más prometedores ha sido el uso de la información genómica y el desarrollo de las nuevas técnicas de ultra secuenciación que han permitido la caracterización del genoma (conjunto de genes y sus variantes) humano.

Desde el inicio del Proyecto Genoma Humano (PGH) hasta su conclusión en 2003 se vaticinó que la posibilidad de elucidar la secuencia genética de los individuos acarrearía enormes avances para el diagnóstico, tratamiento y prevención de enfermedades de manera personalizada. La reducción en los costes para acceder a la información genómica, el coste inicial del PGH fue de unos 3.000 millones de dólares, ha permitido que en la actualidad es sea posible obtener la secuencia de los aproximadamente 20,000 genes (genotipado) de un individuo por aproximadamente 100$ y un genoma completo por alrededor de 1000$. En la actualidad y debido a esa reducción en los costes, la secuenciación ha salido de los grandes centros de investigación para ser ofrecida en hospitales y como un producto directamente accesible para los consumidores a través de proveedores privados mediante lo que se conocen como empresas de genómica directa para los consumidores (más conocidas como Direct-To-Consumer, DTC, genomics en inglés tales como 23&Me, Ancestry.com o en España el Obechip).

En relación con el genoma otro aspecto de gran interés en los últimos años en la investigación biomédica es el microbioma (J. Lederberg, 2000), o conjunto de microorganismos (bacterias, hongos, virus) que conviven en un individuo. En esta área al igual que en el desarrollo del genoma, los desarrollos de secuenciación han permitido acceder a información personalizada acerca de su composición y evolución a lo largo de la vida de un individuo. En paralelo a la posibilidad de obtener información acerca de la composición del microbioma de los individuos se ha avanzado en la comprensión de las interacciones entre las variaciones en los microorganismos que lo conforman y su relación con sus hospedadores. Tal es la relevancia del microbioma en la fisiopatología humana que se le ha llegado a considerar como un segundo genoma o un órgano oculto (Zhu B. et al. 2010) dado que su variaciones y alteraciones tienen efectos en aspectos endocrinos (Burcelin R et al. 2011), inmunológicos (Clemente JC. et al. 2018), neurológicos (Bruce-Keller AJ. et al. 2017) e incluso la longevidad (Santoro A. et al. 2018). En este aspecto, estos autores estudiaron el papel que desempeña el microbiota intestinal a lo largo del proceso de envejecimiento especialmente en las últimas dos décadas de vida. Sin embargo, habría que ser cautos con las conclusiones de este trabajo debido a la escasez de datos disponible.

Figura 1. Algunos efectos del microbioma en la salud



Al igual que ha sucedido con los datos genómicos, el acceso a la información relativa al microbioma también se ha popularizado y se ha comercializado para los consumidores a través de compañías DTC como Ubiome, Thryve microbiome o Second Genome, a unos precios asequibles para determinados sectores de la población ofreciendo la posibilidad de obtener el microbioma de diferentes regiones corporales (oral, genital, intestinal o de la piel). Algunas de estas compañías, mencionan el hecho de que esta información es relevante para la salud y puede ser compartida con los profesionales sanitarios.

2. INFORMACIÓN GENÓMICA Y DE MICROBIOMA EN SEGUROS

Los datos genómicos y del microbioma, se presentan pues como elementos relevantes en el desarrollo de las nuevas aproximaciones de "medicina personalizada" o "medicina de precisión" (Kashyap PC. et al 2017) y deben ser por tanto un elemento de interés para las empresas aseguradoras.

A pesar de su potencial interés, la utilización de datos genómicos por parte de las compañías aseguradoras está limitada por motivos éticos y legales en numerosos países. Un aspecto muy importante que debe tenerse en cuenta al referirse a la información genómica es que esta presenta unas características únicas que la hacen especialmente sensible, como que informa no solo del individuo, si no también acerca de sus familiares consanguíneos pasados, presentes y futuros.

En Europa, los miembros del Consejo Europeo aprobaron en 1997 la Convención de Helsinki por la cual se prohíbe la discriminación en base a la genética y la solicitud de análisis genéticos predictivos por parte de las compañías aseguradoras (Van Hoyweghen I. et al. 2008). En EE.UU por ejemplo la ley "Genetic Information Non discrimination Act" (GINA) de 2008 prohíbe por una parte a las aseguradoras usar datos genéticos de sus clientes para tomar decisiones sobre la cobertura de la póliza, y por otra a las empresas usar esta información para contratar nuevos trabajadores así como para tomar decisiones sobre promoción interna en la compañía), pero un reciente cambio en la legislación ("Preserving Employee Wellness ProgramsAct H.R. 1189") permite a los empleadores pedir a sus asalariados información sobre historial médico familiar como parte de este programa de bienestar incentivado económicamente.

En este contexto la información, el microbioma surge como una nueva fuente de información con repercusiones en la salud con un potencial valor predictivo que se puede añadir a otras relacionadas con la auto-monitorización y la auto-cuantificación y que puede convertirse en un elemento de interés para las compañías aseguradoras. A diferencia de lo que sucede con la información genómica individual y las limitaciones existentes para su utilización en la industria aseguradora hasta donde llega nuestro conocimiento no existen semejantes barreras en el potencial uso de información asociada con el microbioma.

3. MICROBIOMA Y BIG DATA

Un aspecto importante a la hora de considerar el análisis del microbioma como una herramienta de interés para la industria aseguradora es el la cantidad y tipo de nuevos datos e información que debe ser considerada.

El total de microorganismos que conforma el microbioma es cuando menos igual que el número de células humanas en el cuerpo, y cuando se considera el factor de posibles genes presentes en el microbioma la cifra aumenta a 360 veces el número de genes humanos (Sender R. et al. 2016). Estas cifras permiten considerar el análisis del microbioma con un problema de tipo "Big Data". A pesar de ello, para la industria aseguradora la información que debería manejar se reduciría enormemente dado que seguramente los parámetros de interés serían únicamente aquellos que permitiesen catalogar las muestras de acuerdo a su composición en los diferentes grupos de riesgo. En este caso, se reduciría la complejidad a la identificación de los taxones asociados con los riesgos de interés. Nuestro interés en este trabajo se centra en el efecto del microbioma en longevidad y envejecimiento. En este sentido, la identificación de patrones de supervivencia juega un factor clave para determinar la tasa de envejecimiento de una población.

4. METODOLOGÍA ESTADÍSTICA

En este trabajo proponemos el modelado estadístico por medio de modelos paramétricos de tiempo de vida acelerada (AFT). Estos modelos no han sido habitualmente en el análisis de datos de supervivencia. Sin embargo, al estar formulados en términos de la curva de supervivencia en lugar de en la función de azar, ofrecen una gran versatilidad estadística. Los modelos paramétricos AFT representan una alternativa a los tradicionales modelos de azar proporcional (PH) para el modelado estadístico de datos de supervivencia. A diferencia de estos últimos modelos, las técnicas AFT describen de una manera directa los tiempos de supervivencia y al mismo tiempo suministran diversos estadísticos que pueden interpretarse en función de la curva de supervivencia. Por otro lado, los modelos de azar proporcional no permiten establecer una diferencia clara desde una perspectiva paramétrica de los efectos de las variables explicativas a través del tiempo y los efectos de estas mismas variables sobre la probabilidad de supervivencia límite del evento. Asimismo, desde una perspectiva del análisis de la longevidad, los métodos paramétricos AFT presentan unas interesantes propiedades, por ejemplo, estos de igual manera que los modelos de azar proporcional pueden incluir una fracción de supervivencia, pero a diferencia de los modelos PH, esta fracción es independiente de otros parámetros del modelo. Además, los modelos AFT facilitan al investigador unas herramientas muy útiles para el análisis de datos de supervivencia. Para una variable aleatoria que representa el tiempo a un cierto evento T, el modelo paramétrico de tiempo de vida acelerado AFT, establece una relación lineal entre la variable $Y=\log T$ y las variables explicativas, $Y_i=x_i\beta+\varepsilon_i$ en la cual la x_i representa un conjunto de factores y variable explicativas, β es un vector de regresores y ε_i son un conjunto de variables aleatorias independientes e idénticamente distribuidas con media cero que constituyen los residuos (ver Collet 2003). Supongamos que la función de supervivencia de un individuo que recibe un tratamiento experimental en tiempo t, verifica $S(t \mid x,\beta)=S_0\,(t\, e^{-x\beta})$ donde S_0, es la función de supervivencia base. Además la expresión anterior en términos de su esperanza puede reescribirse como $E(Y|x,\beta)=e^{x\beta}\, E_0\,(Y)$, es decir, adopta la forma de un modelo lineal generalizado con función de enlace logarítmica, por tanto la media de la variable se puede expresar de manera proporcional a una transformación exponencial de una combinación lineal de las variables explicativas relacionadas con el análisis del microbioma. En consecuencia, la incorporación de este predictor lineal permite acelerar o desacelerar el tiempo de vida por medio de una constante. .A diferencia de los modelos de regresión clásicos, los modelos AFT asumen una formulación paramétrica sobre la variable asociada a la perturbación (usualmente distribución normal, distribución logística o distribución de valores extremos). La estimación de los parámetros en los modelos de tiempo de vida acelerada puede realizarse de una manera relativamente sencilla por el método de máxima verosimilitud. Además todas las propiedades relacionadas con los estimadores de máxima verosimilitud son de fácil interpretación y por tanto los tests estadísticos usuales relacionados con el nivel de significatividad de los estimadores pueden formularse de una manera sencilla.

5. CONCLUSIONES

En este trabajo hemos examinado las diferentes posibilidades de aplicación del microbioma, como en elemento relevante para definir el riesgo de un individuo para sufrir ciertas patologías, así como su implicación en el análisis de la longevidad. Un detallado análisis bibliográfico de la aplicación de datos asociados con el microbioma se ha presentado. Por otra parte se desarrollan diversas vías de su aplicación potencial en la industria aseguradora, mediante un análisis de diversas técnicas de modelado y análisis de datos para simular y comparar los efectos de la utilización de datos asociados con el microbioma y los métodos tradicionales para el cálculo de los riesgos asociados para los individuos tomadores de seguros.

ACKNOWLEDGMENTS

Este trabajo ha sido financiado por la Fundación MAPFRE mediante una ayuda de investigación Ignacio H. de Larramendi (Convocatoria 2017). El Dr G. Lopez Campos también quiere agradecer el apoyo proporcionado el Wellcome-Wolfson Institute for Experimental Medicine.

REFERENCIAS

Bruce-Keller AJ, Salbaum JM, Berthoud HR. Harnessing Gut Microbes for Mental Health: Getting From Here to There. Biol Psychiatry. 2018 Feb 1;83(3):214-223.

Burcelin R, Serino M, Chabo C, Blasco-Baque V, Amar J. Gut microbiota and diabetes: from pathogenesis to therapeutic perspective. Acta Diabetol. 2011 Dec;48(4):257-273.

Clemente JC, Manasson J, Scher JU. The role of the gut microbiome in systemic inflammatory disease. BMJ. 2018 Jan 8;360:j5145.

Collet, D. (2003) Modelling survival data in medical research, second edition. CRC Press, Boca Raton.

Kashyap PC, Chia N, Nelson H, Segal E, Elinav E. Microbiome at the Frontier of Personalized Medicine. Mayo Clin Proc. 2017 Dec;92(12):1855-1864.

Lederberg, J. (2000) Infectious history. Science 288, 287–293.

Santoro A, Ostan R, Candela M, Biagi E, Brigidi P, Capri M, Franceschi C. Gut microbiota changes in the extreme decades of human life: a focus on centenarians. Cell Mol Life Sci. 2018 Jan;75(1):129-148.

Sender, R. et al. (2016) Are we really vastly outnumbered?. revisiting the ratio of bacterial to host cells in humans. Cell 164, 337–340.

Van Hoyweghen I, Horstman K. European practices of genetic information andinsurance: lessons for the Genetic Information Nondiscrimination Act. JAMA. 2008 Jul 16;300(3):326-7.

Zhu B, Wang X, Li L. Human gut microbiome: the second genome of human body. Protein Cell. 2010 Aug;1(8):718-25.

SPATIO TEMPORAL MODELING MORTALITY FOR AGES AFTER RETIREMENT IN EUROPE

Patricia Carracedo
Universidad Internacional de Valencia
Área de ciencias sociales, España
pcarracedo@universidadviu.com

Ana Debón
Universitat Politècnica de València
Centro de Gestión de la Calidad y del Cambio, España
andeau@eio.upv.es

ABSTRACT

Insurance companies and governments have information about the mortality of different regions over time, it would also be interesting to use this information to classify regions with similar or different mortality over time and covariables which explains these differences using powerful statistical software. For this, the aim of this paper is to select the best panel data model using MATLAB and R software for further comparison. These models take into account the spatial and temporal dependence of the database. The selected models were validated by the measures of goodness of fit and tests over residuals. The panel data used corresponds to the mortality of retirement people between 65 and 110+ years of 26 European countries for the period 1995-2009.

1. INTRODUCTION

The demographic dynamics of population aging in developed European and other less developed countries are proposing reforms in welfare states and pensions in Europe. On the basis of this, it is important for governments in European countries to know whether mortality is concentrated in identifiable geographical areas and thus whether an inequality gap is opening up.

These spatial concentrations are produced by the existence of autocorrelation or spatial dependence, which implies that the mortality of geographically close areas is more related than that of geographically distant areas. The detection of spatial dependence is achieved using spatial statistics.

On the other hand, socioeconomic variables such as sex, age, race and others, depending on the geographic unit considered, can influence the risk of disease and therefore mortality. The observed differences in mortality are confounded by these variables; so in order to compare crude mortality rates from different geographic areas, it is necessary to standardize these rates. The direct standardization method produces the Comparative Mortality Figure (CMF) and the indirect standardization method produces the Standardized Mortality Ratio (SMR) both measures free of size population effect. Thus, the basis of this paper lies in the ability to incorporate spatial-temporal dependence in the mortality for ages after retirement, quantify by means of the CMF, of European countries over time by means of the model that best captures the spatio-temporal dependence using MATLAB and R.

2. MATERIALS AND METHODS

2.1. Data

This study deals with mortality data of European countries for the period 1995 to 2012 and the age range considered is between 65 to more than 110. The data was taken from the (HMD, 2016) for a total of 26 countries: Austria, Belarus, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Netherlands, Norway, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, Switzerland, United Kingdom and Ukraine. The logarithm of the CMF was modeled to correct the asymmetry and to approximate its distribution to the normal distribution.

Thus, the logarithm of the CMF was the dependent variable in the spatio-temporal panel data model. The independent variables to be considered in the model were downloaded from the

World Bank Database (2015). These variables were: the crude birth rate, activity rate, energy consumption of the road sector, GDP, population growth and health expenditure.

Statistical analysis was performed using the R environment for statistical computing (R Core Team, 2015) and MATLAB (MathWorks, 2015). Some R-packages as spdep (Bivand, 2012; Charpentier,2014), splm (Millo and Piras, 2012) and plm (Croissant et al., 2008) and the functions available from Elhorst (2011) in MATLAB.

2.2. Statistic to quantify the mortality

CMF ratio is calculated as the ratio of the number of deaths that would be expected in the standard population if it experienced the death rates of the studied population, to the actual number of deaths in the standard population over a specified time period (Julious, 2001). A ratio greater than 1 represents an unfavourable mortality experience but a ratio less than 1 indicates a favourable mortality experience. This measure is better than SMR for spatial comparison when age-specific mortality rates are known for all studied populations.

2.3. Spatial Panel Data Models

A panel data model is a regression model that uses the temporal and spatial dimension of the data to estimate the parameters of interest. This model can control unobserved heterogeneity produced by both the spatial units studied and by time. The importance of controlling this heterogeneity lies in the fact that it reduces problems of multicollinearity between variables by permitting parameter estimates in panel models to be more efficient. The models that will finally be implemented are:

- Spatial lag model with spatial and temporal fixed effect (SLMSTFE)

$$y_{it} = \alpha + \lambda \sum_{j=1}^{N} W_{ij} y_{jt} + \beta x_{it} + \mu_i + \nu_t + \epsilon_{it}$$

- Spatial lag model with random spatial effect and temporal fixed effect (SLMSTRE)

$$y_{it} = \alpha + \lambda \sum_{j=1}^{N} W_{ij} y_{jt} + \beta x_{it} + \phi + \nu_t + \epsilon_{it}$$

– Spatial Durbin Model with spatial and temporal fixed effect (SDM)

$$y_{it} = \alpha + \lambda \sum_{j=1}^{N} W_{ij} y_{jt} + \beta x_{it} + \mu_i + \nu_t + \delta \sum_{j=1}^{N} W_{ij} x_{jt} + \epsilon_{it}$$

– Spatial Durbin Model with random spatial effect and temporal fixed effect (SDMR)

$$y_{it} = \alpha + \lambda \sum_{j=1}^{N} W_{ij} y_{jt} + \beta x_{it} + \phi + \nu_t + \delta \sum_{j=1}^{N} W_{ij} x_{jt} + \epsilon_{it}$$

where W_{ij} is the matrix of spatial weights. λ is the spatial autoregressive parameter associated with the dependent variable and δ is a vector of spatial parameters associated with independent variables. µi is the fixed spatial effect, ν_t is the fixed temporal effect and Φ is the random spatial effect. In addition, the coefficients of the models will be interpreted including the direct and indirect effects (spatial spillover effects). The two measures of goodness of fit of the models were the coefficient of determination R^2 and the residual variance σ^2.

3. RESULTS

The doctoral thesis Carracedo (2017) show the spatial correlation over time in the mortality of the 26 studied European countries was confirmed. Specifically, two significant clusters are observed, especially for people older than 64: a cluster of high CMF consisting of Eastern European countries (Poland, Lithuania, Latvia, Estonia, Ukraine, Belarus, Slovakia and Hungary) and another cluster of low CMF consisting of Western European countries (The United Kingdom, Austria, Spain, Italy, France, Switzerland, Germany, Luxembourg and Belgium) Non-significant values of the Local Moran's I therefore identify countries which do not belong to any cluster (The Czech Republic, Portugal, Denmark, Finland, Ireland, The Netherlands, Norway, Slovenia and Sweden).

Figure 1. Significant cluster map for men between 65 and over age 110 for years 1995 and 2009 in Europe

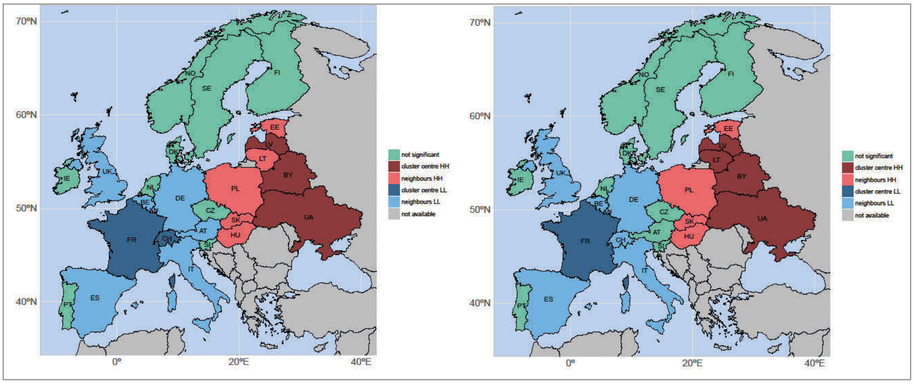
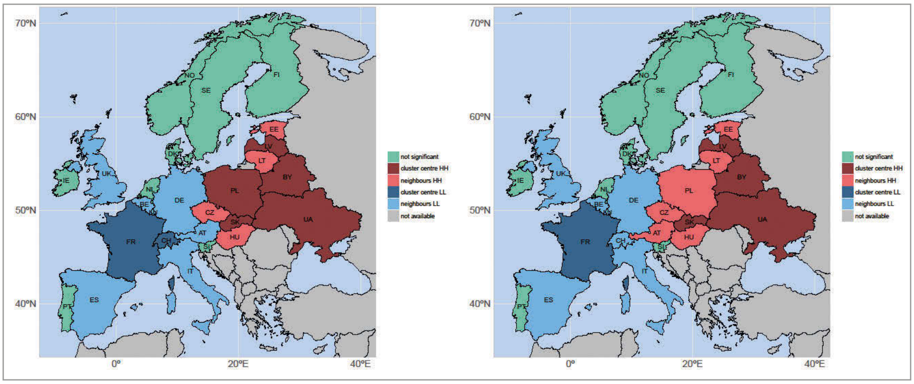


Figure 2. Significant cluster map for women between 65 and over age 110 for years 1995 and 2009 in Europe



On the other hand, one of the main conclusions of this thesis is that the models that include spatial dependence notably increase the R2 with respect to the ordinary regression; therefore we must continue investigating these models.

In our paper Carracedo and Debón (2017a) the SLMSTFE model was implemented in R, where the dependent variable was a logarithm of the SMR. The spatial parameter and all covariates were significant. In addition, the countries that form the different clusters were confirmed in the estimates of the spatial fixed effects of model. The measures of goodness of fit R^2 and σ^2 and residual analysis indicated that it was a good model. In this study, the CMF will be modeled not the SMR since, according to Julious et al. (2001), the CMF is more recommended when specific mortality data are available for each age subgroup for each country and year, as in our case.

Carracedo and Debón (2017b) presented the comparison of both software packages using data from the previous paper Carracedo and Debón (2017a). In this, finally, the best spatial model of panel data that could be obtained with the R software was a SLMSTFE while in MATLAB it was an SDM to model SMR for all ages and both sexes. Unlike, in this study, the CMF is modeled for advanced ages and separately for men and women. In addition, the covariable health expenditure will be included in the model. We hope to obtain similar results respect to last study where the covariate health expenditure will give important information.

4. CONCLUSIONS

This paper has presented a spatial-temporal methodology to select the best panel data model which exploits the geographical dependence between countries over time using MATLAB and R. The panel data used corresponds to the mortality of people between 65 and 110+ years of 26 European countries for the period 1995-2009.

Once confirmed the dependency spatial between these European countries for the period in Carracedo and Debón (2017a) and Carracedo (2017), the next step is to implement a panel data model to control the space and time dependence of these countries during the considered period of time.

The new model taking into account variable health expenditure can change the results obtained by Carracedo and Debón (2017b). The selected models will be validated by the measures of goodness of fit residual variance and coefficient of determination.

ACKNOWLEDGMENTS

This study has been supported by the Fundación MAPFRE (ayudas a la investigación Ignacio H. de Larramendi 2017 Seguro y Previsión Social).

REFERENCES

Bivand, R. (2012). spdep: Spatial Dependence: Weighting Schemes, Statistics and Models. R package versión 0.5-53. http://CRAN.R-project.org/package=spdep.

Carracedo Garnateo, P. (2017). Metodología espacio-temporal con datos de panel. Estudio de la mortalidad europea. Universitat Politècnica de València. doi:10.4995/Thesis/10251/89080.

Carracedo, P. and Debón, A. (2017a). Spatial statistical tools to assess mortality differences in Europe. Springer (Ed.), Proceedings of the second International Congress on Actuarial Science and Quantitative Finance of Springer Proceedings in Mathematics & Statistics (PROMS), 49-74.

Carracedo, P. and Debón, A. (2017b). Selection of spatiotemporal models with panel data in MATLAB and R, Ret@. Accepted.

Charpentier, A. (2014). Computational Actuarial Science with R. Chapman y Hall/CRC

Croissant, Y., Millo, G., et al. (2008). Panel data econometrics in R: The plm package. Journal of Statistical Software, 27(2):1-43.

Human Mortality Database (2016). University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Avaliable at www.mortality.org or www.humanmortality.de (data downloaded on 12th july 2016).

Elhorst, J.P. (2011). Matlab software to estimate spatial panels. Version 2011-04-11, http://www.regroningen.nl/elhorst/software.shtml (routines downloaded on 6th May 2015).

Elhorst, J.P. (2014). Spatial econometrics: from cross-sectional data to spatial panels. Springer.

Julious, S., Nicholl, J., and George, S. (2001). Why do we continue to use standardized mortality ratios for small area comparisons? Journal of Public Health, 23(1):40-46.

MathWorks, T. (2015). MATLAB - The Language of Technical Computing, Version 8.5.0.197613.Natick,Massachusetts.http://www.mathworks.com/products/matlab/.

Millo, G., and Piras G. (2012). "splm: Spatial panel data models in R". Journal of Statistical Software 47.1. http://www.jstatsoft.org/v47/i01/, 1-38.

R Core Team (2015). R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria. http://www.R-project.org/.

The World Bank Database (2015). World Development Indicators. Data download on 9th January 2015 in http://data.worldbank.org/.

MACRO-PRUDENTIAL RISK MANAGEMENT IN INSURANCE-REINSURANCE NETWORKS. INFLUENCE OF MARKET CONCENTRATION

Anna Castañer
Universitat de Barcelona
Departament de Matemàtica Econòmica, Financera i Actuarial, Spain
acastaner@ub.edu

M. Mercè Claramunt
Universitat de Barcelona
Departament de Matemàtica Econòmica, Financera i Actuarial, Spain
mmclaramunt@ub.edu

Claude Lefèvre
Université Libre de Bruxelles
Département de Mathématique, Belgium
clefevre@ulb.ac.be

Stéphane Loisel
Université de Lyon 1
Institut de Science Financière et d'Assurances, France
stephane.loisel@univ-lyon1.fr

ABSTRACT

Partially Schur-constant models are naturally involved in top-down macro-risk management. In a simplified insurance-reinsurance network, we consider an insurance regulator concerned with systemic risk created by potentially large events like nuclear terrorism or mega-earthquakes. Within our setting, the regulator uses a model that provides a distribution of the random total loss for the insurance industry if one catastrophic event occurs, as well as a joint model for the aggregate insurance losses that reflects dependencies. Our main interest

is the probability of default and the probability of insolvency of each of the insurers and the reinsurers that participate in the network. In this paper, we focus our attention on the influence that the number of participants in the network has on the previous probabilities.

1. INTRODUCTION

In a simplified insurance-reinsurance network, we consider an insurance regulator concerned with systemic risk created by potentially large events like nuclear terrorism or mega-earthquakes. Our main interest is the probability of default and the probability of insolvency of each of the insurers and the reinsurers that participate in the network. Several probabilities of contagion are also computed. We use partially Schur-constant models as claim amount models and find that they are naturally involved in top-down macro-risk management. The partially Schur-constant models have been defined and studied in Castañer et al. (2018). In that paper we developed an application to an insurance-reinsurance network with 8 insurers and 2 reinsurers. Although we briefly define partially Schur-constant models in the next section, we refer to Castañer et al. (2018) for the details about them. In the present work, we focus on the influence that the number of participants in the network has on the insolvency and contagion probabilities.

2. PARTIALLY SCHUR-CONSTANT MODELS

In a discrete setting, Schur-constant models are studied in Castañer et al. (2015) and Lefèvre et al. (2017). A vector of lifetimes $\boldsymbol{X}=(X_1,\dots,X_n), n\geq 2$, of arithmetic non-negative random variables has a Schur-constant (S-C) joint survival function $S(x)$: $\mathrm{N}_0^n \to [0,1]$ if

$$P(X_1 \geq x_1,\dots,X_n \geq x_n)=S(x_1+\dots+x_n), \tag{1}$$

for all $\boldsymbol{x}=(\boldsymbol{x}_1,\cdots,\boldsymbol{x}_n) \in \mathbf{N}_0^n$.

An important characteristic of these models is that the joint distributions and dependences inside X are all identical and such assumption could be restrictive or unrealistic for certain practical purposes. To overcome this inconvenient, we consider a set of n lifetimes that is partitioned into m homogeneous groups,

$$\boldsymbol{X}_1=(X_{1,1}, \cdots, X_{1,\mathrm{n}1}), \cdots, X_{\mathrm{m}}= (X_{\mathrm{m},1}, \cdots, X_{\mathrm{m,nm}}),$$

where $n_1,\cdots,n_m$ are positive integers. The model (1) is then generalized by assuming that their joint survival function can be written under the form

$$P(\boldsymbol{X}_1 \geq \boldsymbol{x}_1,\cdots, \boldsymbol{X}_m \geq \boldsymbol{x}_m) = S(|\boldsymbol{x}_1|,\cdots,|\boldsymbol{x}_m|), \qquad (2)$$

for all $\boldsymbol{x}_j=(x_{j,1}, \cdots, x_{j,nj}) \in \mathbf{N}_0^{nj}$ with $|\boldsymbol{x}_j|=x_{j,1} + \cdots + x_{j,nj}$, $1 \leq j \leq m$. In this new model (2), S represents some admissible m-variates function that, among other characteristics must be $\boldsymbol{n}$ –monotone, $\boldsymbol{n} \in N^m$, on N_0^m. Multiple monotonicity is a well-known concept that has found many applications in the framework of univariate real functions (Williamson (1956), McNeil and Nešlehová (2009) and Constantinescu et al. (2011)). Ressel (2014) deals with the multiple monotonicity in multivariate real functions but, as far as we know Castañer et al. (2018) is the first paper that considers the discrete multivariate case.

It can be shown that a vector is partially Schur-constant if and only if it can be represented as a mixed multivariate m-variate multinomial distribution with some specific properties that are developed in Castañer et al. (2018).

3. INSURANCE-REINSURANCE NETWORK

We consider a simplified network that includes two groups of insurers (n_1 large insurers and n_2 small insurers) and two reinsurers (large reinsurer (A) and small reinsurer (B)). Each insurer can have a reinsurance contract with each of the reinsurers and B has a reinsurance contract with reinsurer A. We consider an insurance regulator concerned with systemic risk created by potentially large events. The regulator has only partial information about the insurer exposures and knows only the number of insurers in each group and the common market share of each insurer inside each group. The regulator has also a model that provides a distribution for Z, the random total loss for the insurance industry if one catastrophic event occurs, and a joint model for (Z_1, Z_2), the two aggregate insurance losses for the two classes of insurers. With these assumptions, the most natural approach to the multivariate model for the bulk claims amounts for insurers is the partially Schur-constant model.

In the default model we consider that reinsurers can be solvent, insolvent or defaulted depending on the payments and the buffers they have. Reinsurers have two kinds of capital buffers to avoid recapitalization calculated with the value at risk of their costs at the 95% and 99.6%. If one reinsurer is defaulted it applies a proportional rule to its payments. Regarding the insurers, they have initial capitals calculated with the value at risk at the 90% of their losses without taking into account potential defaults of reinsurers. If an insurer has a net insurance loss, after taking into account potential defaults of reinsurers A and B, bigger than its initial capital, then it is insolvent.

4. RESULTS OF THE PRACTICAL APPLICATION

In this section, we calculate (with simulations) the probability of insolvency of each insurer and reinsurer and the conditional probability that reinsurer B defaults given that reinsurer A defaults. We are also interested in the contagion probability from reinsurers to insurers defined as the conditional probability that at least one insurer becomes insolvent due to the default of at least one reinsurer, i.e. the insurer would not be insolvent if the reinsurers were not defaulted but it is insolvent because the reinsurers are defaulted (at least one of them). We see that the number of participants in the networks influences these probabilities.

We consider that $Z_1 = 3Z_0 + 3Z_1$ and $Z_2 = Z_0 + Z_2^{\perp}$ where $Z_0, Z_1^{\perp}$ and $Z_2^{\perp}$ are i.i.d. Zipf-distributed (with parameter 2.1). Let consider that the 75% of the market corresponds to large insurers (each one with the same market share) whereas the rest corresponds to small insurers (each one with the same market share). Reinsurers use non-proportional l *xs* r contracts: large insurers use treaty 98 xs 24 with reinsurer A and 6 xs 7 with reinsurer B, small insurers use 32 xs 8 treaty with reinsurer A and 2 xs 2 with reinsurer B and reinsurer B uses treaty 4 xs 12 with reinsurer A. The initial capital of large and small insurers is 7 and 2 respectively. The capital buffers of reinsurers are 0 and 8 for reinsurer A and 2 and 10 for reinsurer B (the second capital buffer is used for defaults whereas the first one is used for insolvencies). These values for the different parameters are calculated, using the value at risk as explained before, in a network with 2 large and 2 small insurers, and in this application we consider that the parameters of the reinsurance contracts and the capital buffers of insurers and reinsurers do not vary when the number of insurers that participates on the network increases. Other analysis are possible, but the results should be interpreted differently.

We consider that both the number of small (n_2) and large insurers (n_1) can vary from 2 to 5. Tables 1, 2, 3, 4 and 5 show the values of the probabilities of interest depending on the number of participants in the network.

Table 1. Probability of insolvency of a large insurer as a function of the number of participants

n_2 \ n_1	2	3	4	5
2	0.0099	0.0058	0.0039	0.0028
3	0.0100	0.0058	0.0039	0.0029
4	0.0099	0.0058	0.0039	0.0028
5	0.0098	0.0057	0.0039	0.0028

Table 2. Probability of insolvency of a small insurer as a function of the number of participants

n_2 \ n_1	2	3	4	5
2	0.0099	0.0100	0.0098	0.0098
3	0.0057	0.0058	0.0057	0.0056
4	0.0039	0.0039	0.0038	0.0038
5	0.0028	0.0028	0.0028	0.0028

Table 3. Probability of default of reinsurer A / reinsurer B as a function of the number of participants

n_2 \ n_1	2	3	4	5
2	0.0038 / 0.0038	0.0031 / 0.0044	0.0026 / 0.0041	0.0024 / 0.0037
3	0.0036 / 0.0041	0.0028 / 0.0045	0.0023 / 0.0042	0.0020 / 0.0038
4	0.0034 / 0.0041	0.0026 / 0.0045	0.0021 / 0.0042	0.0018 / 0.0038
5	0.0033 / 0.0040	0.0025 / 0.0044	0.0020 / 0.0041	0.0017 / 0.0038

Table 4. Conditional probability that reinsurer B defaults given that reinsurer A defaults as a function of the number of participants

n_2 \ n_1	2	3	4	5
2	0.4514	0.5986	0.6398	0.6469
3	0.5131	0.6430	0.6898	0.7031
4	0.5496	0.6817	0.7271	0.7442
5	0.5736	0.7030	0.7545	0.7610

Table 5. Probability of contagion as a function of the number of participants

n_2 \ n_1	2	3	4	5
2	0.3194	0.4818	0.5732	0.6257
3	0.3891	0.5480	0.6309	0.6878
4	0.4332	0.5852	0.6662	0.7243
5	0.4615	0.6099	0.6962	0.6847

Table 1 shows that when the number of large insurers increases, the probability of insolvency of a large insurer decreases. This is due to a more balanced distribution of the 75% market share when there are more large insurers in the network, so the risk is also more diversified. Contrarily, if the number of small insurers is larger, the probability of insolvency remains virtually unaltered given that the probability of insolvency of the large insurers is not affected. Table 2 can be interpreted in a similar way.

Table 3 shows the probability of default of reinsurers A and B according to the number of participants. As regards reinsurer A, the probability of default decreases as the number of participants increases. This would be a consequence of a more diversified risk resulting from a larger number of companies. For reinsurer B, the probability of default differs from that of reinsurer A. In this case, the probability of default depends not only on the number of participants but it is also affected by the potential default of reinsurer A. Then, the probability of default not always decreases with an increasing number of participants in the network.

If instead we consider the conditional probability of default of reinsurer B when reinsurer A defaults, as in Table 4, we find that a larger number of participants (both of small and large reinsurers) leads to a higher probability of default.

The probability of contagion from reinsurers to insurers, Table 5, is quite big and increases when the number of participants in the network increases, with a different effect depending on the type of insurer. Then, the market concentration decreases the probability of contagion.

5. CONCLUSIONS

In an insurance-reinsurance network, using a loss model that take into account dependencies between the losses of the insurers, we have computed the effect that the market concentration has on the probability of insolvency of insurers and reinsurers and other probabilities. One of the conclusions is that the contagion probability between reinsurers and insurers increases when the number of participants increase.

ACKNOWLEDGMENTS

The authors received support from the Cátedra ICEA-UB de Seguros y Fondos de Pensiones. C.L. received support from the Chair Generali Actuariat Responsable sponsored by the French Fondation du Risque. S.L. received support from the Research Project LoLitA of the French Agence Nationale de la Recherche, the Chair DAMI sponsored by BNP Paribas Cardif and the Chair Actuariat Durable sponsored by Milliman.

REFERENCES

Castañer, A., Claramunt, M.M., Lefèvre, C., Loisel, S. (2015). "Discrete Schur-constant models". Journal of Multivariate Analysis, 140, 343-362.

Castañer, A., Claramunt, M.M., Lefèvre, C., Loisel, S. (2018). "Partially Schur-constant models". Submitted.

Constantinescu, C., Hashorva, E., Ji, L. (2011). "Archimedian copulas in finite and infinite dimensions- with application to ruin problems". Insurance: Mathematics and Economics, 49, 487-495.

Lefèvre, C., Loisel, S., Utev, S. (2017). "Markov property in discrete Schur-constant models". Methodology and Computing in Applied Probability, https://doi.org/10.1007/s11009-017-9564-5

McNeil, A., Nešlehová, J. (2009). "Multivariate archimedian copulas, d-monotone functions and l1-norm symmetric distributions". Annals of Statistics, 37, 3059-3097.

Ressel, P. (2014) "Higher order monotonic functions of several variables". Positivity, 18, 257-285.

Williamson, R. (1956). "Multiple monotone functions and their lapace transforms". Duke Mathematica Journal, 23, 189-207.

UNA FAMILIA DE PRINCIPIOS DE PRIMA BASADA EN UNA DISTRIBUCIÓN DE RIESGO AJUSTADA

Antonia Castaño-Martínez
Universidad de Cádiz, Facultad de Ciencias
Departamento de Estadística e Investigación Operativa, España
antonia.castano@uca.es

Gema Pigueiras
Universidad de Cádiz, Facultad de Ciencias
Departamento de Estadística e Investigación Operativa, España
gema.pigueiras@uca.es

Miguel A. Sordo
Universidad de Cádiz, Facultad de Ciencias
Departamento de Estadística e Investigación Operativa, España
mangel.sordo@uca.es

RESUMEN

Existen multitud de métodos para obtener principios de prima en las ciencias actuariales. Uno de ellos consiste en definir distribuciones de riesgo ajustadas para luego tomar la esperanza y así obtener principios de prima. En este artículo describimos una familia de principios de prima obtenida a través de la esperanza de una distribución de riesgo ajustada que verifica una serie de propiedades deseables al condicionarla respecto de la distribución de riesgo original. De este modo obtenemos una secuencia de principios de prima que incorporan la aversión al riesgo de la aseguradora a través de dos parámetros. Además, esta familia de principios de prima puede ser representada mediante tres enfoques diferentes muy utilizados en la literatura.

1. INTRODUCCIÓN

En términos generales, un principio de prima es una regla que asigna una cantidad (a la que llamamos prima) a un riesgo que va a ser asegurado. Formalmente, dada una variable aleatoria X representando un riesgo o una pérdida, un principio de prima $(T(X))$ no es más que una función que asigna un número real a cada valor de X. Para un estudio más detallado ver Goovaerts et al. (1984) y Denuit et al. (2005).

En la literatura existen diferentes métodos por los que se obtienen principios de prima. El primero de ellos consiste en obtener el principio de prima como la suma de la pérdida media más una cierta cantidad (llamada carga de riesgo) que está estrechamente relacionada con la variabilidad del riesgo o pérdida. Algunos principios de prima tradicionales obtenidos mediante este método, son el principio de prima del valor esperado ($T(X)=E[X] + \alpha E[X]$, con $\alpha \geq 0$), el principio de prima de varianza ($T(X)=E[X] + \alpha\, Var[X]$, para algún $\alpha \geq 0$), etc.

El segundo consiste en un método de caracterización a través del cual se obtiene el principio de prima como una función que satisface una serie de propiedades deseables desde el punto de vista de la aseguradora. Algunas de estas propiedades son:

a. Independencia: $T(X)$ depende sólo de la distribución acumulada de X (la prima depende solo de la pérdida monetaria del riesgo y de la probabilidad de que ocurra dicha pérdida).

b. Carga de riesgo: $T(X) \geq E[X]$ (el principio de prima debe cargar al menos el pago esperado del riesgo X).

c. No existencia de carga de riesgo injustificada: Si X es igual a una constante c, entonces $T(X)=c$.

d. Pérdida máxima: $T(X) \leq \max[X] = F_X^{-1}(1)$.

e. Invarianza por traslación: $T(X+c) = T(X)+c$ para todo riesgo X y toda constante c (si incrementamos el riesgo mediante una cantidad constante c, entonces la prima se incrementará en esa misma cantidad).

Para más propiedades ver Young (2004).

El tercer método consiste en obtener el principio de prima a través de una teoría económica particular del riesgo. Como ejemplo de principio de prima obtenido mediante este método podemos destacar el principio de prima distorsionado (Wang, 1996) o el principio de prima de Esscher (Bühlmann, 1980).

Estos métodos no son mutuamente excluyentes sino que algunos principios de prima surgen de aplicar más de un método a la vez.

En este artículo describiremos la familia de principios introducida por Sordo et al. (2016). Dicha familia puede ser obtenida a través de los tres enfoques anteriores y además incorpora unos parámetros que controlan la aversión al riesgo de la aseguradora. También estudiaremos algunas de sus propiedades.

A lo largo de este artículo consideraremos variables aleatorias no negativas (o riesgos) con función de distribución continua.

2. UNA FAMILIA DE PRINCIPIOS DE PRIMA

En esta sección describimos una familia de principios de prima dependiente de parámetros que controlan la aversión al riesgo de la aseguradora. Para ello, sea X una distribución de riesgo con función de distribución F. Consideremos una muestra aleatoria de reclamaciones independientes X_1, X_2, ... ,X_n, con la misma distribución que X, y sus correspondientes estadísticos ordenados $X_{1:n}, X_{2:n},..., X_{n:n}$ con $1 \leq i \leq n$ y $n \geq 1$. Se sabe que la función de distribución de $X_{i:n}$ $(i, 1, ..., n)$ viene dada por

$$F_{i:n}(x) = \beta_{i,n-i+1}\big(F(x)\big), \quad x \geq 0$$

siendo

$$\beta_{i,j}(t) = \int_0^t \frac{(i+j-1)!}{(i-1)!\,(j-1)!}\, u^{i-1}(1-u)^{j-1}\,du, \quad 0 \leq t \leq 1$$

la función beta incompleta de Pearson (1934) de parámetros (i,j). Si la variable aleatoria es absolutamente continua entonces la función de densidad de X_(i:n) viene dada por

$$f_{i,n}(x)=\frac{n!}{(i-1)!(n-i)!}\ [F(x)]^{i-1}\,[1-F(x)]^{n-i}\,f(x),\quad x\geq 0$$

Supongamos que, a cambio de asegurar el riesgo X, la aseguradora desea cobrar al menos el pago esperado del riesgo $X_{i:n}$ para algún $i=1,\ldots,n$ con $n\geq 1$. Observemos que en este caso el parámetro i refleja la aversión al riesgo de la aseguradora; a mayor i, mayor aversión al riesgo. El siguiente paso consiste en obtener una distribución de riesgo ajustada Y que verifique una serie de propiedades deseables al condicionarla respecto a $X_{i:n}$. Dichas propiedades son:

(P1) La distribución condicionada $[Y\,|\,X_{i:n}=x]$ depende sólo de la cola de X. Esta propiedad es necesaria ya que, en caso contrario, el principio de prima no satisfaría la propiedad de independencia.

(P2) $E[Y\,|\,X_{i:n}=x]\geq x$ para todo x. De este modo aseguramos que se verifique la propiedad de carga de riesgo, esto es, la prima media que se recibe en el caso de una reclamación de tamaño x es mayor o igual que x.

(P3) $[Y\,|\,X_{i:n}=x_1]\leq_{st}[Y\,|\,X_{i:n}=x_2]$ para todo $x_1\leq x_2$. Bajo esta propiedad se tiene que $\mathrm{E}[Y\,|\,X_{i:n}=x]$ es creciente en x, con lo cual la aseguradora obtiene mayores primas con el objeto de proporcionar mayores coberturas de seguro.

(P4) Si X es de cola pesada, la función $E[Y\,|\,X_{i:n}=x]-x$ es creciente en x. Esta propiedad tiene como objeto proteger a la aseguradora frente a grandes reclamaciones.

Lema 1. Sea X una distribución de riesgo con función de supervivencia $\bar{F}$. La variable aleatoria $Y_{X,i,n}$ definida por

$$[Y_{X,i,n}\,|\,X_{i:n}=x]=^{d}[X\,|\,X>x]\quad \textit{para todo}\ \ x\geq 0$$

verifica las propiedades (P1), (P2) y (P3). Más aún, si $\bar{F}$ es log-convexa, lo cual es una manera de formalizar la noción de cola pesada, $Y_{X,i,n}$ también verifica la propiedad (P4).

Definida esta distribución de riesgo ajustada, obtenemos la secuencia de principios de prima dada por

$$T_{i,n}(X) = E[Y_{X,i,n}] \tag{1}$$

para todo $1 \leq i \leq n$ y $n \geq 1$.

En primer lugar mostramos que, efectivamente, los parámetros i y n reflejan la actitud frente al riesgo de la aseguradora.

Teorema 2. Dado un riesgo X, se tiene para todo $1 \leq i \leq n-1$

$$Y_{X,i,n} \leq_{st} Y_{X,i+1,n}$$

$$Y_{X,i,n} \leq_{st} Y_{X,i,n-1}$$

Como resultado inmediato del Teorema 2 obtenemos que, fijado $i \geq 1$, a mayor n (con $n \geq i$), menor aversión al riesgo (esto es $T_{i,n}(X) \leq T_{i,n-1}(X)$). Por el contrario, fijado $n \geq 1$, a medida que aumenta i (con $1 \leq i \leq n$) aumenta la aversión al riesgo (esto es $T_{i,n}(X) \leq T_{i+1,n}(X)$).

Teorema 3. Sea X una distribución de riesgo con función de supervivencia $\bar{F}$ y $E[X]<\infty$. La familia de principios de prima dada por (1) toma la forma

$$T_{i,n}(X) = \frac{n}{n-i} E[X] - \frac{1}{n-i} \sum_{j=1}^{i} E[X_{j:n}] = \frac{1}{n-i} \sum_{j=i+1}^{n} E[X_{j:n}]$$

para $n \geq 2, 1 \leq i \leq n$ y

$$T_{n,n}(X) = n \left[T_{1,1}(X) - \sum_{j=2}^{n} \frac{1}{j(j-1)} E[X_{j:j}] \right], \quad n > 1$$

donde $T_{1,1}(X) = E[X] + \epsilon(X)$ y $\epsilon(X)$ es la entropía residual acumulada introducida por Rao et al. (2004).

El principio de prima dado por (1) se puede representar desde tres enfoques diferentes. El primero de ellos es como principio de prima distorsionado con distorsión cóncava dada por

$$h_{i,n}(t) = 1 - \frac{n!}{(i-1)!(n-i)!} \int_t^1 \int_p^1 (1-u)^{i-1} u^{n-i-1} \, du \, dp \, ,$$

para todo $n \geq 1$ e $1 \leq i \leq n$.

El segundo es como mixtura de TVaRs, es decir, $T_{i,n}(X)$ puede ser escrita como un área ponderada bajo la curva definida por el $TVaR_p(X)$. Esto es

$$T_{i,n}(X)=\int_0^{\infty} E[X \mid X > \mathrm{t}]\, dF_{X_{i:n}}(t)=\int_0^1 TVaR_p(X)\ _{d\beta i,n\text{-}i+1}(p).$$

En este caso la aversión al riesgo versa sobre la función $\beta_{i,n\text{-}i+1}(p)$.

El tercer y último enfoque consiste es escribir el principio de prima como

$$T_{i,n}(X) = E[X]+\epsilon_{i,n}(X),$$

donde, por el Teorema 3, obtenemos que la carga de riesgo $\epsilon_{i,n}(X)$ viene dada por

$$\epsilon_{i,n}(X)=\frac{1}{n-1}\sum_{j=i+1}^{n}\left[E[X_{j:n}] - E[X]\right]=\frac{1}{n-1}\sum_{j=1}^{i}\left[\ E[X] - E[Xj{:}n]\right],$$

para $n \geq 2, 1 \leq i \leq n$ y

$$\epsilon_{n,n}(X) = n\epsilon(X) - n\sum_{\mathrm{j}=2}^{n}\frac{[E[X_{j:j}] - E[X]]}{j(j\text{-}1)},$$

para $n \geq 1$.

Además, sabemos que $\epsilon_{1,n}(X)$ es consistente con $\epsilon(X)$, en el sentido de que si n es un número real, entonces $\epsilon_{1,n}(X)$ converge a $\epsilon(X)$ cuando n converge a 1.

Podemos encontrar una expresión alternativa en el caso i=n con n≥1, en términos de un desarrollo en serie, esto es $\bar{F}$

$$\mathrm{T}_{n,n}(X) = n\sum_{j=n+1}^{\infty}\frac{E[X_{j:j}]}{(j\text{-}1)j}$$

De este modo, y teniendo en cuenta que $T_{n,n+1}(X) \leq T_{n,n}(X)$, podemos acotar tanto $Tn,n\ (X)$ como la carga de riesgo para $i = n$ y $n \geq 1$ del siguiente modo

$$0 < E[X_{n+1:n+1}] - E[X] \leq \epsilon_{n,n}(X) \leq min\{F_X^{-1}(1) - E[X], n\epsilon(X)\}$$

En este caso, a diferencia de otros principios de prima que pueden ser expresados de esta misma manera, la carga de riesgo no es fija sino que depende de los parámetros i y n, reflejando por tanto la actitud hacia el riesgo de la aseguradora.

Además, la carga de riesgo $\epsilon_{i,n}(X)$ satisface las siguientes propiedades

(i) $\epsilon_{i,n}(X+k) = \epsilon_{i,n}(X)+k$ para todo X y todo k constante,

(ii) $\epsilon_{i,n}(0) = 0$ y $\epsilon_{i,n}(\alpha X) = \alpha\epsilon_{i,n}(X)$ para todo X y todo $\alpha>0$,

(iii) $\epsilon_{i,n}(X) \geq 0$ para todo X, con $\epsilon_{i,n}(X)=0$ si X es una variable aleatoria degenerada en c,

(iv) $X \leq_{disp} Y$ implica que $\epsilon_{i,n}(X) \leq \epsilon_{i,n}(Y)$, donde $\leq_{disp}$ denota el orden dispersivo (ver Shaked y Shanthikumar, 2007).

Por tanto, $\epsilon_{i,n}(X)$ es una medida de variabilidad en el sentido de Bickel y Lehmann (1976); $\epsilon_{i,n}(X)$ es una medida de dispersión alternativa a la varianza.

3. CASOS PARTICULARES

La mayoría de los actuarios coinciden en que las reclamaciones en el caso de distribuciones de cola pesada son más peligrosas que las de cola ligera. Un modo de reconocer las distribuciones de cola pesada es a través de la función de tasa de fallo.

$$r(\mathrm{x}) = {}^{\mathrm{f}(x)}\big/_{\bar{F}(x)}, \quad x \geq 0$$

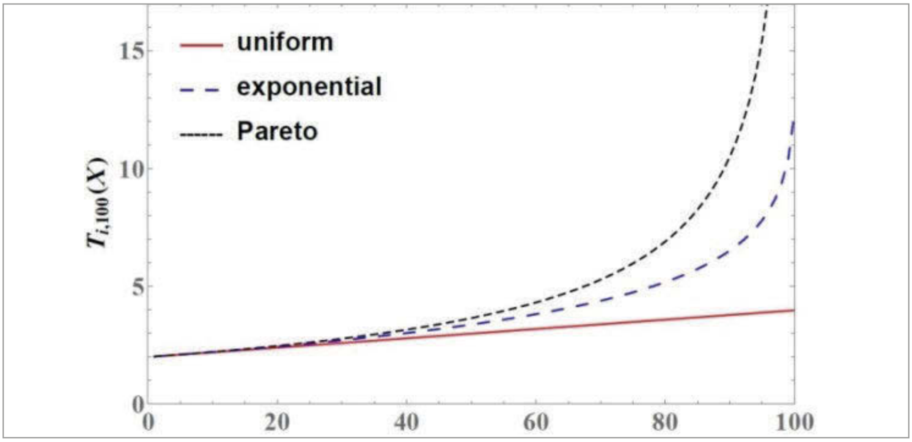
Si $r(x)$ es pequeña, la distribución es de cola pesada mientras que si $r(x)$ es grande, la distribución es de cola ligera. Siguiendo este razonamiento, cualquier distribución uniforme es menos arriesgada que cualquier distribución exponencial y esta a su vez que cualquier Pareto, en el caso de que todas ellas tengan la misma media.

En esta sección calculamos expresiones explícitas para $T_{i,n}(X)$ en el caso concreto de tres distribuciones particulares como son la distribución uniforme, la exponencial y la Pareto. Para ello basta utilizar las expresiones obtenidas en el Teorema 3 y las ya conocida expresiones para la esperanza de los estadísticos ordenados (ver Balakrishnan y Nevzorov, 2003). Además realizamos una comparativa aplicando el principio de prima a estas distribuciones en el caso de que todas tengan media 2.

Tabla 1. $T_{i,n}(X)$ para las distribuciones uniforme, exponencial y Pareto para $i = 1,\dots n$ y $n \geq i$.

$U \sim U(0,a)$	$W \sim \text{Pareto}(\alpha,\beta)$	$Z \sim Exp(\lambda)$
$T_{i,n}(U) = \frac{a(n+i+1)}{2(n+1)}$	$T_{i,n}(W) = \frac{n!\ \alpha\beta\Gamma(n-i+1-1/\alpha)}{(n-i)!(\alpha-1)\Gamma(n+1-1/\alpha)}$	$Ti,n\ (Z) = \lambda\left(\sum_{j=1}^{i}\frac{1}{n-j+1}+1\right)$

Figura 1. $T_{i,100}(X)$, con $1 \leq i \leq n$, para las distribuciones de Pareto, uniforme y exponencial de media 2



Observamos en la Figura 1 como la distribución de Pareto siempre da mayores primas que la exponencial, y esta a su vez que la uniforme.

4. CONCLUSIONES

Basándonos en los estadísticos ordenados y a través de una distribución de riesgo ajustada que verifica una serie de buenas propiedades para la aseguradora, hemos obtenido una familia de principios de prima. Dicha familia incorpora el grado de aversión al riesgo de la aseguradora, lo que juega un papel crucial en la economía del seguro. Finalmente la secuencia $\{T_{i,n}(X)\}$ ha sido presentada desde tres enfoques diferentes; como principio de prima distorsionado, como área ponderada bajo el $TVaR_p(X)$ y finalmente como una suma del riesgo medio más una cierta carga de riesgo $\epsilon_{i,n}(X)$.

AGRADECIMIENTOS

Esta investigación ha sido financiada por el Ministerio de Economía y Competitividad (España) con los proyectos MTM2014-57559-P y MTM2013-46962-C2-2-P.

REFERENCIAS

Balakrishnan, N., y Nevzorov, V. B. (2003). "A primer on statistical distributions". John Wiley & Sons, Inc.

Bickel, P.J., y Lehmann, E.L. (1976). "Descriptive statistics for nonparametric models. III. Dispersion". The Annals of Statistics, 4, 1139-1158.

Bühlmann, H. (1980). "An Economic Premium Principle". ASTIN Bulletin, 11, 52-60.

Denuit, M., Dhaene, J., Goovaerts, M., y Kaas, R. (2005). "Actuarial Theory for Dependent Risks". John Wiley & Sons.

Goovaerts, M., De Vylder, F., y Haezendonk, J. (1984). "Insurance Premiums". North Holland, Amsterdam.

Pearson, K. (1934). "Tables of the incomplete beta function". Cambridge University Press, Cambridge, UK.

Rao, M., Chen, Y., Vemuri, B.C., y Wang, F. (2004). "Cumulative residual entropy: A new measure of information". IEEE Transactions on Information Theory, 50, 1220–1228.

Shaked, M., y Shanthikumar, J.G. (2007). "Stochastic Orders". In: Series: Springer Series in Statistics, Springer.

Sordo, M.A., Castaño, A., y Pigueiras, G. (2016). "A family of premium principles based on mixtures of TVaRs". Insurance: Mathematics and Economics, 70, 397-405.

Young, V. (2004). "Premium Principles". In: Encyclopedia of Actuarial Science, Wiley, New York.

Wang, S. (1996). "Premium calculation by transforming the layer premium density". ASTIN Bulletin, 26, 71–92.

POWER TO INVEST

Jorge Castro
MACDA Research Group
castro@cs.upc.edu

Joaquim Gabarro
ALBCOM Research Group. Computer Science Dept.
gabarro@cs.upc.edu

Maria Serna
ALBCOM Research Group. Computer Science Dept
Universitat Politècnica de Catalunya, Barcelona. mjserna@cs.upc.edu

ABSTRACT

In this post-recession time it is important to measure the possibilities offered by a society in relation to investments. To do that, we consider an investment schema $S=(R; R_1,\dots,R_n)$, where R is a lower bound on the desired return and the R_i's are the return of the assets (to invest in). We introduce the power to invest, denoted as $Power(S)$, to measure the capability of the schema to fulfil the requirement R. The power to invest is inspired in the Coleman's power of a collectivity to act. We consider the angel-daemon approach to uncertainty and extend it to investment schemas. The approach tries to tune cases in-between the worst and the best scenarios and analyse them through game theory. We show how to use the power to invest to assess uncertainty in such situations and develop several examples.

1. INTRODUCTION

Harry Markowitz (1952) introduced the mean-variance approach [6]. Consider a set $N=[n]=\{1,\dots,n\}$ of assets with expected returns $R_1,\dots,R_n$ as part of an investment schema

S=(R; R_1,...,R_n). A portfolio $w = (w_1,\ldots,w_n) \in \Delta_n$ provides a probability distribution on N, i.e., positive weights with $\sum_{1\leq i\leq n} w_{i=1}$. The expected return (the mean) is $E_{S,w} = \sum_{1\leq i\leq n} w_i R_i$. The variance is $Var(S,w) = \sum_{1\leq i,j\leq n} w_i w_j \sigma_i \sigma_j$, where σ_i is the standard deviation for asset i and $\rho_{i,j}$ is the correlation coefficient for assets i and j. The Markowitz approach considers points $(E(S,w),Var(S,w))$ for different portfolios. James Coleman (1971) introduced the formal definition of the power of a collectivity to act. In this paper we adapt this idea to provide investors with information about his degree of freedom to choice. J. Gabarro et al. (2014, 2017) and G. Fragnito (2017) modelled uncertainty through strategic situations in-between the worst and the best scenarios. Here we use the variance to define such scenarios. This strategic approach is used to extend the power to invest to take into account uncertainty. In this way, the power to invest can be used to study the behaviour, as a whole, of the investing capabilities of a given market.

2. POWER TO INVEST

For n assets with returns $R_1,\ldots,R_n$, we roughly identify possible investments by the subsets of N. In order to associate a unique return to any $I \subseteq N$, we have to take in addition a probability distribution on the elements in I. To exemplify our approach, we consider in the remaining of the paper the uniform distribution which grants a maximal variety and diversification among investments. Under equiprobable weights, for an investment $I \subseteq N$, the expected return is $E(I) = \frac{1}{\#I}\sum_{i\in I} R_i$.

An *investment schema* is a tuple $S = (R; R_1,\ldots,R_n)$, where $R > 0$ is the minimal acceptable return. Given an investment schema, an investment I is feasible (or acceptable) for S iff $E(I) \geq R$. The set of all feasible investments for schema S is $F(S) = \{I \mid E(I)\geq R\}$. The empty investment I=∅ is never a feasible investment because $E(I) = 0$ (it is not possible to get a return if there is no investment at all). Therefore, $\#F(S) \leq 2n - 1$. We define the power to invest of S as

$$Power(S) = (\#F(S))/(2^n-1).$$

The power to invest provides a rough estimation of the capabilities to invest in an environment described by S and the associated probability distributions. It measures the dynamicity of the society to fulfil R. For a moderate R, in an active society, there should be many different ways to get a return R. Observe that the size of $F(S)$ is a measure of this fact. As $0 \leq \#F(S) \leq 2^n - 1$, $0 \leq Power(S) \leq 1$. The power to invest gives precise mathematical meaning to some basic facts. Let $S = (R; R_1, \ldots, R_n)$, and $S'=(R'; R_1', \ldots, R_n')$, with $R' \geq R$. When the minimal return is low, the power to invest is high. This is translated as follows, when $R \leq min_{i \in N} R_i$, $Power(S)=1$. When the minimal return is high, the power is low. When R is too high, it could be impossible to fulfil it. Observe that $Power(S)$ is zero when $R > max_{i \in N} R_i$. When the minimal return increases, the power to invest cannot increase. That is, when R'>R and $R_i = R_i'$, $1 \leq i \leq n$, $Power(S) \geq Power(S')$. When productivity increases globally, the power to invest cannot decrease. This translates into, when $R_i < R_i'$, $1 \leq i \leq n$, and $R = R'$, $Power(S) \leq Power(S')$.

Let us consider some highly stylized investment schemes. First of all consider a case where all the assets have the same return. To denote n assets all of them with the same return R we write $n{:}R$. Then, let $S = (R'; R, \ldots, R) = (R'; n{:}R)$. Consider an investment I containing k assets, $0 < k \leq n$, $I = (k{:}R)$. $E(I)$ is independent of the value of k as $E(I) = (\mathrm{kR})/k = R$. Thus, $\#F(S) = 2^n - 1$, if $R' \leq R$, and 0, otherwise. Then, $Power(S) = 1$ if $R' \leq R$, and 0, otherwise.

Let us consider another stylized investment schema $S = (R; n_1{:}R_1, n_2{:}R_2)$. Assume without loss of generality that $R_1 \geq R_2$. Only the case $R_1 \geq R \geq R_2$ is interesting because, when $R \leq R_2$ the power is 1 and when $R_1 < R$ the power is 0. For an investment $I = (k_1{:}R_1, k_2{:}R_2)$ with $k_1 + k_2 > 0$, $E(I) = (k_1 R_1 + k_2 R_2)/(k_1 + k_2)$. The feasible investments are defined by pairs (k_1, k_2) in the following set

$$F(S) = \{(k_1, k_2) \mid (k_1 + k_2)\,\mathrm{R} \leq k_1 R_1 + k_2 R_2,\ k_1 \leq n_1,\ k_2 \leq n_2,\ k_1 + k_2 > 0\}$$

A pair $(k_1, k_2) \in F(S)$ allows for $\binom{n_1}{k_1}\binom{n_2}{k_2}$ different feasible investments. Therefore,

$$Power(R, n_1{:}R_1, n_2{:}R_2) = \frac{1}{2^{n_1} + n_{2-1}} \sum_{(k_1, k_2) \in F(S)} \binom{n_1}{k_1}\binom{n_2}{k_2}.$$

Taking a numerical example, for example $S = (0.10; 7{:}0.15, 4{:}0.05)$. The restriction $(k_1 + k_2)R \leq k_1 R_1 + k_2 R_2$ gives $0.10(k_1 + k_2) \leq 0.15\mathrm{k\_1} + 0.05k_2$ equivalent to $k_1 \geq k_2$. Therefore, $F(S) = \{(k_1, k_2) \mid k_1 \geq k_2, k_1 \leq 7, k_2 \leq 4, k_1 + k2 > 0\}$ and $Power(S) = 0.818$.

Now we compute the power to invest of data taken from the foundational work of H. Markowitz (1959). We consider the years 1943 and 1944 with really impressive returns (World War II) and the average over 18 years (1937-54) with more moderate (average) returns as given in the following table.

Year	1 AmT	2 AT&T	3 USS	4 GM	5 AT&	6 CC	7 Bdn.	8 Frstn	9 SS
1943	0.428	0.300	.0149	0.225	0.313	0.351	0.341	0.580	0.639
1944	0.192	0.103	0.260	0.290	0.637	0.233	0.227	0.473	0.282
1937-54	0.066	0.062	0.146	0.173	0.198	0.055	0.128	0.190	0.116

We consider the associated investment schemas:

S_{43} (R) = (R; 0.428,0.300,0.149,0.225,0.313,0.351,0.341,0.580,0.639)

S_{44} (R) = (R; 0.19,0.103,0.260,0.290,0.637,0.233,0.227,0.473,0.282)

$S37\text{-}54$ (R) =(R; 0.066,0.062,0.146,0.173,0.198,0.055,0.128,0.190,0.116)

The power to invest, for different values of R, is given in the following table.

R	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$Power(S_{43}(R))$	1.0	0.996	0.878	0.287	0.019	0.003	0.0
$Power(S_{44}(R))$	1.0	0.962	0.497	0.048	0.003	0.001	0.0
$Power(S_{37\text{-}54}(R))$	0.906	0.0	0.0	0.0	0.0	0.0	0.0

According to this table, in 1943 there was more power to invest than in 1944. On the average returns the power is impressively smaller.

3. UNCERTAINTY

Returns are strongly volatile H. Markowitz (1959) studied the volatility of AmT from 1937 to 1954. The expected return was $R_{AmT} = 0.066$ and $\sigma_{AmT} = 0.231$. Using the variance to bound the spread of a perturbation, we can see that it affects the return as R_{AmT} – σ_{AmT} $= -0.165$ or $R_{AmT}+\sigma_{AmT}$ $= 0.297$. To deal with the volatility in the power to invest, we adapt the notion of uncertainty profile given in Fragnito et al. (2017) to this framework. A uncertainty profile captures one volatile scenario. It is a tuple $U=(S,A,D,\delta_a, \delta_d, b_a,b_d)$ where, S = $(R; R_1,\ldots,R_n)$, is an investment schema; $A,D \subseteq [n]$ are the sets of assets whose returns may be subject to angelic and daemonic perturbations, respectively; $\delta_a : A \rightarrow R$ and $\delta_d : \mathrm{D} \rightarrow \mathrm{R}$ are the strength of the potential return's perturbations; $b_a, b_d \in N, b_a \leq \#A$ and $b_d \leq \#D$ represent the spread of the angelic and daemonic perturbations. Based in the mean-variance approach proposed by H. Markowitz (1959), we take δ_a $(i)=\sigma_i$ and δd (i) $= -\sigma i$. The perturbation is exerted though joint actions (a,d), where $a \subseteq A$ and $d \subseteq D$, $\#a = ba$ and $\#d = b_d$, providing the perturbed schema $S[a,d] = (R; R'_1,\ldots, R'_n)$, with $R'_i = R_i + x_a(i)\,\sigma_i - x_d(i)\,\sigma_i$, where $x_a(i) = 1$, if $i \in a$, 0 otherwise, and $x_d(i) = 1$, if $i \in d$, 0 otherwise. Let us consider an example. The following table is adapted from Table 8.1 in Arratia (2014).

	1 BKE	2 FCEL	3 GG	4 OII	5 SEB
R_i	0.027	0.068	0.021	0.018	0.010
σ_i	0.15	0.270	0.189	0.164	0.124

For instance, taking $S = (R; 0.027,0.068,0.021,0.018,0.010)$ and considering $(a,d)=(\{BKE\},\{GG\}) = \{\{1\},\{3\}\}$, $S[a,d] = (R; 0.177,0.068,\text{-}0.168,0.018,0.010)$.

We are interested to know how perturbations affect the power of an investment. Given $U = (S,A,D,\delta_a,\delta_d, b_a, b_d)$, we associate an strategic angel/daemon (or a/d) game as follows. $G(U) = (\{a,d\}, A_a, A_d, u_a, u_d)$ has two players, the angel a and the daemon d. The respective sets of player's actions are $Aa = \{a \subseteq A | \#a = b_a\}$ and $A_d = \{d \subseteq D | \#d = b_d\}$. The utilities are defined as u_a (a,d) $= Power(S[a,d])$ and u_d (a,d) $= -u_a$ (a,d), for $(a,d) \in A_a \times A_d$. Observe that a/d games by definition are zero-sum games. Consider U_1 with $A = D = \{GG,OII\}$, $b_a = b_d = 1$, and $R = 0.015$ and U_2 with $R = 0.02$, $A = \{BKE,FCEL\}$, $D = \{GG,OII\}$ and $b_a = b_d = 1$.The a/d game $G(U_1)$ has $A_a = A_d = \{\{GG\},\{OII\}\}$ while the a/d game $G(U_2)$ has $A_a = \{\{BKE\},\{FCEL\}\}$ and $A_d = \{\{GG\},\{OII\}\}$. Zero-sum games can be represented by giving the values of u_a, as shown in the following tables, there a is the row player and d is the column player.

$G(U_1)$	{GG}	{OII}
{GG}	0.935	0.709
{OII}	0.580	0.935

$G(U_2)$	{GG}	{OII}
{BKE}	0.516	0.548
{FCEL}	0.580	0.612

Notice that, in an a/d game the set of strategy profiles is $A_a \times A_d$. In general the angel and daemon choice of actions can be done probabilistically. Mixed strategies, for a and d, are probability distributions $\alpha{:}A_a \to [0,1]$ and $\beta{:}A_d \to [0,1]$ respectively. A mixed profile is a tuple (α,β). As usual utilities are extended to mixed profiles to $u_a(\alpha,\beta) = \sum_{(a,d)\in A_a \times A_d} \alpha(a) u_a(a,d)\beta(d)$ and $u_d(\alpha,\beta) = -u_a(\alpha,\beta)$. By definition, $u_a(a,d) = Power(S[a,d])$, thus it makes sense to extend Power to invest under mixed strategies defining $Power(S[\alpha,\beta]) = u_a(\alpha,\beta)$. Let Δ_a and Δ_d denote the set of mixed strategies for a and d, respectively. A pure strategy profile (a,d) is a special case of mixed strategy profile (α,β) in which $\theta(a)=1$ and $\theta(d)=1$. A mixed strategy profile (α,β) is a Nash equilibrium if, for any $\alpha' \in \Delta_a$, $u_a(\alpha,\beta) \geq u_a(\alpha',\beta)$ and, for any $\beta' \in \Delta_d$, $u_d(\alpha,\beta) \geq u_d(\alpha,\beta')$. A pure Nash equilibrium, PNE, is a Nash equilibrium where α and β are pure strategies. The preceding $G(U_1)$ has no PNE. Game $G(U_1)$ has a (mixed) Nash equilibrium given by $\alpha(\{GG\})=0.388$, $\alpha(\{OII\})=0.612$, $\beta(\{GG\})=0.611$ and $\beta(\{OII\})=0.389$. In this case $Power(S[\alpha,\beta]) = 0.796$. The strategy $(\{FCEL\},\{GG\})$ is the only PNE of $G(U_2)$ having a power of 0.580. J. von Neumann and O. Morgenstern (1953) shown that all the Nash equilibria of a zero-sum game G have the same value $\nu(G)$ corresponding to the utility of the row player. For an a/d game $G(U)$,

$$\nu(G(U)) = max_{\alpha\in\Delta_a} min_{\beta\in\Delta_d} Power(S[\alpha,\beta]) = min_{\beta\in\Delta_d} max_{\alpha\in\Delta_a} Power(S[\alpha,\beta]).$$

Considering a/d games, we extend the definition of the power to invest to uncertainty profiles defining $Power(U)=\nu(G(U))$. In the preceding examples $Power(U_1) = 0.796$ and $Power(U_2) = 0.580$.

4. CONCLUSIONS AND OPEN TOPICS

We have introduced the notion of power to invest as a measure of the freedom to invest in different assets. We apply this notion to a variety of cases and showing its workability. We considered equiprobable weights but other distributions could also be worth to analyse. In particular, the repetition of asset values allows us to model other distributions.

We also adapted the notion of uncertainty profile to deal with returns' volatility. Using the a/d approach we show that the power to invest is well shaped to deal with uncertainty profiles. In our examples the roles of a and d are symmetric, the angel increases the return by σ_i and the daemon decreases it by $-\sigma_i$. Nevertheless, asymmetrical views are also possible (in order to emphasize disasters, we can take $\delta_a=\sigma_{i/2}$ and $\delta_d=-2\sigma_i$). We have not taken into account correlations. However, correlation can help to design the sets A and D in U. Finally, let us remind that the power to invest is inspired by the Coleman's power to act in cooperative game theory. In game theory there are many other power indices. In particular, Shapley (1962) introduced the Banzhaf or the Shapley-Shubik. The incorporation of such indices to this framework is and interesting open problem.

ACKNOWLEDGMENTS

J. Castro was partially supported by the Spanish Ministry for Economy and Competitiveness (MINECO) and the European Union (FEDER funds) under grant (TIN2017-29244-R). J. Gabarro and M. Serna were partially supported by MINECO and FEDER funds under grant GRAMM (TIN2017-86727-C2-1-R) and Generalitat de Catalunya, Agència de Gestió d'Ajuts Universitaris i de Recerca, under project 2017 SGR 786 (ALBCOM). M. Serna was also supported by MINECO under grant BGSMath (MDM- 2014-044).

REFERENCES

A. Arratia (2014). Computational finance. An introductory course with R. Atlantis Press, Paris.

J. Coleman (1971). Control of collectivities and the power of a collectivity to act. In Social choice, pages 269–300. Gordon and Breach, 1971. Reedited, 2011.

G. Fragnito, J. Gabarro, and M. Serna (2017). An angel-daemon approach to assess the uncertainty in the power of a collectivity to act. In Proc. ECSQARU 2017, LNCS 10369:318–328. Springer, 2017.

J. Gabarro, M. Serna, and A. Stewart (2014). Analysing web-orchestrations under stress using uncertainty profiles. The Computer Journal, 57(11):1591–1615, 2014.

J. Gabarro and M. Serna (2017). Uncertainty in basic short-term macroeconomic models with angel-daemon games. Int. J. Data Analysis Techniques and Strategies, 9(4):314–330, 2017.

H. Markowitz (1959). Portfolio Selection. John Wiley, London, 1959.

L. Shapley (1962). Simple games: An outline of the descriptive theory. Systems Research and Behavioral Science, 7(1):59–66, 1962.

J. von Neumann and O. Morgenstern (1953). Theory of games and economic behavior. Princeton University Press, Princeton and Oxford, 1953.

QUANTIFYING CREDIT PORTFOLIO LOSSES UNDER MULTI-FACTOR MODELS

Gemma Colldeforns Papiol
Centre de Recerca Matemàtica, Spain, gcolldeforns@crm.cat

Luis Ortiz-Gracia
University of Barcelona, School of Economics, Riskcenter-IREA, Spain
luis.ortiz-gracia@ub.edu

Cornelis W. Oosterlee
Centrum Wiskunde & Informatica, Amsterdam, Holanda, c.w.oosterlee@cwi.nl

ABSTRACT

In this work, we investigate the challenging problem of estimating credit risk measures of portfolios with exposure concentration under the multi-factor Gaussian and multi-factor t-copula models. It is well-known that Monte Carlo (MC) methods are highly demanding from the computational point of view in the aforementioned situations. We present efficient and robust numerical techniques based on the Haar wavelets theory for recovering the cumulative distribution function (CDF) of the loss variable from its characteristic function. To the best of our knowledge, this is the first time that multi-factor t-copula models are considered outside the MC framework. The analysis of the approximation error and the results obtained in the numerical experiments section show a reliable and useful machinery for credit risk capital measurement purposes in line with Pillar II of the Basel Accords.

1. BACKGROUND

This work is an abridged version of the manuscript Colldeforns-Papiol et al. (2017). Financial institutions need to evaluate and manage the risk arising from its business activities. Credit risk is the risk of losses from the obligor's failure to honour the contractual agreements. It is usually the main source of risk in a commercial bank. Banks are subject to *regulatory capital* requirements under Basel Accords and they are forced to keep aside a cushion to absorb potential losses in the future. The capital needed in order to remain solvent at a certain confidence level is called *economic capital*. The basic regulatory risk measure in credit risk is Value-at-Risk (VaR) and it is a quantile of the loss distribution computed at 99.9% confident level with a one-year time horizon. Although it is still the regulatory measure, the VaR value has mainly two drawbacks that may impede a robust credit risk measurement. One of these two disadvantages is that VaR is not sub-additive and contradicts the idea of diversification. The second is that VaR gives no indication about the severity of losses beyond the computed quantile. This is the reason why the Expected Shortfall (ES) might be used in place of the VaR value for internal risk capital assessment (i.e., for economic capital calculation).

The Vasicek model forms the basis of the Basel II approach. It is a Gaussian one-factor model where default events are driven by a latent common factor that is assumed to follow a Gaussian distribution, also called the *Asymptotic Single Risk Factor* (ASRF) model. Under this model, loss only occurs when an obligor defaults in a fixed time horizon. If we assume certain homogeneity conditions, this one factor model leads to a simple analytical asymptotic approximation for the loss distribution and VaR value. This approximation works well for a large number of small exposures but can underestimate risks in the presence of exposure concentrations (see Ortiz-Gracia and Masdemont (2014)). Concentration risks in credit portfolios arise from an unequal distribution of loans to single borrowers (*exposure or name concentration*) or different industry or regional sectors (*sector or country concentration*). While regulatory capital is estimated by means of the ASRF model under Pillar I, economic capital takes into account concentration risks and is calculated under Pillar II. Monte Carlo simulation either with one-factor or multi-factor models (to account for sector concentration or for modelling complicated correlation structures) is a standard method for measuring the risk of a credit portfolio. However, this method is time-consuming when the size of the portfolio increases. Computations can become unworkable in many situations, taking also into account that financial companies have to re-balance their credit portfolios frequently. On top of that, when using MC methods the variance is always an issue when estimating the risk measures at high confidence levels. For the aforementioned reasons, numerical methods are appealing in this field.

Different techniques can be found in the literature for estimating the risk with multi-factor Gaussian copula models, like MC methods in Glasserman et al. (2007), Hermite approximations in Owen et al. (2015), where the main application is for large loan or mortgage portfolios, a hierarchical factor model in Fok et al. (2014) where closed-form solutions are derived under the assumption that the number of sectors in the portfolio is large, and an extension of the granularity adjustment technique to a new dimension is developed in Pykhtin (2004). However, as pointed out in Kang and Shahabuddin (2005), some works suggested that default events driven by t-distributed random variables provide better empirical fit to the observed data. This is the so-called t-copula model, where default events are expressed as the ratio of a normal and a scaled chi-square random variable. The bivariate version of this last type of models is tackled with simulation in Chan and Kroese (2010) and in Rutkowski and Tarca (2015), and a complicated multi-factor version in Kang and Shahabuddin (2005).

2. METHODOLOGY

In the present work, we develop numerical techniques to contribute to the efficient measurement of VaR and ES values for small or big portfolios in the presence of exposure concentration under high-dimensional models. It is worth remarking that small and/or concentrated portfolios are particularly challenging cases, since asymptotic methods usually work out well for large and diversified portfolios. We model the dependence among obligors by means of multi-factor Gaussian copula and multi-factor t-copula models. To the best of our knowledge, this is the first time that multi-factor t-copula models are considered outside the MC framework. We estimate the risk measures in a procedure composed of two main parts. The first part is the numerical computation of the characteristic function associated to the portfolio loss variable. We tackle this part with different techniques depending on the underlying model. For the (bivariate) t-copula model we perform a double integration with Gauss-Hermite and generalized Gauss-Laguerre quadrature, while the multi-factor Gaussian model is treated with the quadratic transform approximation (QTA) method put forward in Glasserman and Suchintabandid (2012), where the authors calculate the price of a collateralized debt obligation. We derive the characteristic function for the most challenging model, this is, the multi-factor t-copula, by conditioning on the chi-square random variable of the model and applying the QTA method at each discretization point of the resulting one-dimensional integral. This last model is by far the most involved in terms of computing effort. Once the characteristic function for the loss variable has been obtained, then the second part of the procedure consists of a Fourier inverson to recover its CDF. For this purpose, we use a method based on Haar wavelets developed in Masdemont and Ortiz-Gracia (2014) for the one-factor Gaussian copula model. Moreover, we have improved the efficiency of this method by computing the coefficients of the expansion by means of an FFT algorithm and we have shown that this method outperforms the well-known numerical Laplace transform inversion method of Abate et al. (2000) and Abate and Whitt (1995) used in risk management in Glasserman and Ruiz-Mata (2006) and in Glasserman and Suchintabandid (2012) in terms of efficiency and robustness. The numerical experiments carried out in this work show the high accuracy and speed of the method. Another point of importance is the robustness of the wavelet approach. We show how the scale of approximation (this is, the number of terms used to approximate the CDF) is related to the absolute error of the method. All these features make the proposed methodology an efficient and reliable machinery to be used in practice.

3. FINDINGS, CONCLUSIONS AND FUTURE WORK

In this work we have investigated efficient numerical methods to obtain the VaR and ES values for portfolios with exposure concentration under multi-factor Gaussian and t-copula models. It is well-known that MC methods are highly demanding from the computational point of view when dealing with big sized and high-dimensional models for the estimation of VaR and ES values at high confidence levels.

These methods are composed of two main parts. The first part is the numerical computation of the characteristic function associated to the portfolio loss variable. We tackle this part by different techniques depending on the underlying model. For the (bivariate) t-copula model we perform a double integration with Gauss-Hermite and generalized Gauss-Laguerre quadrature and call this method WA-HL, while the multi-factor Gaussian model is treated with the QTA method and called WA-QTA. The characteristic function for the most challenging model, this is, the multi-factor t-copula, is derived by conditioning on the chi-square random variable of the model and computed by applying the QTA method at each discretization point of the resulting one-dimensional integral. This last model is by far the most involved in terms of computing effort. Once the characteristic function for the loss variable has been obtained then the second part of the procedure comes into play. This second step is the Fourier inversion method called WA, which is based on Haar wavelets and it was developed in Masdemont and

Ortiz-Gracia (2014) and in Ortiz-Gracia and Masdemont (2014) to recover the CDF of the loss variable. We have improved the efficiency of the WA method by computing the coefficients of the expansion by means of an FFT algorithm and we have shown that this method outperforms the NLTI inversion method in terms of efficiency and robustness.

The overall CPU time of the numerical methods employed is impressive taking into account their size and dimension as well as the confidence levels considered. This may be the first time that multi-factor t-copula model is considered outside the MC framework. This research opens the door to calculate the risk contributions to the VaR and ES risk measures under the same model assumptions and we will consider this problem in our future work.

ACKNOWLEDGEMENTS

The research leading to these results has received funding from La Caixa Foundation. G. C.-P. acknowledges AGAUR-Generalitat de Catalunya for funding under its doctoral scholarship programme. G. C.-P. and L. O.-G. acknowledge the Spanish Ministry of Economy and Competitiveness for funding under grants ECO2016-76203-C2-2 and MTM2016-76420-P (MINECO/FEDER, UE).

REFERENCES

Abate, J., Choudhury, G. L., and Whitt, W. (2000). "An introduction to numerical transform inversion and its application to probability models". In Computational Probability, ed. W.K. Grassman, Kluwer, Norwell, MA.

Abate, J., and Whitt, W. (1995). "Numerical inversion of Laplace transforms of probability distributions". ORSA Journal on Computing, 7(1), 36-43.

Chan, J. C. C., and Kroese, D. P. (2010). "Efficient estimation of large portfolio loss probabilities in t-copula models". European Journal of Operational Research, 205, 361-367.

Colldeforns-Papiol, G., Ortiz-Gracia, L., and Oosterlee, C. W. (2017). "Quantifying credit portfolio losses under multi-factor models". Submitted for publication, available at www.ssrn.com.

Fok, P. W., Yan, X., and Yao, G. (2014). "Analysis of credit portfolio risk using hierarchical multifactor models. Journal of Credit Risk, 10(4), 45-70.

Glasserman, P., Kang, W., and Shahabuddin, P. (2007). "Large deviations in multifactor portfolio credit risk. Mathematical Finance, 17(3), 345-379.

Glasserman, P., and Ruiz-Mata, J. (2006). "Computing the credit loss distribution in the Gaussian copula model: a comparison of methods". Journal of Credit Risk, 2(4), 33-66.

Glasserman, P., and Suchintabandid, S. (2012). "Quadratic transform approximation for CDO pricing in multifactor models". SIAM J. Financial Math., 3(1), 137-162.

Kang, W., and Shahabuddin, P. (2005). "Fast simulation for multifactor portfolio credit risk in the t-copula model". In Proceedings of the 37th conference on Winter simulation, pages 1859-1868. Winter Simulation Conference.

Masdemont, J. J., and Ortiz-Gracia, L. (2014). "Haar wavelets–based approach for quantifying credit portfolio losses". Quantitative Finance, 14(9), 1587-1595.

Ortiz-Gracia, L., and Masdemont, J. J. (2014). "Credit risk contributions under the Vasicek one-factor model: a fast wavelet expansion approximation". Journal of Computational Finance, 17(4), 59-97.

Owen, A., Bryers, J., and Buet-Golfouse, F (2015). "Hermite approximations in credit portfolio modeling with probability of default-loss given default correlation". Journal of Credit Risk, 11(3), 1-20.

Pykhtin, M. (2004). "Multi-factor adjustment". Risk, March, 85-90.

Rutkowski, M., and Tarca, S. (2015). "Regulatory capital modeling for credit risk". International Journal of Theoretical and Applied Finance, 18(5), 1550034, 44 pages.

REMARKS ON BENEFIT SHARING

Freddy Delbaen
Departement für Mathematik, ETH Zürich, Rämistrasse 101, 8092 Zürich, Switzerland, delbaen@math.ethz.ch
Institut für Mathematik, Universität Zürich, Winterthurerstrasse 190, 8057 Zürich, Switzerland

Delia Coculescu
Institut für Mathematik, Universität Zürich, Winterthurerstrasse 190, 8057 Zürich, Switzerland

ABSTRACT

We use the theory of coherent measures to look at the problem of benefit sharing in an insurance business. The benefit share of an insured is calculated by the surplus premium in the contract. The theory of coherent risk measures and the resulting capital allocation gives a way to divide the benefit between the insured and the capital providers, i.e. the shareholders.

1. INTRODUCTION

We use the following setup. There are N agents to be insured, indexed $i = 1,...,N$. There is one insurer denoted by the index 0. There is also a "super"-reinsurer whose role will be explained later. The agents have liability that they want to insure. This liability for agent i is $X_i \geq 0$. Without insurance her position will be $-X_i$. There are different premium principles which will be described in the examples below. The utility functions of the agents are denoted by u_i. The coherent utility function of the insurer is u_0 and to reduce complexity, supposed to be commonotonic. The utility functions of the agents are more restrictive than the insurer's utility function u_0. We mean by that when a random variable is not acceptable for the insurer, then it is not acceptable for the agents. Equivalently we can say that acceptable elements for the agent i are acceptable for the insurer. This is a translation of the fact that the agent i feels a need for insurance.

The insurer also brings in an initial capital k_0. He will take the insurance only if he can obtain a better outcome. Because u_0 is coherent the total premium, π_0, must be at least $\pi_0 = -u_0\left(-\sum_{i=1}^N X_i\right)$. We suppose that the scenario set S, for u_0 is weakly compact so that there is a $Q_0 \in S$ such that

$$E_{Q_0}\left[\sum_{i=1}^N X_i\right] = \sup_{QS} E_Q\left[\sum_{i=1}^N X_i\right] = -u_0\left(-\sum_{i=1}^N X_i\right).$$

In case u_0 is commonotonic, we can even choose Q_0 so that for all Y commonotonic with $\sum_i X_i$ and such that the distribution of Y has no other points of increase as $\sum_i X_i$, we have $u_0(Y) = E_Q[Y]$.

From [1],[2] using the capital allocation principle we find that the individual "fair" premia should be

$$\pi_i = E_{Q0}[X_i].$$

For the agent i this is a good deal since $-\pi_i \geq u_i(-X_i)$. What she pays for insurance is better than paying X_i. Indeed for each i we have $-\pi_i \geq u_i(-X_i)$ which implies that $\pi_i - X_i$ is not acceptable for the insurer and hence not acceptable for the agent i.

2. MODEL 1

In this example we suppose that the insurer has limited liability. We take for the total premium π_0. We distinguish several cases:

1. $\sum_i X_i > \pi_0 + k_0$ In this case the total claim size exceeds the available capital. The excess is supposed to be covered by for instance the government and this at no cost. The initial capital should then be sufficiently high to make the deal acceptable for the government. The determination of this level is beyond the contents of this paper. The problem is the subject of ongoing research of the present authors together with Artzner and Eisele. We denote by A the set $A = \left\{\sum_i X_i > \pi_0 + k_0\right\}$.

2. $\pi_0 \leq \sum_i X_i \leq \pi_0 + k_0$ In this case there is no benefit and the insurer will lose part of his investment. We denote by the set $B = \{\pi_0 \leq \sum i \leq \pi_0 + k_0\}$.

3. $\pi_0 > \sum_i X_i$ In this case there is a benefit. The insurer will keep the benefit. The agents do not get a benefit share. This can be defended since they already "gained" from the allocation principle which is their share when entering the insurance. Also they do not take any risk. We denote by C the set $C = \{\pi_0 > \sum_i X_i\}$.

Is this deal acceptable for the insurer? The insurer will accept the deal if

$$u_0 = (1_{B \cup C}(\pi_0 + k_0 - \sum_i X_i)) \geq k_0$$

This is easily proved. By definition of π_0, we have $u_0\ (\pi_0 - \sum_i X_i)=0$, hence $u_0\ (\pi_0 - \sum_i X_i + k_0)=k_0$, therefore by monotonicity $u_0\ ((\pi_0 - \sum_i X_i + k_0)^+) \geq k_0$. We leave it as an exercise to find good conditions under which the last inequality is strict.

3. MODEL 2

This is almost the same as example 1, but this time we require a premium for covering the excess. We also assume that the reinsurer has no default. The reinsurance premium will be calculated by the same coherent utility function, i.e. the same set S. It is here that we use the commonotonicity. The retention will be denoted by R and this results in the splitting:

$$\sum_i X_i = \left(\sum_i X_i\right) \wedge R + \left(\sum_i X_i - R\right)^+$$

The two terms are commonotonic and hence the premium satisfies

$$\pi_0 = \pi^R + \rho^R; \pi^R = E_{Q_0}\left[\left(\sum_i X_i\right) \wedge R\right]; \rho^R = E_{Q_0}\left[\left(\sum_i X_i - R\right)^+\right]$$

The retention R must satisfy:

$$\pi^R + k_0 = R;.$$

The existence and uniqueness of R follows from an easy analysis of the function

$$R_+ \to R_+; R \to R - E_{Q_0}\left[\left(\sum_i X_i\right) \wedge R\right]$$

This function is strictly increasing after it leaves zero, is 0 at 0 and tends to ∞ for $R \to +\infty$. The available capital is R, which is also the maximum the insurer has to pay out. The surplus is therefore

$$R - \left(\sum_i X_i\right) \wedge R$$

The agents again do not take any risk and hence they should not participate in the benefit. The insurer finds it a good deal if

$$u_0\left(R - \left(\sum_i X_i\right) \wedge R\right) \geq k_0$$

But the definition of R shows that $u_0\left(R - \left(\sum_i X_i\right) \wedge \mathrm{R}\right) \wedge R\right) = k_0$.That means there is no incentive to do business and the insurer must get all the profit to have an equivalent outcome.

4. MODEL 3

This is an extension of the previous models. The agents pay a premium equal to $pi \geq \pi_i$. This has the advantage that the insurer can announce the premium without having to calculate the total premium necessary to cover the total losses. Of course this procedure should lead to a premium greater than the fair premium as calculated in the previous models. An example of such a premium calculation could be the amount $\sup_{Q\in S} E_Q[X_i] = -u_0(-X_i)$. The extra premium can be seen as a contribution to the capital. This time the agents can lose money and hence are entitled to a share in the benefit. All investors must be treated in the same way and hence the share of agent is proportional to her contribution, namely: $\frac{p_i - \pi_i}{\sum_j (p_j - \pi_j) + k_0}$. The investor gets: $\frac{k_0}{\sum_j (p_j - \pi_j) + k_0}$. These fractions are paid out regardless of having caused a claim or not.

The re tention is now defined by the relation

$$\sum_i (p_i - \pi_i) + k_0 + \pi^R = R$$

The existence and uniqueness of R are proved in the same way. This time we must see whether this is a good deal for the insurer as well as for the agents. For the insurer we must check the inequality

$$u_0\left(\frac{k_0}{k_0+\sum_i (p_i-\pi_i)}\left(R-\sum_i X_i\right)^+\right)\geq k_0$$

By homogeneity of u_0 this is the same as:

$$u_0\left(R-\left(\sum_i X_i\right)^+\right)\geq k_0+\sum_i (p_i-\pi_i)$$

¡As in the previous example this follows from the definition of R which implies:

$$u_0\left(R-\left(\sum_i X_i\right)\wedge R\right)=R-\pi^R=k_0+\sum_i (p_i-\pi_i)$$

For agent we must check:

$$u_i\left(-p_i+\frac{p_i-\pi_i}{k_0+\sum_j (p_j-\pi_j)}\left(R-\sum_i X_i\right)^+\right)\geq u_i(-X_i)$$

This is equivalent to

$$u_i\left(\frac{p_i-\pi_i}{k_0+\sum_j (p_j-\pi_j)}\left(R-\sum_i X_i\right)^+\right)\geq u_i(p_i-X_i)$$

The left hand side is positive whereas the right hand side is negative provided the premium p_i is not too big. For instance if $p_i \leq \sup_{Q\in S} E_Q\,[X_i]$ we have $u_0(p_i - X_i) \leq 0$ and hence also $u_i(p_i - X_i) \leq 0$. In any case the agent i will not pay a premium p_i hat is bigger than $-u_i\,(-X_i)$. Counting on benefit participation is not realistic since the benefit depends also on the claims incurred through the other agents.

5. DISCUSSION OF THE MODELS

There are different shortcomings of the models used. The premiums paid by the agents are augmented by the administration or handling costs. Here we might argue that in case of a claim, the agents incur these costs themselves. In the handling costs there is also included the commission paid out to the intermediaries or the brokers.

The utility functions of the agents cannot be supposed to be positively homogeneous. They should be concave. The assumption can be relaxed but is used here to avoid too many technicalities. However if commonotonicity is used we must suppose that the utility functions are positively homogeneous as this is a consequence of commonotonicity. So we use coherence for the utility function of the insurer.

We supposed the presence of a reinsurer who is default free. In some cases a government can provide a guarantee but in general we must include the possibility of default of the reinsurer. We also supposed that the reinsurer and the insurer have the same premium calculation premium principle. This is not true in practice and more general situations can be handled. For the sake of simplicity these are not yet included in the paper.

To limit the liability of the insured, there might be a rule that in case of default of the insurer and the reinsurer, the agent is not liable for the remaining losses.

In our models there is a reward for those who take the risk. This is in contradiction with some life insurance practice where only the amount of the total premium is important.

REFERENCES

Delbaen, F (2002). "Coherent Risk Measures". Lectures given at the Cattedra Galileiana at the Scuola Normale Superiore di Pisa, March 2000, Published by the Scuola Normale Superiore di Pisa.

Delbaen, F (2011). "Monetary Utility Functions". Lectures held in 2008 and published in the series "Lecture Notes of the University of Osaka".

INDICADORES DE MORTALIDAD PARA COLOMBIA

Gisou Díaz
Universidad del Tolima, Colombia
Departamento de Matemáticas y Estadística
gdiazrj@ut.edu.co

Ana Debón
Universitat Politecnica de València, Spain
Centro de Gestión de la Calidad y del Cambio
andeau@eio.upv.es

Vicent Giner-Bosch
Universitat Politecnica de València, Spain
Centro de Gestión de la Calidad y del Cambio
vigibos@eio.upv.es

ABSTRACT

El análisis de indicadores de mortalidad de un país es fundamental para evaluar su situación social, económica y de salud. En Colombia, como en muchos países de América Latina, se han dado cambios demográficos importantes como la disminución de la tasa de mortalidad, el aumento de la esperanza de vida, la disminución de la tasa de natalidad y cambios en la estructura por edades de la población, fenómenos que han contribuido a que se presenten los primeros signos de envejecimiento poblacional. El objetivo de este trabajo fue calcular indicadores de mortalidad como esperanza de vida y el índice de Gini, que resuman y caractericen la mortalidad en Colombia y sus cambios, tanto para edades infantiles como avanzadas, con el fin de poder analizar fenómenos demográficos como la longevidad.

1. INTRODUCCIÓN

En las últimas décadas, en Colombia se han dado grandes cambios a nivel demográfico: incremento constante de la población, disminución en la tasa de mortalidad infantil, aumento de la esperanza de vida al nacer y primeros signos de envejecimiento poblacional. Estos cambios demográficos son consecuencia del crecimiento y desarrollo del país, así como del aumento del nivel educacional, mejoras en la salud y en las condiciones en calidad de vida de la población.

Las tablas de mortalidad permiten medir las probabilidades de vida y de muerte de una población. Asimismo, aportan información sobre algunos indicadores como la mortalidad infantil, la esperanza de vida al nacer y la estructura de la mortalidad por sexo y edad, elementos básicos para el análisis de la situación actual, en el pasado reciente y en futuro próximo del país.

Los indicadores de mortalidad y en especial la esperanza de vida han servido para resumir el comportamiento de la mortalidad en cuanto a tendencia, sin embargo, cabe preguntarse el comportamiento de la mortalidad en cuanto a dispersión a través de medidas como el índice de Gini. Un conjunto apropiado de indicadores para el estudio de la longevidad debe incluir un indicador de la mortalidad infantil, la esperanza de vida, la edad modal de muerte, la curva de Lorenz y el índice de Gini.

2. INDICADORES DE MORTALIDAD

La esperanza de vida se puede calcular a partir de la tasa de mortalidad. Representa el número medio de años que les quedan por vivir a los supervivientes a la edad x en caso de prevalecer las condiciones de mortalidad existentes (INE, 2017). En este trabajo calculamos la esperanza de vida al nacer y la esperanza de vida a los 65 años.

La edad modal de muerte constituye un indicador de longevidad. Representa la edad a la cual se produce el máximo de defunciones de una población. En una tabla de mortalidad indica la edad a la cual fallecen la mayoría de los individuos de la cohorte ficticia inicial (Canudas-Romo, 2008).

La curva de Lorenz de mortalidad representa la distribución de la edad de muerte de los individuos de una población. Para obtener la curva se ubica la proporción de fallecidos antes de la edad x en las abscisas frente a la proporción acumulada de años que esos individuos han vivido en las ordenadas. Luego se unen los puntos, quedando la curva siempre por debajo de la diagonal. Cuando el número de años vividos está repartido por igual en toda la población, la curva de Lorenz coincide con la diagonal. El caso opuesto de que el número de años vividos esté concentrado en un solo individuo, la curva de Lorenz recorrerá los ejes inferior y derecho (Llorca et al., 2000).

El índice de Gini se considera la medida más útil para analizar la desigualdad en la esperanza de vida y resume la curva de Lorenz de mortalidad (Singh et al., 2017). Se calcula como una función adicional de la tabla de mortalidad, que evalúa la desigualdad entre los individuos correspondiente a los años vividos por una persona hasta la muerte. Si el índice de Gini de mortalidad es cercano a cero indica que todos los individuos mueren aproximadamente a la misma edad, mientras que si es cercano a uno indica que hay grandes diferencias en la edad de muerte, por consiguiente, una gran cantidad de individuos mueren a una edad muy temprana y muy pocos consiguen sobrevivir más que la media.

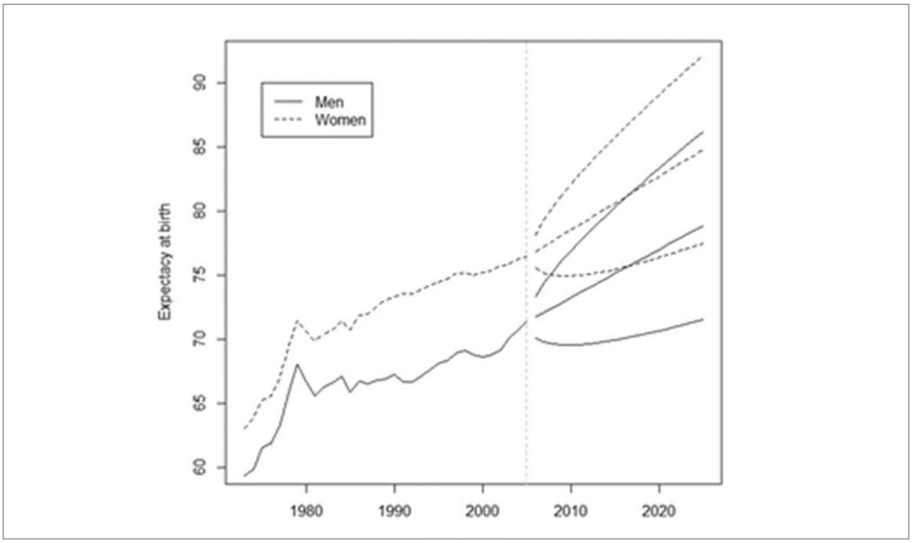
La razón de dependencia económica es un indicador demográfico que relaciona la población en edades económicamente improductivas con el resto de la población (Lora, 2008). En Colombia, al igual que para la mayoría de los países, la población considerada como dependiente son los menores de 15 años, $P_{<15}$, y mayores de 65, $P_{>65}$, y la población económicamente productiva o "potencialmente activa" corresponde a personas con edad entre 15 a 64 años, P_{15-64}.

3. CÁLCULO DE INDICADORES PARA COLOMBIA

En este estudio se utilizaron datos provenientes de tablas de mortalidad abreviadas calculadas para Colombia en el periodo 1973-2005, utilizando información proveniente de Latin American Human Mortality Database (Urdinola and Queiroz, 2017). Las edades se encuentran agrupadas: [0-1[, [1-5[, [5-10[y el resto en grupos quinquenales de edad hasta los 85 años. Como solo se disponía de datos de población para los últimos cuatro censos (1973, 1985, 1993, 2005) se completó la información haciendo uso de la interpolación lineal para calcular la población entre censos (1974 a 1984, 1986 a 1992 y de 1994 a 2004).

Se calcularon los indicadores de mortalidad para el periodo analizado para hombres y mujeres por separado y, además para alguno de ellos, se realizaron proyecciones hasta 2025 con el fin de poder analizar sus tendencias en los próximos años y su relación con los cambios demográficos que se vienen presentando en Colombia. Para las proyecciones se utilizó el modelo de Lee-Carter con dos términos para calcular las probabilidades de muerte en el periodo 2006-2025 mediante el ajuste de un modelo de serie temporal (ARIMA) para cada proceso temporal. Para el análisis de los datos se utilizó el programa R, versión 3.2.3 (R Core Team, 2015) y las librerías gnm (Turner and Firth, 2015) y forecast (Hyndman, 2016).

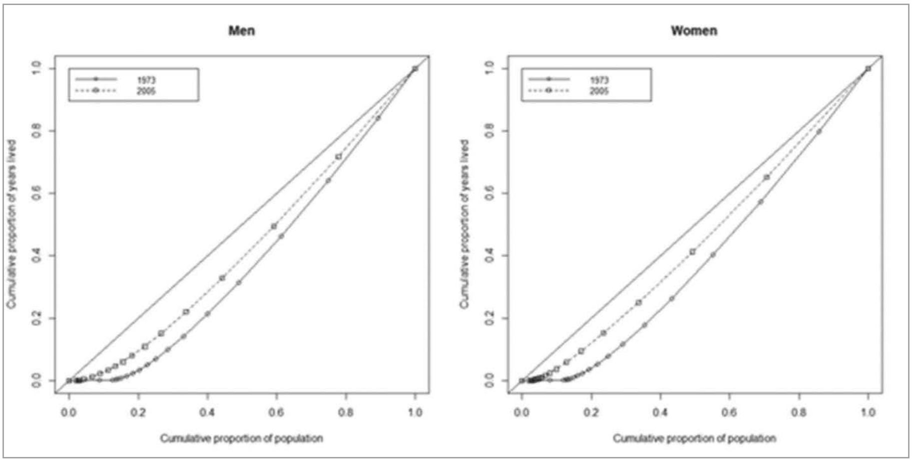
Figure 1. Evolución (1973-2005) y predicción (2006-2025) para la esperanza de vida al nacer para Colombia



La Figura 1 muestra la evolución de la esperanza de vida al nacer de 1973 a 2005 y la predicción para 2006-2025. La esperanza de vida aumentó en ambos sexos. En los hombres el aumento fue alrededor de 10 años y en las mujeres de 13 años durante el periodo estudiado. Además, podemos decir que la esperanza de vida sigue con esta tendencia de aumentar en ambos sexos, y para 2025 el aumento sería de 7.5 años en promedio partiendo de los 70 años y 75 años para hombres y mujeres respectivamente.

La curva de Lorenz para los años 1973 y 2005 se muestran en la Figura 2. En ambos sexos se observa una leve tendencia hacia la diagonal para el 2005, siendo en las mujeres este acercamiento más notable. Conjuntamente se puede apreciar como las edades infantiles y juveniles tienen una contribución pequeña en la distribución a los años vividos, lo que muestra una desigualdad en la edad de muerte (o la esperanza de vida) de la población colombiana.

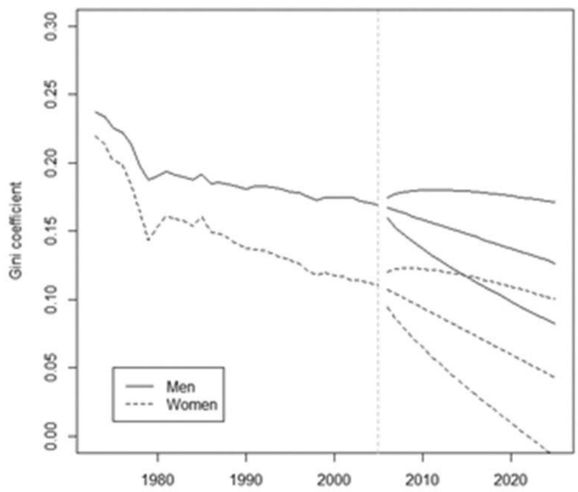
Figure 2. Curva de Lorenz de mortalidad para Colombia, 1973 y 2005



En la Figura 3 observamos el comportamiento del índice de Gini de mortalidad, el cual disminuye en ambos sexos durante el periodo analizado, siendo la disminución más marcada en las mujeres. Podemos indicar por tanto que las desigualdades en cuanto a la edad de muerte son mayores en los hombres que en las mujeres durante todo el periodo analizado, y la proyección es que esta tendencia continúe hasta 2025.

Para la edad modal de muerte, podemos decir que en los hombres durante el periodo 1973 a 1989 fue el intervalo [75,80[años, y entre 1990 a 2005, la edad modal de muerte aumentó, ubicándose en el intervalo [80,85[años. En las mujeres la edad modal de muerte estuvo en el intervalo [75,80[años para el periodo 1973 a 1983, y en el periodo 1984 a 2005, la edad modal de muerte aumentó al intervalo [80,85[años.

Figure 3. Evolución (1973-2005) y predicción (2006-2025) del índice de Gini de mortalidad para Colombia



En cuanto a la razón de dependencia para el periodo analizado, vemos como disminuye de manera notable en ambos sexos (de 0.94 a 0.60 en los hombres y de 0.85 a 0.56 en las mujeres), permaneciendo en cada año los valores más elevados en los hombres.

4. CONCLUSIONES

El cálculo y la predicción de los indicadores de mortalidad permite afirmar que la población colombiana se ha visto inmersa en un fenómeno de mejoramiento gradual de sus condiciones de vida. Además de calcular la esperanza de la vida, medida más utilizada para analizar la longevidad, que refleja los cambios de la mortalidad en el tiempo, en el presente trabajo se estudiaron otros indicadores como la edad modal de muerte, la curva de Lorenz de mortalidad, el índice de Gini de mortalidad y la razón de dependencia. Entre las conclusiones alcanzadas en el periodo estudiado podemos mencionar que la esperanza de vida se ha incrementado en el tiempo y esta tendencia se mantiene para los próximos años. Además, se observa mayor longevidad en las mujeres que en los hombres dado que tienen una esperanza de vida mayor y un índice de Gini menor. El aumento de la edad modal de muerte y la mejora en la curva de Lorenz confirman los cambios demográficos favorables en cuanto longevidad y a una distribución de la esperanza de vida proporcional en la población.

AGRADECIMIENTOS

La investigación llevada a cabo en este trabajo ha sido financiada por el Ministerio de Economía y Competitividad de España, con el proyecto MTM2013-45381-P.

REFERENCIAS

Canudas-Romo, V. (2008). "The modal age at death and the shifting mortality hypothesis". Demographic Research, 19, 1179-1204.

Hyndman R.J. (2015). "Forecast: Forecasting functions for time series and linear models". R package version 6.2, http://github.com/robjhyndman/forecast.

INE (2017). "Indicadores Demográficos Básicos". Instituto Nacional de Estadística, España.

Llorca, J., Prieto, M. D., Alvarez, C. F., and Delgado-Rodríguez, M. (1998). "Age differential mortality in Spain, 1900-1991". Journal of Epidemiology & Community Health, 52:259-261.

Lora, E. (2008). "Técnicas de medición económica. Metodología y aplicaciones en Colombia". Alfaomega Clombiana S.A, Bogotá D.C., fourth edition.

Singh, A., Shukla, A., Ram, F., and Kumar, K. (2017). "Trends in inequality in length of life in India: a decomposition analysis by age and causes of death". Genus, 73(1):5.

Turner, H. and Firth, D. (2015). "Generalized nonlinear models in R: An overview of the gnm package". R package versión 1.0-8, http://CRAN.R-project.org/package=gnm.

Urdinola, B. and Queiroz, B. (2017). www.lamortalidad.org. Latin American Human Mortality Database, 21 de Noviembre de 2015.

TIME-VARYING RISK AVERSION. AN APPLICATION TO EUROPEAN OPTIMAL PORTFOLIOS

Antonio Díaz
Universidad de Castilla-La Mancha, Albacete, Spain
Facultad de CC. Económicas y Empresariales | Plaza de la Universidad,1 | 02071
Antonio.Diaz@uclm.es

Carlos Esparcia
Universidad de Castilla-La Mancha, Albacete, Spain
Facultad de CC. Económicas y Empresariales | Plaza de la Universidad,1 | 02071
Carlos.Esparcia@uclm.es

ABSTRACT

Despite the influence of risk aversion in the optimal portfolio context, there are not many studies which have explicitly estimated the risk aversion parameter. Instead of that, researchers almost always choose random fixed values to reflect the common levels of risk aversion. However, the above could generate optimal portfolios, which not reflect the actual investor's attitude towards risk. Otherwise, as it is well known, an individual is more or less risk averse according to the economic and political circumstances. Given the above, we model the risk aversion attitude so that it changes over time, in order to take into account the variability in agents' expectations. Therefore, the aim of this paper is to shed light on the choice of the risk aversion parameter that correctly represents the investors' behaviour. For that purpose, we build optimal portfolios for different types of investment profiles in order to compare whether is better to use a constant risk aversion parameter or a dynamic one. In particular, our proposal is based on estimating the time-varying risk aversion parameter as a derivation of the market risk premium. For that purpose, we implement several statistical univariate and multivariate models. Specifically, we use conditional variance and correlation models, such as GARCH (1, 1), GARCH-M (1, 1) and DCC-GARCH.

1. INTRODUCCIÓN

In accordance with the mean-variance approach, we can partially order the set of investment opportunities, reducing the choice of investors to those portfolios located on the efficient frontier. However, with this approach, the investors cannot compare which alternatives are dominant among themselves; therefore, they are not allowed to select the investment portfolio that best meets their economic objectives. To find this portfolio, we must use a different criterion, incorporating the individual risk attitude. Although these preferences are very complex (they depend on, for instance, the age, gender, education level, and income of the individual), to make their implementation easier, they are represented by a single parameter that summarizes the personal level of risk aversion, the risk aversion parameter.

In spite of playing a key role in the optimal portfolio construction, there are few studies that have explicitly estimated the risk aversion of an investor. Instead, they choose random values to reflect the common levels of risk aversion. The equity literature on risk aversion developed based on the review by Arrow (1971), who affirmed that the risk aversion parameter should be approximately 1. Otherwise, in the equity context, several studies have been published that differ in their estimations of risk aversion. For instance, Mehra and Prescott (1985) argued that this parameter should be greater than 10. Moreover, Ghysels et al. (2005) affirmed the risk attitude should be between 1.5 and 2, on average, while Guo and Whitelaw (2006) established the mentioned parameter of 4.93.

However, common sense tells us that the use of fixed arbitrary values for this parameter could yield optimal portfolios that do not reflect the actual investor´s attitude towards risk. An individual is more or less risk averse according to the economic and political circumstances. For instance, we are currently in a period in which even the most adventurous investor has had to reduce his optimistic expectations. Given that, it seems reasonable to model the risk aversion parameter so that it changes over time, to consider the variability in the agents' expectations. In this context, there are some studies in the financial literature that refer to time-varying risk aversion. For instance, Kim (2014) proposes a consistent indicator of conditional risk aversion in consumption-based CAPM. Other studies have differed widely in their estimates of time-varying risk aversion, such as Dionne (2014), who aims to extend the concept of orders of conditional risk aversion to orders of conditional dependent risk aversion. However, our motivation follows the framework proposed by Frankel (1982) and revised by Giovanni and Jorion (1989) and Cotter and Hanly (2010), which is based on estimating the risk aversion parameter as a derivation of the CRRA[1].

According to the last paragraphs, the aim of this paper is to reveal the optimal parameter choice that provides a better representation of the investors' attitude towards risk. In particular, our proposal is based on estimating the time-varying risk aversion parameter as a derivation of the mentioned CRRA and strongly related to the market risk premium context. For that purpose, we build a well-diversified portfolio through the selection of ten risky equities traded on the Eurostoxx-50 index. From here, we introduce the time-varying modeling of probability distribution moments, to consider the optimal portfolios changing over time. To reach the above aim, and focusing on the optimal portfolio problem, we propose the application of conditional variance and correlation schemes such as GARCH (1, 1) and DCC-GARCH.

Otherwise, we estimate the CRRA from the market risk premium, which depends on the mean and variance of the market[2]. This estimation allows us to obtain the risk aversion attitude of an investor in a single number. However, in this research we are more interested in the time-varying risk aversion, not in a constant parameter. Thus, we model the market mean and variance through conditional models such as the GARCH (1, 1)-M specification. Further, we aim to assess whether it is better to work with a constant or changing risk aversion parameter. For that purpose, we build optimal portfolios for different types of investment profiles, a conditional one associated with the CRRA, and one based on constant risk aversion. Note that we assess the portfolios for different time frames, ones related to calm periods and others related to economic recession. Finally, to conclude this research, we present the main results and conclusions obtained from the study. Some important implications are revealed, including the best fit of the dynamic risk aversion attitude to the economic and political circumstances. The above could suggests that time-varying risk aversion models perform better than constant ones.

---

1 This tern refers to the changes in relative risk aversion, which is a way to express the risk aversion attitude of an investor through his utility function.

2 Note that we use the daily closing prices of EuroStoxx-50 Index as market portfolio and the 3-month German Treasury Bills as risk free rate.

2. HOW TO BUILD TIME-VARYING RISK AVERSION PORTFOLIOS

2.1. The optimal portfolio construction. An extension of the CARA function

The investor's problem is based on determine the equity weights that maximize the expected utility of the investor, given the constraint that these weights sum the unity:

$$\max_{w} U\,(E_P,\sigma_P^2) = (w'E - \frac{\alpha}{2}\; w'\,Vw) \tag{1}$$

$$s.a.\ \sum_{i=1}^{n} w_i = 1$$

We estimate the conditional variance matrix, V, from the application of univariate and multivariate schemes such as GARCH (1, 1) and DCC-GARCH.

2.2. Model A. Dynamic risk aversion and the market risk premium

We follow the approach revised by Cotter and Hanly (2010) which is based on estimating the observed risk aversion through a derivation of the CRRA and applies it to generate utility maximizing based on the unleaded gasoline market. Specifically, the derivation of the CRRA is based on the market risk premium. In this case, we use the mentioned estimation, but for the case of EuroStoxx-50 data[3]:

$$\alpha(CRRA) = \frac{E(R_m) - Rf}{\sigma_m^2} \tag{2}$$

In this case, we use the GARCH in mean schemes to estimate the conditional mean ($E(R_m)$) and variance (σ_m^2) simultaneously.

2.3. Model B. Constant risk aversion. An application of the Sharpe ratio

Focusing on the case of constant risk aversion, we must set a criterion for choosing an appropriate parameter according to the risk aversion attitude in Europe. In particular,

[3] We use daily closing prices and we transform them into returns by applying logarithms.

our proposal is based on choosing several values according to each one of the literatures mentioned in Section 1. Once we have selected these values, we keep the ones that make the optimal portfolio has better performance at each day according to the Sharpe ratio. We use the 3-month German Treasury Bills as risk-free rate[4]. Lately, we calculate the average of the optimal parameters obtained at each day in the market, in order to reach a single risk parameter. Finally, the obtained average parameter is 8.51, and it is the one used to model the optimal portfolios at each time of the market.

3. MAIN RESULTS AND FINDINGS. THE CERTAINTY EQUIVALENT

In this section, we show a well-known performance measure to assess our portfolio management. Figure 1 shows the Certainty-Equivalent.

Figure 1. Certainty-Equivalent

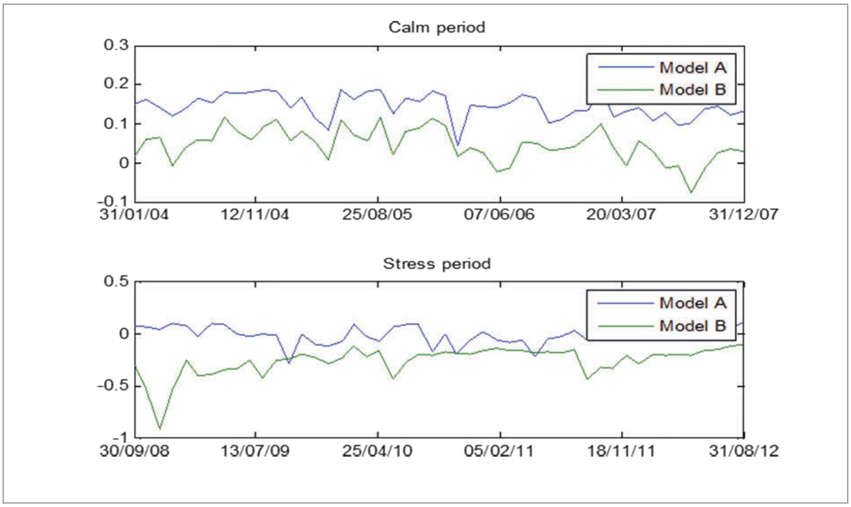


Figure 1 We plot the monthly evolution of the Certainty-Equivalent for both studied models (dynamic and constant risk aversion). This performance measure is expressed in annual terms. Note that we generate different kind of time-varying portfolios for two sample periods. Top graph represents a calm period, which runs from 01/01/2004 to 31/12/2007. Moreover, the second timeframe is a more stressed one (located at the bottom) and comes from 01/09/2008 to 31/08/2012.

[4] Remember that we are working with daily equity prices, but we can evaluate the performance of our portfolio in a monthly way by ascertaining the returns and the variance at monthly frequency.

Then, analysing the performance results from Figure 1, we have tested that the highest risk premium offered to exchange our portfolio, is the one showed by Model A (dynamic risk aversion), since it is the one that usually offers us the greatest relationship over time. We obtain evidence that portfolios with better performances based on Certainty-Equivalent ratio are those ones associated to the time-varying risk aversion attitude, while those with a constant risk aversion parameter (Model B), have a negative risk-return relationship and a very unstable trend throughout the whole studied period (calm and stress).

In addition, we implement a mean-difference test (parametric test) for two independent samples, that is to say, we compare whether the differences between the averaged ratios of the dynamic and constant models are significant or not. Then, if we had to invest some money in a risky portfolio, we would choose the one associated with Model A. However, according to the results obtained in the mentioned test, we find that there are few differences between the different selected models. In fact, if we prefer a less complex method (by calculating the conditional risk aversion attitude through different mathematical equations), we can select the constant risk aversion scheme. As we have mentioned before, this is because the differences between the best model (Model A, based on time-varying risk aversion) and the worst one (Model B, based on a constant risk attitude parameter) are not significant.

REFERENCES

Arrow, K., 1971. Essays in the theory of risk bearing, Markham, New York.

Cotter, J., Hanly, J., 2010. Time-varying risk aversion: An application to energy hedging. Energy Economics, 432-441.

Dionne, G., 2014. When can expected utility handle first-order risk aversion?. Journal of Economic Theory, volume 154, 403-422.

Frankel, J., 1982. In search of the exchange risk premium: A six currency test assuming mean-variance optimization. Journal of International Money and Finance 1, 255 – 274.

Guo, H., Whitelaw, R., 2006. Uncovering the risk–return relation in the stock market. The Journal of Finance 3, 1433 – 1463.

Ghysels, E., Santa-Clara, P., Valkanov, R., 2005. There is a risk-return trade off after all. Journal of Financial Economics 76, 509-548.

Giovannini, A., Jorion, P., 1989. The time-variation of risk and return in the foreign exchange and stock markets. Journal of Finance 2, 307 – 325.

Kim, K.H., 2014. Counter-cyclical Risk Aversion. Journal of Empirical Finance 29, 384-401.

Mehra, R., Prescott, E., 1985. The equity premium: A puzzle. Journal of Monetary Economics 15, 145-161.

SIMPLIFICANDO LA CREDIBILIDAD DE HACHEMEISTER

Zuleyka Díaz Martínez
Universidad Complutense de Madrid, España
Departamento de Economía Financiera y Actuarial
zuleyka@ccee.ucm.es

José Fernández Menéndez
Universidad Complutense de Madrid, España
Departamento de Organización de Empresas
jfernan@ccee.ucm.es

Óscar Rodríguez Ruiz
Universidad Complutense de Madrid, España
Departamento de Organización de Empresas
orr@ccee.ucm.es

ABSTRACT

La aplicación de la Teoría de la Credibilidad a modelos de regresión fue llevada a cabo por Hachemeister en 1975. Sin embargo el modelo que planteaba, pese a su plena corrección formal, daba lugar en ocasiones a resultados inesperados y poco intuitivos. Bühlmann y Gisler mostraron que estos problemas podían ser evitados tomando como origen para la variable independiente el baricentro o centro de masas de sus valores. Esto también resulta complejo y los resultados obtenidos pueden ser difíciles de interpretar. En este trabajo se plantea una alternativa sencilla e intuitiva que proporciona resultados razonables y que se basa en calcular de forma independiente los factores de credibilidad para la pendiente y para el intercepto de los modelos de regresión. Se comprueba la validez del modelo planteado usando los mismos datos de Hachemeister.

1. INTRODUCCIÓN

La Teoría de la Credibilidad permite calcular la prima para una póliza, o un subgrupo de pólizas dentro de una cartera más amplia, combinando la información disponible para la póliza o subgrupo con la del conjunto de la cartera. La prima será una suerte de media ponderada de las que corresponderían al subgrupo y a la cartera y donde la del subgrupo ponderará tanto más cuanto mayor sea la información disponible específica para él. Si M es la prima media para el conjunto de la cartera y X la prima para el subgrupo calculada teniendo en cuenta exclusivamente la información disponible para él, entonces la prima de credibilidad será $R=z\cdot X+(1-z)\cdot M$, donde z es el denominado factor de credibilidad, que toma valores entre 0 y 1 y tanto más próximos a 1 cuanta más información haya disponible para el subgrupo.

En el modelo de credibilidad de Bühlmann se asume que cada subgrupo dentro de una cartera se indexa por medio de un parámetro θ que es una variable aleatoria. Existirá un parámetro μ que caracteriza el riesgo de cada subgrupo, luego puede ser expresado como $\mu(\theta)$, y el objetivo es obtener un estimador de credibilidad de ese parámetro para cada subgrupo, estimador al que denominaremos $\hat{\hat{\mu}}(\theta_i)$. Se dispone de un estimador insesgado $\hat{\mu}(\theta_i)$ para $\mu(\theta_i)$ construido con la información disponible para el subgrupo i-ésimo y el objetivo es calcular $\hat{\hat{\mu}}(\theta_i)$ como una combinación lineal convexa de $\hat{\mu}(\theta_i)$ y de un parámetro global para toda la cartera de la forma $\mu=E(\mu(\theta_i))$. Los parámetros que caracterizan la cartera en su conjunto se denominan parámetros estructurales, y son, además de μ, la varianza de $\mu(\theta_i)$, $T = var(\mu(\theta_i))$ y $S_i = E(var(\beta|\theta_i))$, siendo β un estadístico suficiente para $\mu(\theta_i)$ que resume los valores de las observaciones de manera que el estimador de credibilidad depende de ellas sólo a través de β.

Para obtener el estimador de credibilidad, $\hat{\hat{\mu}}(\theta)$, a partir de un estimador insesgado $\hat{\mu}(\theta)$, se proyectará $\mu(\theta)$ sobre el subespacio vectorial $L(1,\hat{\mu}(\theta))$ de variables aleatorias de la forma $\lambda_0\cdot 1+\lambda\cdot\hat{\mu}(\theta)$. Los elementos de $L(1,\hat{\mu}(\theta))$, serán elementos de L^2, el espacio de Hilbert de variables aleatorias de cuadrado integrable. En L^2 se puede definir el producto escalar entre dos variables aleatorias como $\langle X,Y\rangle = E(X\cdot Y) = \int XYdP$. Este producto escalar da lugar a una norma tal que $\|X\|^2 = E(X^2) = \int X^2\, dP$. La proyección de $\mu(\theta)$ sobre $L(1,\hat{\mu}(\theta))$ será el elemento $\hat{\hat{\mu}}(\theta)$ de $L(1,\hat{\mu}(\theta))$ que minimiza la norma de $\hat{\mu}(\theta)-\hat{\hat{\mu}}(\theta)$. Se tendrá que $\hat{\mu}(\theta)-\hat{\hat{\mu}}(\theta)$ es ortogonal a $L(1,\hat{\mu}(\theta))$, con lo cual será ortogonal tanto a 1 como a $\widehat{\mu}(\theta)$. Éstas son las “ecuaciones normales” que permiten obtener el valor de $\widehat{\widehat{\mu}}(\theta)$. El estimador de credibilidad será entonces $\widehat{\widehat{\mu}}(\theta) = \left(1 - \frac{T}{S+T}\right)\mu + \frac{T}{S+T}\ \widehat{\mu}\ (\theta)$.

2. EL MODELO DE HACHEMEISTER

El modelo de Hachemeister es un caso particular de credibilidad de Bühlmann aplicado a un modelo de regresión lineal. Típicamente se tiene una prima que va evolucionando a lo largo del tiempo y se plantea un modelo de regresión lineal que describe esa evolución. Se trata de un problema multidimensional en el que los parámetros que deben ser estimados serán los coeficientes de la recta de regresión, y los estimadores de credibilidad de esos parámetros se obtendrán como combinación de los coeficientes de la regresión para cada subgrupo y los coeficientes para toda la cartera.

En un caso como éste el estimador de credibilidad $\widehat{\widehat{\mu}}(\theta)$ será un vector con dos componentes, que son el intercepto y pendiente de la recta de regresión que muestra la evolución de la prima a lo largo del tiempo. El espacio vectorial $L(1, \widehat{\mu}\ (\theta))$ sobre el que se proyecta $\mu(\theta)$ para obtener $\widehat{\widehat{\mu}}(\theta)$ está formado por el conjunto de vectores de dos componentes de la forma $\begin{pmatrix} \lambda_{01} & 0 \\ 0 & \lambda_{02} \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix}\begin{pmatrix} \widehat{\mu_0}(\theta) \\ \widehat{\mu_1}(\theta) \end{pmatrix}$, donde $\widehat{\mu}_0\ (\theta)$ y $\widehat{\mu}_1\ (\theta)$ son estimadores insesgados del intercepto y la pendiente de la recta de regresión para la prima del subgrupo.

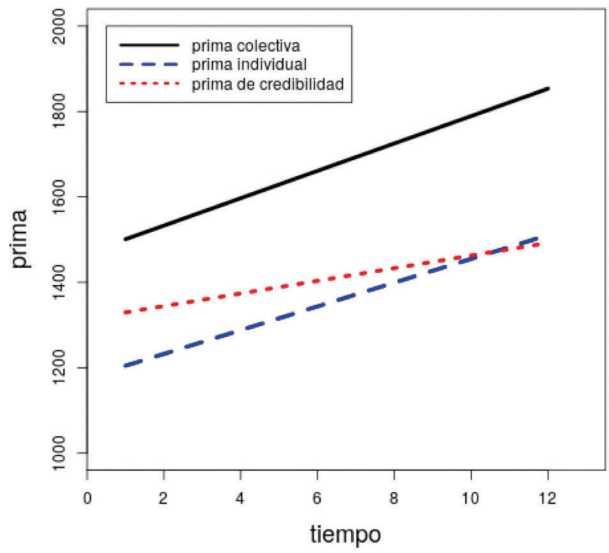
El problema con este enfoque es que cada estimador de la prima de credibilidad, $\widehat{\mu}_i\ (\theta)$ se ve afectado tanto por $\widehat{\mu}_0\ (\theta)$ como por $\widehat{\mu}_1\ (\theta)$, y estas interrelaciones entre pendientes e interceptos pueden dar lugar a resultados contraintuitivos y difícilmente justificables, cosa que ya resultó evidente para Hachemeister. Esto se puede observar en la Figura 1, construida con los mismos datos usados por Hachemeister y disponibles en el paquete de R actuar. Lo esperable sería que la evolución a lo largo del tiempo de la prima de credibilidad fuese algún tipo de promedio entre la prima colectiva y la individual, pero para el caso concreto del estado nº 4 de los usados por Hachemeister (que es el representado en la Figura 1), esto no ocurre, y las primas de credibilidad estimadas para periodos futuros en modo alguno pueden ser vistas como una combinación lineal convexa de las primas individual y colectiva. En un trabajo más de 20 años posterior al de Hachemeister, Bühlmann y Gisler mostraron que esto puede solventarse reescalando la variable independiente (habitualmente el tiempo), para que su origen quede en el baricentro de su conjunto de valores. No obstante los resultados que se obtienen pueden ser difíciles de interpretar y, además, si los valores de las variables independientes son distintos para cada subgrupo también habrá que calcular baricentros distintos para cada uno de ellos, complicando aún más la situación.

3. UNA ALTERNATIVA MÁS SENCILLA

El problema con el modelo de Hachemeister deriva del hecho de que los dos parámetros para cada subgrupo, parámetros que serán la pendiente y el intercepto de cada una de las rectas que determinan las primas de credibilidad, se estiman conjuntamente, proyectando sobre un subespacio que tiene en cuenta ambos parámetros. Las interrelaciones entre los dos parámetros hacen que la recta que determina la prima de credibilidad pueda en algunos casos tener un comportamiento no esperado. El mecanismo de estimación de los parámetros no tiene en cuenta la estructura geométrica del modelo (es decir, no tiene en cuenta que un parámetro es un intercepto y otro una pendiente de un modelo lineal) y eso puede dar lugar a resultados inesperados.

Una alternativa que permite obtener resultados razonables consiste en estimar cada uno de los dos parámetros que determina la recta de la prima de credibilidad de forma independiente del otro e imponiendo, de ser necesario, alguna condición que excluya soluciones no deseadas. Una situación como ésta se puede observar en la Figura 2, en donde se consideran dos factores de credibilidad independientes, z_0 para la estimación del intercepto de la prima de credibilidad como $\hat{\hat{\beta}}_0 = z_0\,\beta_0+(1-z_0)\,B_0$, y $\boldsymbol{z_1}$ para la estimación de la pendiente de la recta que determina la prima de credibilidad como $\hat{\hat{\beta}}_1 = z_1\,\beta_1+(1-z_1)\,B_1$, donde la restricción de que tanto $\boldsymbol{z_0}$ como $\boldsymbol{z_1}$ tomen valores entre 0 y 1 nos garantiza un comportamiento razonable para la prima de credibilidad, ya que la prima de credibilidad en el origen tendrá un valor entre las primas individual y colectiva en el origen, y la pendiente de la prima de credibilidad también tendrá un valor intermedio entre las pendientes de las primas individual y colectiva; en la Figura 2 esto se traduce en que la recta correspondiente a la prima de credibilidad estará entre las dos líneas grises de puntos (cuyas pendientes son la de la prima individual en un caso y la colectiva en otro).

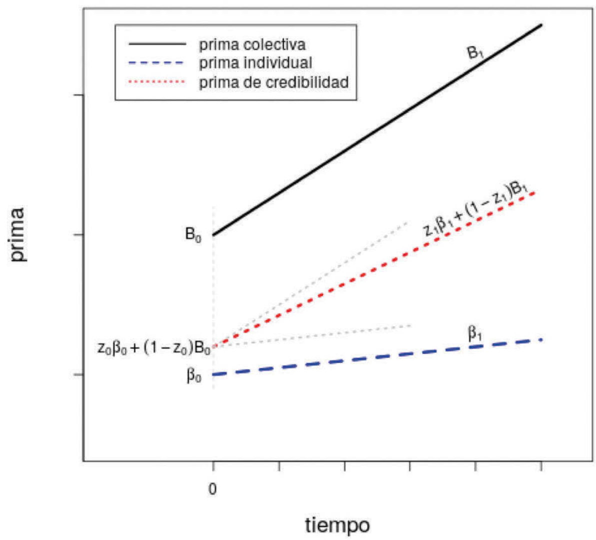
Figura 1. Problemas en el modelo de Hachemeister



En el caso de que se quiera obtener una recta para la prima de credibilidad que sea razonable para cualquier valor de la variable independiente la única solución es que esta recta forme parte del haz de rectas definido por las rectas de la prima individual y la colectiva. En este caso la prima de credibilidad pasará por el punto en que se cruzan estas dos rectas y sólo habrá que estimar su pendiente $\hat{\hat{\beta}}_1 = z_1 \beta_1 + (1-z_1) B_1$, siendo $z_1 \in [0,1]$. Esto garantiza unos valores totalmente razonables para la prima de credibilidad y además, en este caso, para todos los posibles valores de la variable independiente.

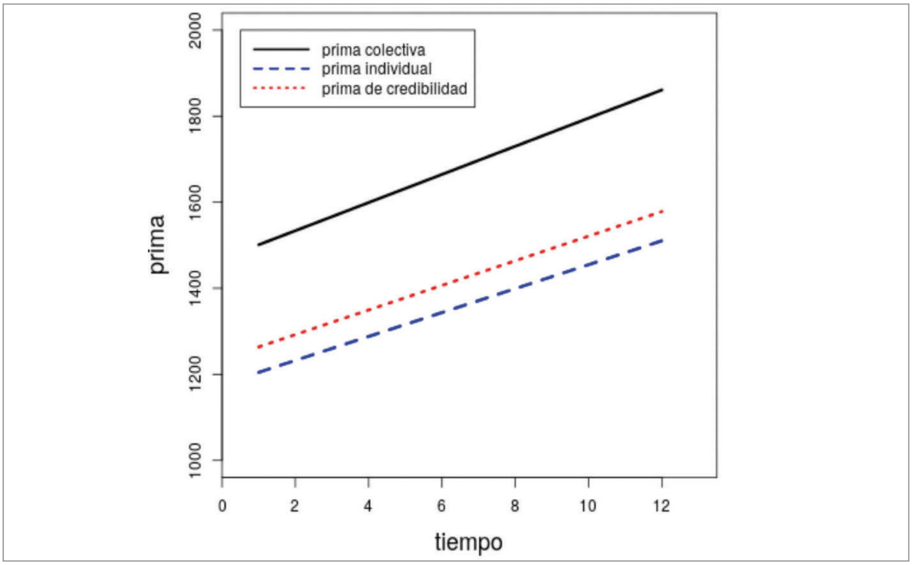
Vamos a aplicar este planteamiento a los datos usados por Hachemeister. Para ello será necesario calcular el intercepto y la pendiente que caracterizan la recta que representa la prima de credibilidad. Estos dos parámetros se estiman de forma independiente (aunque se utilizan los mismos datos para estimarlos, todos los valores de las observaciones). Será necesario estimar los valores de los parámetros estructurales. Se tendrá para ellos entonces que $S = \sum w_i \cdot var(\hat{\hat{\beta}}_1)$, siendo w_i la ponderación de cada subgrupo y $var(\hat{\hat{\beta}}_1)$ la varianza del coeficiente estimado, intercepto o pendiente, de la recta de regresión correspondiente a la prima de cada subgrupo, que T se calcula a partir del estimador $\sum w_i/w \left(\hat{\beta}_i - \overline{\beta}_i\right)^2$, ajustándolo convenientemente para que resulte insesgado, y en donde w es la suma de los w_i y $\overline{\beta}_i$ es la media ponderada de los $\widehat{\beta}_i$, y que μ, el parámetro correspondiente al colectivo, es la media ponderada de los correspondientes parámetros de cada subgrupo tomando como ponderaciones los factores de credibilidad.

Figura 2. Estimando independientemente la pendiente y el intercepto



Procediendo de esta manera se obtienen unos resultados razonables e intuitivos para las primas de credibilidad, como se puede observar por ejemplo, en la Figura 3, en donde se representa la prima de credibilidad obtenida para el estado 4 de Hachemeister. Los factores de credibilidad obtenidos en este caso son de 0.8000048 para el intercepto y de 0.8373196 para la pendiente. Con esto se consigue que tanto el intercepto como la pendiente de la recta que representa la evolución de la prima de credibilidad tengan unos valores intermedios entre los correspondientes a las primas individual y colectiva, y se consigue así evitar comportamientos anómalos para la prima de credibilidad.

Figura 3. Prima de credibilidad para el estado 4 de Hachemeister



AGRADECIMIENTOS

Este trabajo ha sido parcialmente financiado a través de los proyectos de investigación SANTANDER-UCM de referencias PR26/16-20305 y PR26/16-4B-3.

BIBLIOGRAFÍA

Bühlmann, H. and Gisler, A. (1997). "Credibility in the Regression Case Revisited". ASTIN Bulletin, 27(1), 83-98.

Bühlmann, H. and Gisler, A. (2005). "A Course in Credibility Theory". Springer.

Dutang, C., Goulet, V. and Pigeon, M. (2008) "actuar: An r package for actuarial science". Journal of Statistical Software, 25 (7), 1-37.

Hachemeister, C. A. (1975). Credibility for Regression Models with Application to Trend". En P. M. Kahn (ed.), "Credibility: Theory and Practice", 129-163. Academic Press.

ENVIRONMENTAL RISK: ENERGY EFFICIENCY IN SPAIN

Claudia García García
University of Granada
PhD Student at Quantitative Methods in Economics and Business
garciaclaudia@ugr.es

Catalina B. García García
University of Granada
Department of Quantitative Methods in Economics and Business
cbgarcia@ugr.es

Ainara Rodríguez Sánchez
University of Granada
PhD Student at Quantitative Methods in Economics and Business
arsanchez@ugr.es

Román Salmerón Gómez
University of Granada
Department of Quantitative Methods in Economics and Business
romansg@ugr.es

ABSTRACT

The Europe 2020 strategy aims to achieve rapid, sustainable and inclusive growth in European Union (EU) countries. In terms of climate change and energy, one of the objectives for 2020 is 20% increase in energy efficiency. The policies adopted in Spain in this area should be framed within the Europe 2020 strategy, so one of the goals in our country concerning environmental issues should be to increase energy efficiency. With this work, the methodology known as "Data Envelopment Analysis" (DEA) is applied to observe energy efficiency in Spain since the implementation of the Europe 2020 strategy in the EU.

Once the relevant values have been obtained, it is concluded that that the expected results are not really being achieved, so our policies should be aimed at improving this aspect.

1. INTRODUCTION

Tal y como dice el Ministerio de Agricultura y Pesca, Alimentación y Medio Ambiente de España[1], el estudio del clima es un campo de investigación complejo y en rápida evolución, debido a la gran cantidad de factores que intervienen. Según esta misma fuente, en la actualidad existe un consenso científico, casi generalizado, en torno a la idea de que nuestro modo de producción y consumo energético está generando una alteración climática global, que provocará, a su vez, serios impactos tanto sobre la tierra como sobre los sistemas socioeconómicos: predicciones de falta de agua potable, grandes cambios en las condiciones para la producción de alimentos y un aumento en los índices de mortalidad debido a inundaciones, tormentas, sequías y olas de calor, entre otras cosas.

En este marco, surge el concepto de riesgo medioambiental (Delgado, 2008; Batista y Bustos, 2009). Este tipo de riesgo se puede definir como la probabilidad de daños a una comunidad o grupo humano en un lugar dado, debido a las amenazas propias del ambiente y a la vulnerabilidad de los elementos expuestos. La vulnerabilidad, a su vez, se define como la propensión al cambio que tiene un sistema por no ser suficientemente capaz de ajustarse a los cambios producidos por una emergencia ambiental.

Con este contexto, es necesario que las economías del mundo hagan cambios en su estructura económica y política para afrontar los nuevos retos que se plantean en nuestro planeta. Para Greenpeace[2] o la NASA[3], la solución está en el sector energético con el uso de energías renovables, y dejando de lado el uso de energías sucias tales como el petróleo o el carbón.

---

1 http://www.mapama.gob.es/es/cambio-climatico/temas/que-es-el-cambio-climatico-y-como-nos-afecta/

2 https://es.greenpeace.org/es/trabajamos-en/cambio-climatico/

3 https://climate.nasa.gov

La Unión Europea (UE) está trabajando en la estrategia Europa 2020 para la década 2010-2020, consistente en un crecimiento rápido, sostenible e inclusivo. Esta estrategia se utiliza como marco de referencia para las actividades en la UE y en los niveles nacional y regional. En materia de cambio climático y energía, los objetivos para 2020 son:

- Reducir en un 20% las emisiones de efecto invernadero con respecto a los niveles de 1990.

- Incrementar en un 20% el uso de energías renovables.

- Aumentar un 20% la eficiencia energética.

En nuestro país, cabe destacar el Sistema Español de Inventario y Proyecciones de Emisiones a la Atmósfera (SEI)[4], que se trata de un mecanismo esencial para España en la lucha contra los Gases de Efecto Invernadero (GEI). Lo que básicamente hace el SEI es elaborar un inventario sobre las emisiones y evoluciones futuras de los GEI de manera que permite evaluar el cumplimiento de los compromisos adquiridos por nuestro país en materia medioambiental, elaborar políticas y medidas de mitigación de la contaminación atmosférica y valorar la efectividad de los resultados y objetivos logrados.

Según el Artículo 4 del Tratado de Funcionamiento de la Unión Europea, el medio ambiente se trata de una competencia compartida entre la Unión y los Estados miembros. Como consecuencia, podríamos decir que las políticas adoptadas en España en esta materia deberían enmarcarse dentro de la estrategia Europa 2020.

En este trabajo, nos centraremos en el tercero de los objetivos de la estrategia Europa 2020 en materia de cambio climático y energía: aumentar la eficiencia energética. Con ello, pretendemos utilizar la metodología conocida como “Data Envelopment Analysis” (DEA), la cual se explica brevemente en el siguiente apartado, para aplicarla a este contexto y así saber si somos o no somos medioambientalmente eficientes.

---

4 http://www.mapama.gob.es/es/calidad-y-evaluacion-ambiental/temas/sistema-espanol-de-inventario-sei-/

2. METODOLOGÍA

Tal y como se ha dicho en la Introducción, lo que usaremos en este trabajo es la metodología conocida como DEA o análisis envolvente de datos. Este método fue inicialmente desarrollado por Charnes et al. (1978) y se trata de un enfoque no paramétrico basado en la Frontera de Posibilidades de Producción (FPP) con el cual se calcula la eficiencia (productiva) relativa de un conjunto de entidades comparables llamadas "Decision Making Units" o DMUs a través de datos divididos en dos tipos de factores: inputs y outputs.

La metodología clásica sugiere que los inputs deben ser minimizados mientras que los outputs deben ser maximizados, sin embargo, en la práctica aparecen algunos factores "especiales" de manera que puede haber inputs que se quieran maximizar, como por ejemplo productos reciclados, y también puede haber outputs que no se quieran maximizar, como por ejemplo la contaminación (outputs no deseables). Con esta nueva visión, aparecen versiones de los modelos DEA clásicos.

A continuación se va a explicar la metodología que se va a usar para obtener los resultados del siguiente apartado.

Supongamos que tenemos $k = 1, \ldots, K$ DMUs, que tenemos $n = 1, \ldots, N$ inputs no energéticos (x), $l = 1, \ldots, L$ inputs energéticos (e), $m = 1, \ldots, M$ outputs deseables (y) y $j = 1, \ldots, J$ outputs no deseables. Supongamos también que cada una de las DMUs posee la siguiente función "environmental DEA technology" (función de tecnología que posee cada DMU teniendo en cuenta tanto outputs deseables como no deseables):

$$
\begin{aligned}
& T = \Big\{ \big(x, e, y, u\big) : \\
& \textstyle\sum_{k=1}^{K} z_k x_{nk} \leq x_n, n = 1, \cdots, N \\
& \textstyle\sum_{k=1}^{K} z_k e_{lk} \leq e_l, l = 1, \cdots, L \\
& \textstyle\sum_{k=1}^{K} z_k y_{mk} \geq y_m, m = 1, \cdots, L \\
& \textstyle\sum_{k=1}^{K} z_k u_{jk} = u_j, j = 1, \cdots, J \\
& z_k \geq 0, k = 1, \cdots, K \Big\}
\end{aligned}
$$

Esta función sigue rendimientos constantes a escala y medidas de eficiencia radiales[5].

En esta situación, si queremos medir la eficiencia energética de la DMUo basándonos en la metodología DEA, entonces tendremos que resolver el siguiente problema lineal:

$$
\begin{aligned}
&\min \theta_o \\
&\sum_{k=1}^{K} z_k x_{nk} \leq x_{no}, n = 1, \cdots, N \\
&\sum_{k=1}^{K} z_k e_{lk} \leq \theta_o e_{lo}, l = 1, \cdots, L \\
&\sum_{k=1}^{K} z_k y_{mk} \geq y_m, m = 1, \cdots, L \\
&\sum_{k=1}^{K} z_k u_{jk} = u_j, j = 1, \cdots, J \\
&z_k \geq 0, k = 1, \cdots, K
\end{aligned}
$$

Los distintos valores de θ_k obtenidos para cada DMU se pueden considerar un índice agregado y estandarizado de eficiencia energética, cuyos valores se situarán entre los valores 0 y 1, donde 0 representa la máxima ineficiencia y 1 la máxima eficiencia.

En este trabajo, nos vamos a centrar en la versión que proponen Zhou y Ang (2008) para calcular la eficiencia energética de 21 países de la OCDE cubriendo datos desde 1997 a 2001. Ramanathan (2006) aplica la metodología DEA para obtener un índice de eficiencia que combina el crecimiento, las emisiones de CO2 y el consumo de energía para el periodo 1980–2001.

Zhou y Ang (2008) utilizan como output deseable el PIB de cada país, como output no deseable las emisiones de dióxido de carbono (CO2), como inputs no energéticos el capital social de los países y la población activa y, finalmente, como inputs energéticos utilizan el consumo energético dividido en cuatro grupos (carbón, petróleo, gas y otros tipos de energía).

3. DATOS Y RESULTADOS

Para este trabajo, los datos que se van a utilizar son datos anuales de España para los años 2000 a 2014. La razón de utilizar datos anuales es que se quiere ver la evolución de nuestro país en cuanto a la eficiencia energética.

5 Las medidas de eficiencia radiales suponen que existe un cambio proporcional en el ratio input/output, es decir, no existen ni déficits ni excesos de inputs o outputs, respectivamente.

Respecto a los factores que se van a utilizar:

- Output deseable: PIB a precios de mercado. Fuente: *Instituto Nacional de Estadística* (INE).

- Output no deseable: Emisiones de CO_2 (kt). *Fuente: Banco Mundial*.

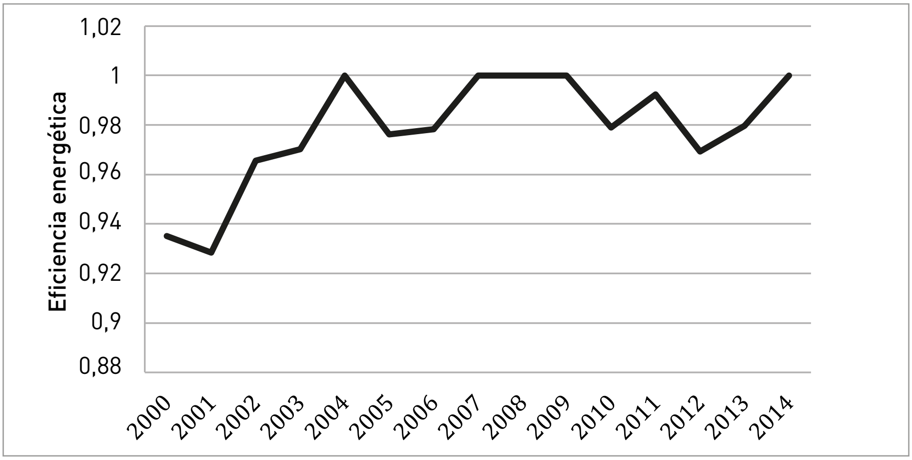
- Inputs no energéticos: valor agregado de la industria (valores constantes) y empleo en la industria (% del empleo total). *Fuente: Banco Mundial*.

- Input energético: Uso de energía (kg de equivalente de petróleo per cápita). *Fuente: Eurostat*.

En este caso, lo que hacemos es tratar a los diferentes años como DMUs. Además, el índice obtenido para cada año ( ) no tiene en cuenta efectos de combinaciones energéticas, por lo que podría tratarse como un índice de eficiencia puramente técnico en el consumo de energía (Zhou y Ang, 2008).

En la figura 1 se puede observar la evolución de los resultados obtenidos con la metodología brevemente explicada en el apartado anterior.

Figura 1 . Evolución de la eficiencia energética relativa para España entre los años 2000 y 2014



Con estos resultados, se evalúa la eficiencia energética relativa de España durante el periodo 2000-2014. Con esto, se puede observar que parece que hay una cierta tendencia creciente, es decir, parece que desde el año 2000 existe una evolución positiva respecto a la eficiencia energética de nuestro país. Sin embargo, si observamos los valores obtenidos, vemos que son bastante similares (y altos – valores por encima del 0,9). Lo que quiere decir esto es que realmente nos estamos manteniendo en un ritmo bastante contante y realmente no parece que haya ninguna mejora. Es importante remarcar aquí que una eficiencia relativa alta o incluso igual a 1 no significa que sean eficientes sino que son los que resultan más eficientes con respecto al resto de unidades analizadas, en este caso con respecto al resto de años analizados, Guccio et al. (2012) y Cavalieri et al. (2017).

4. CONCLUSIONES

Con todo lo anterior, podemos decir entonces que desde la implantación de la Estrategia Europa 2020, España no ha experimentado grandes cambios en la eficiencia energética. La eficiencia relativa se ha mantenido estable desde años anteriores de su implantación hasta la actualidad, llegando incluso a empeorar en algunos de los años analizados (2012, por ejemplo).

Por tanto, podemos decir que el tercero de los objetivos de la Estrategia en materia de cambio climático y energía (aumentar la eficiencia energética en un 20%), realmente no se está consiguiendo, al menos en España. Si fuese así, los resultados obtenidos mostrarían valores más bajos al principio del periodo estudiado y aumentos de la eficiencia energética año a año. En conclusión, se debe trabajar en este sentido para conseguir valores de eficiencia energética mayores.

Como futuras líneas de investigación sería muy interesante analizar la eficiencia energética del resto de países de la UE, y comparar tanto entre años en cada uno de los países (como se hace aquí para España), como entre países. Con esto, conseguiríamos tener una visión mucho más amplia y completa de los resultados que se están consiguiendo realmente en materia de eficiencia energética en Europa con la Estrategia Europa 2020.

REFERENCES

Batista, R., and Bustos, X. (2009). "Sistema de información geográfica y teledetección. Determinación de vulnerabilidad urbana. Caso estado Vargas-Venezuela", Terra Nueva Etapa, 25 (38), 167-190.

Cavalieri, M., Guccio, C., and Rizzo, I. (2017). "On the role of environmental corruption in healthcare infrastructures: An empirical assessment for Italy using DEA with truncated regression approach", Health Policy, 121 (5), 515–524.

Charnes, A., Cooper, W., and Rhodes, E. (1978). "Measuring the efficiency of decision making units". European Journal of Operational Research, 2 (6), 429-444.

Delgado, J. (2008). "La vulnerabilidad humana: del paradigma de la resistencia al paradigma de la resilencia", Tesis Doctoral en Arquitectura, Facultad de Arquitectura y Urbanismo (FAU), Universidad Central de Venezuela (UCV).

European Commission (2010). "Communication from the Commission - EUROPE 2020. A strategy for smart, sustainable and inclusive growth", COM(2010) 2020 final, 3/3/2010.

European Parliament (2012). "Consolidated version of the Treaty on the Functioning of the European Union", Official Journal of the European Union, C 326/47, 26/10/2012.

Guccio, C., Pignataro G., and Rizzo, I. (2012). "Measuring the efficient management of public works contracts: a non-parametric approach", Journal of Public Procurement, 12 (4), 528-546.

Ramanathan, T. (2006). "A multi-factor efficiency perspective to the relationships among world GDP, energy consumption and carbon dioxide emissions", Technological Forecasting & Social Change, 73 (5), 483-494.

Zhou, P., and Ang, B. (2008). "Linear programming models for measuring eonomy-wide energy efficiency performance". Energy Policy, 36 (8), 2911-2916.

ON THE USE OF THE GENERALIZED POISSON DISTRIBUTION IN ACTUARIAL STATISTICS

Emilio Gómez-Déniz
Department of Quantitative Methods and T i DES Institute
University of Las Palmas de Gran Canaria, Spain

Francisco José Vázquez-Polo
Department of Applied Economics Analysis
University of Las Palmas de Gran Canaria, Spain

José María Pérez-Sánchez
Department of Applied Economics Analysis
University of Las Palmas de Gran Canaria, Spain

Enrique Calderín-Ojeda
Centre for Actuarial Studies, Department of Economics
The University of Melbourne, Australia

ABSTRACT

In this work a particular case of the generalized Poisson distribution is considered. This particular case derives in a one parameter distribution, more specifically the Borel-Tanner distribution, which result simple and appropriate in insurance setting. The distribution is unimodal with a zero vertex and overdispersed (mean larger than the variance). We revise some of its properties which have not been showed in the statistical literature and the estimation of the parameter, including the case when regression is considered. Expected frequencies were calculated for some examples, providing a very satisfactory fit. Finally, a conjugate distribution is considered and the resulting mixture distribution is studied.

Keywords: Conjugate, Generalized Poisson Distribution, Premium, Prior Distribution.

Mathematics Subject Classification (2000): 60E05, 62E99, 60J80

1. INTRODUCCIÓN

Many of these phenomena, such as individual automobile insurance claims, are characterized by two features: (i) Overdispersion, i.e., the variance is greater than the mean; (ii) Zero–inflated, i.e. the presence of a high percentage of zero values in the empirical distribution. In view of this, many attempts have been made in statistical literature, and particularly in the actuarial field, to find a probabilistic model for the distribution of the number of counts.

The generalized Poisson distribution proposed by Consul and Jain (1973) has received a lot of attention in the past, particularly in the field of regression analysis when we want to find an alternative to the Poisson, negative binomial and Poisson--inverse Gaussian distributions. See Consul and Famoye (1992), Wang and Famoye (1997), Famoye and Singh (2006) and Gupta et al. (1996); among others. Some applications in the analysis of actuarial data have been mentioned by Ambagaspitiya and Balakrishnan (1994) and Scollnik (1998), among others. On the other hand, the bivariate version of the generalized Poisson distribution has been also used in actuarial setting by Vernic (1997). One of the advantages of the new distribution is its simplicity (its pmf contains no special function) and flexibility.

To the best of our knowledge, most of the properties which we present in this work concerning to the discrete distribution presented here has not been previously addressed in statistical and actuarial literature.

The structure of the paper is as follows. In Section 2 we present the proposed distribution and study some of its more important properties, such as moments, skewness, kurtosis, mode, estimation, etc. Stochastic interpretation of the parameter is showed in Section 3. In Section 4 we give a lower bound for the probability density function of the total claim amount which is used into the collective risk model. A mixture model is showed in Section 5. Applications are given in Section 6 and discussion and conclusions in the last Section.

2. THE MAIN MODEL

The probability function of the generalized Poisson distribution (see Consul, 1989; Consul and Jain, 1973) is given by

$$p_x = \Pr(X = x) = \theta_1(\theta_1 + x\theta_2)^{x-1}\frac{\exp[-(\theta_1 + x\theta_2)]}{x!}, \quad x = 0,1,\ldots, \tag{1}$$

for $\theta_1 > 0$ and $|\theta_2| < 1$.

In this work we propose to take in (1) $\theta_1 = \theta_2 = \theta$, being $0 < \theta < 1$, from which the probability function (1) reduces to

$$p_x = \Pr(X = x) = \frac{(1+x)^{x-1}}{x!}\theta^x \exp[-(1+x)\theta], \quad x = 0,1,\ldots, \tag{2}$$

Henceforward, we will write $X \sim GPo(\theta)$ when a discrete random variable has the probability function (2).

Consider the probability function corresponding to the Borel-Tanner distribution introduced by Haight and Breuer (1960)

$$p(y;r,\theta) = A(y,r)\theta^{y-r}\exp(-\theta y), \quad y = r, r+1,\ldots, \tag{3}$$

where

$$A(y,r) = \frac{r}{(y-r)!}y^{y-r-1},$$

then, it is straightforward to see that the probability function (2) can be obtained from (3) by taking $r = 1$ and $x = y - 1$.

Although the probabilities are easy to calculate we can compute them by the recursive relation given by

$$p_x = \left(\frac{1+x}{x}\right)^{x-1}\theta\exp(-\theta)p_{x-1}, \quad x = 1,2,\ldots,$$

where $p_0 = \exp(-\theta)$. Observe that p_0 is equal to the value of the Poisson distribution in $x = 0$. The probability generating function and the moment generating function can be written in terms of the Lambert's Ω function. See Ambagaspitiya and Balakrishnan (1994) for details. They offers also an integral equation for getting the probability density function of the compound generalized Poisson distribution when the claim sizes are absolutely continuous and a recurrence equation when are discrete.

The mean and variance of the probability function in (2) are given by

$$E(X) = \mu = \frac{\theta}{1-\theta},$$
$$var(X) = \mu_2 = \frac{\theta}{(1-\theta)^3},$$

from which we can ge the index of dispersion given by $ID = var(X)/E(X) = (1-\theta)^{-2} > 1$. Then the parameterization of the generalized Poisson distribution in the form of (2) shows overdispersion (variance greater than the mean). Additionally, since $d\mu/d\theta > 0$ and $d\mu_2/d\theta > 0$ we have that the mean and variance increase with the value of the parameter θ.

Since probability function (2) can be written as

$$p_x = h(x)\exp[\eta(\theta)T(x) - \vartheta(\theta)],$$

where $h(x) = (1+x)^{x-1}/x!$, $\eta(\theta) = \log\theta - \theta$, $\vartheta(\theta) = \theta$ and $T(x) = x$, it can be seen that the probability function (2) is a member of the natural exponential family of distributions (nor in its canonical form), where $0 < \theta < 1$ is the parameter of the family. Furthermore, as was pointed out by Consul and Jain (1973), it is a member of the discrete Lagrangian family of distributions.

Furthermore, probability function (2) can also be rewritten as

$$p_x = \frac{a_x \lambda^x}{g(\lambda)},$$

where $a_x = 1/x!(1+x)^{x-1} = 1/x!\sum_{j=0}^{x-1}\binom{x-1}{j}x^j$, $\lambda = \theta\exp(-\theta)$ and $g(\lambda) = \exp(\theta)$.

Thus, it is also a power series distribution (see Johnson et al. (2005), p. 75). Thus, we have a new distribution, together with the Bernoulli, binomial, geometric, negative binomial, Poisson and logarithmic series, within this interesting class of distributions.

After simple computations it is easy to obtain that

$$\frac{dp_x}{d\mu} = \frac{dp_x}{d\theta}\frac{d\theta}{d\mu} = p_x(x-\mu)\frac{1}{\mu_2}, \qquad (4)$$

therefore, by using a result provided in Bardwell (1960) (see also Amidi, 1976) we get

$$\mu_i = \mu_2\left[\frac{d\mu_{i-1}}{d\mu} + (i-1)\mu_{i-2}\right], \quad i = 3,4,\ldots, \qquad (5)$$

where μ_i is the ith moment about the mean.

Since probability function (2) is a member of the family discussed by Bardwell (1960) we have the following result.

Proposition 1 If a discrete random variable X *follows the probability function (2) then it is verified that:*

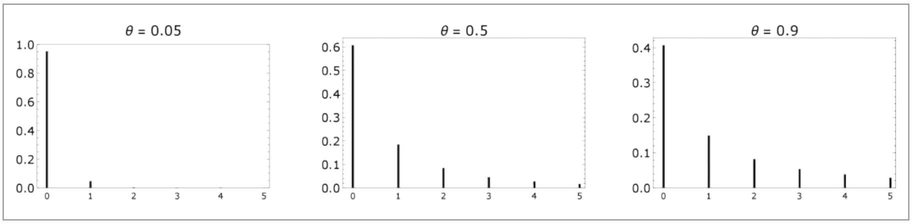
1. $Mean\ deviation = E|X-\mu| = -2\theta(1-\theta^2)\frac{\partial}{\partial\theta}F([\mu],\theta).$
2. $\Pr(X=x;\theta) = \eta\exp\left\{\int \frac{x-\mu}{\mu_2}\frac{d\mu}{d\theta}d\theta\right\}.$

Here, $F([\mu],\theta) = \sum_{x=0}^{[\mu]}\Pr(X=x;\theta)$, where $[\cdot]$ represents the integer part and η is a constant of normalization.

Proof: The results follow directly using Theorems 2 and 3 in Bardwell (1960).

In addition, the probabilities for different values of x were calculated and graphs were plotted for various values of the parameter. These are shown in Figure 1. This figure shows that the proposed distribution is unimodal with a zero vertex.

Figura 1. Some examples of the probability function (2) for different values of parameter θ



According to Bardwell (1960), a characteristic of the family of functions obeying (4) is that each has a single maximum which occurs when $x = \mu$. In practice, the modal value is achieved at $x_{mode} = [\mu] - 1$. We can see that, as the parameter θ increases, the mass of the probabilities tends to be less concentrated in zero.

The following result shows that the distribution with probability function given in (2) is infinitely divisible.

Proposition 2 *The distribution with probability function given in (2) is infinitely divisible.*

Proof: Firstly, we have that $p_0 \neq 0,\ p_1 \neq 0$. Then, we must prove that $\{p_j/p_{j-1}\},\ j = 1,2,\ldots$ forms a monotone increasing sequence with j. If we define p_x also for non-integer values of x, we have that for $x \geq 1$

$$\frac{d}{dx}\frac{p_x}{p_{x-1}} = \log\left(1 + \frac{1}{x}\right) + \frac{1-x}{x+1}\frac{1}{x}.$$

Now, using the inequality $\log(1+z) > z/(z+1),\ z > -1$, it is easy to get that

$$\frac{d}{dx}\frac{p_x}{p_{x-1}} > \frac{1}{x(x+1)} > 0,$$

for all $x \geq 1$. Now, the result follows by applying Theorem 2.1 in Warde and Katti (1971).

The fact that $\{p_j/p_{j-1}\},\ j = 1,2,\ldots$, forms a monotone increasing sequence requires that be a decreasing sequence (see Johnson and Kotz, 1982, p. 75), which is congruent with the zero vertex of the new distribution. Moreover, as any infinitely divisible distribution defined on non-negative integers is a compound Poisson distribution (see Proposition 9 in Karlis and Xekalaki, 2005), we conclude that the probability function given in (2) is a compound Poisson distribution.

Furthermore, the infinitely divisible distribution plays an important role in many areas of statistics, for example, in stochastic processes and in actuarial statistics. When a distribution G is infinitely divisible then for any integer $x \geq 2$, there exists a distribution G_x such that G is the x-fold convolution of G_x, namely, $G = G_x^{*x}$.

Since the new distribution is infinitely divisible, an lower bound for the variance can be obtained (see Johnson and Kotz, 1982, p. 75), which is given by

$$var(X) \geq \frac{p_1}{p_0} = \theta \exp(-\theta).$$

2.1 Skewness and kurtosis

Some important indices of the shape of the distribution, apart from the mean and the variance, are the skewness ($\beta_1 = \mu_3/(\mu_2)^{3/2}$) and the kurtosis ($\beta_2 = \mu_4/(\mu_2)^2$).

Using (5) we get that

$$\mu_3 = \frac{\theta(-2\theta^2 + 4\theta + 1)}{(1-\theta)^5},$$

$$\mu_4 = \frac{\theta(-4\theta^3 + 3\theta^2 + 15\theta + 1)}{(1-\theta)^7}.$$

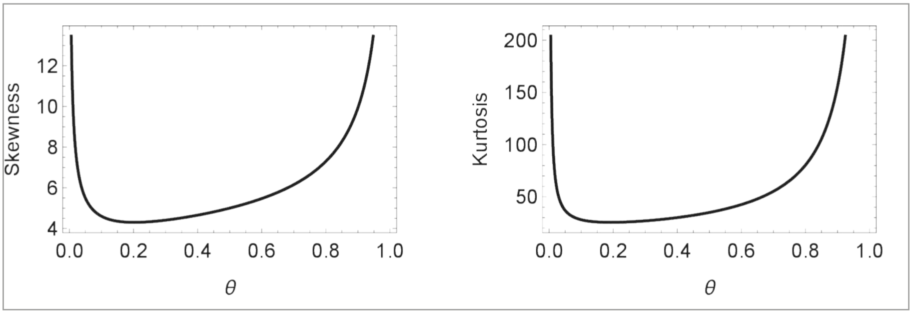
From these expressions we get

$$\beta_1 = \frac{-2\theta^2 + 4\theta + 1}{1-\theta}\sqrt{\frac{1-\theta}{\theta}},$$

$$\beta_2 = 1 + 4\theta + \frac{1}{\theta} + \frac{15}{1-\theta}.$$

In Figure 2, these two indices are used to show the skewness and kurtosis of the proposed distribution. In contrast with Poisson distribution which has non-increasing skewness and kurtosis functions the GP$o(\theta)$ distribution presents convex skewness and kurtosis functions showing intervals with decreasing behavior and increasing otherwise.

Figura 2. Skewness (left panel) and kurtosis (right panel) of the distribution



2.2 Estimation

Let now $\tilde{x} = (x_1, \ldots, x_n)$ a sample taken from the distribution (2) it is easy to see that the moment estimate of the parameter θ coincides with the maximum likelihood estimate and it is given by $\hat{\theta} = \bar{x}/(1+\bar{x})$, where $\bar{x} = (1/n)\sum_{i=1}^{n} x_i$ is the sample mean. The second partial derivative of the log-likelihood function evaluated in the maximum likelihood estimate is $d\ell(\theta;\tilde{x})/d\theta\big|_{\theta=\hat{\theta}} = -n\bar{x}/\hat{\theta} < 0$, then the maximum likelihood estimated is guaranteed. A little algebra provides the element of the Fisher's information matrix, given by $E\left[-d^2\ell(\theta;\tilde{x})/d\theta^2\big|_{\theta=\hat{\theta}}\right] = n/(\hat{\theta}(1-\hat{\theta}))$.

Therefore $var(\hat{\theta}) = \hat{\theta}(1-\hat{\theta})/n$.

However, instead of moment estimation we can also use the fact that

$$E\left[\frac{a_{X-1}}{a_X}\right] = \theta \exp(-\theta),$$

which is obtained by using a result appearing in Papathanasiou (1993). Then, if we can use the sample value $\tilde{p} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{1+x_i}\right)^{x_i-1}$ the estimator of the parameter θ is just the solution of the equation $\tilde{p} - \theta\exp(-\theta) = 0$, which can be solved numerically in an easy way.

The rest of the paper will contain the stochastic interpretation of the parameter, a lower bound for the aggregate distribution, a mixture and conjugate study where an EM algorithm is implemented to estimate the parameters in the mixture with the truncated gamma distribution; finally some applications and the main conclusions are given.

ACKNOWLEDGMENTS

This work was partially funded by grants ECO2013-47092 (Ministerio de Economía y Competitividad, Spain) and ECO2017-85577-P (Ministerio de Economía, Industria y Competitividad, Agencia Estatal de Investigación, Spain).

REFERENCES

Ambagaspitiya, R. and Balakrishnan, N. (1994). On the compound generalized Poisson distributions. ASTIN Bulletin, 24:255–263.

Bardwell, E. (1960). On certain characteristics of some discrete distributions. Biometrika, 47(3/4):473–475.

Consul, P. (1989). Generalized Poisson Distributions. Properties and Applications. Marcel Dekker, Inc., New York.

Consul, P. and Famoye, F. (1992). Generalized Poisson regression model. Communications in Statistics-Theory and Methods, 21:89–109.

Consul, P. and Jain, G. (1973). A generalization of the Poisson distribution. Technometrics, 15(4):791–799.

Denuit, M., Goovaerts, M., and Kaas, R. (2005). Actuarial Theory for Dependent Risks. John Wiley & Sons.

Famoye, F. and Singh, K. (2006). Zero-inflated generalized Poisson regression model with an application to domestic violence data. Journal of Data Science, 4:117–130.

Gupta, P., Gupta, R., and Tripathi, R. (1996). Analysis of zero-adjusted count data. Computational Statistical and Data Analysis, 23.

Haight, F. and Breuer, M. (1960). The Borel-Tanner distribution. Biometrika, 47(1,2):143–150.

Johnson, N. and Kotz, S. (1982). Developments in discrete distribution, 1969-1980. International Statistical Review, 50:71–101.

Karlis, D. and Xekalaki, E. (2005). Mixed Poisson distributions. International Statistical Review 73:35–58.

Papathanasiou, V. (1993). Characterizations of power series and factorial series distributions. Sankhyā, 55(1):164–168.

Ross, S. (1996). Stochastic Processes. Second Edition. John Wiley & Sons, Inc.

Scollnik, D. (1998). On the analysis of the truncated generalized Poisson distribution using a Bayesian method. ASTIN Bulletin, 28:135–152.

Shaked, M. and Shanthikumar, J. (2007). Stochastic orders. Series: Springer Series in Statistics. Springer, New York.

Vernic, R. (1997). On the bivariate generalized Poisson distribution. ASTIN Bulletin, 27(1):23–31.

Wang, W. and Famoye, F. (1997). Modeling household fertility decisions with generalized Poisson regression. Journal of Population Economics, 10:273–283.

I'M JUST AN INDUSTRY WHOSE INTENTIONS ARE GOOD

O Lord, please don't let me be misunderstood!

James Jones
Illinois State University, Katie School, Normal
IL 61790-5490, USA
jrjone2@ilstu.edu

Krzysztof Ostaszewski
Illinois State University, Actuarial Program, Normal
IL 61790-4520, USA
krzysio@ilstu.edu

ABSTRACT

In this work, we note a significant reputational challenge faced by the insurance industry, which is viewed mostly negatively by general public, its customers and its potential employees. We note that this negative perception originates partly in the complex structure of insurance products, and partly in a misperception of the role insurance plays in the society, a misperception that the industry itself perpetuates.

We propose that the insurance industry needs to emphasize the true nature of its social function, which is to encourage rational risk taking, and to encourage proper pricing of risk, bringing about better and more sustainable risk management practices.

1. INTRODUCTION

The insurance industry's negative image also extends to prospective employees. In a 2014 report about insurance employment the consulting firm Deloitte noted "insurance remains last in popularity rankings across all industries surveyed", (in the period surveyed being from 2010 till 2014), with insurance being the least popular industry to work for during the entire period. Deloitte also found that, consistently, no insurance company placed among the most popular employers among young people. In a survey published by Indeed.com (2016), for employers in the United States, not a single insurance company was listed among the ten more attractive employers in the U.S.

An industry's reputation is not just a "nice to have' but is in fact a "must have" for its long term sustainability. We believe that is not an overstatement, when one considers that insurance is an intangible product, based on a promise to make a payment in exchange for a premium paid. The success of the industry offering such a promise is based in no small part on its reputation. There is a strong correlation between insurers that have high customer satisfaction and their profitability. Additionally, the value of an insurance firm is based on the technical expertise of its workers to properly price, sell and then deliver on this promise to pay. The reputation of the industry affects whether it can attract and retain talented workers who can provide the skills and knowledge to serve its core functions.

Without addressing the reputational issue the industry will face sustainability challenges ranging from a hampered ability to attract talent, to threats from alternative capital (like catastrophe bonds, sidecars, and other capital market solutions), new shared-economy solutions (like Lemonade Insurance), increasing disintermediation, and a loss of investor confidence. To help address this it is important to understand the very real reasons why insurance is misunderstood and the significant reasons why it benefits society.

2. DIAGNOSING THE REPUTATIONAL CHALLENGES

So what accounts for the insurance industry's reputation, and what can be done about it? First, we must recognize customer expectations are not just set by other insurers, but by experiences that customers have in other domains as well. For decades, conventional wisdom held that those customers who experienced claims, and were satisfied with their claim service, would be more loyal than those who had never had a claim. However, in today's modern world of purchases made instantaneously on Amazon via mobile apps, the mere process of filing a claim and talking with a claims professional is enough to drive customers to want to switch insurers. The changing perception of what is an “onerous process”, despite the actual outcome, is certainly a challenge for insurers who must by law and practice document and investigate claims, and verify payouts. The industry can benefit by addressing some of the underlying causes of this reputation and by better promoting the benefits that insurance provides society.

The key characteristics that affect the negative reputation of the industry that will be considered in this paper are: the Aleatory Nature of Insurance, the Contract of Adhesion, and the Principle of Indemnity.

Aleatory (depending on chance, random) means the payout for insurance is based on a triggering of a random event such as a tornado, or car accident, an illness, or loss of life. Unlike the payout for an annuity based on amounts contributed over time, or an investment which is accumulated over time with a general idea of the payout at the end, typical insurance payouts are based on a contingent event set forth in an insurance contract (policy), which may or may not be understood by the policy holder. And unlike gambling, or investments in the stock market, insurance only pays out when there is a loss. So there is no “fun” or psychic reward.

Consider how this plays out for homeowners insurance. In an average year, only about 5 percent of homeowners will have an insurance claim. The amounts paid out to these homeowners are based on the contractual wording of their policies and NOT based on how much insureds have paid in insurance over time. It is easy to understand then why those 95 percent who pay premiums and collect nothing in exchange except for the “peace of mind” that they could have been paid if they had suffered a covered loss, may find insurance to have been a waste. This aleatory nature of insurance creates a somewhat intractable reputation problem, because the very nature of insurance provides that those who don’t have claims, in effect subsidize those who do have claims. And in order for the insurance mechanism to work affordably, only a small percentage of policyholders should be paid each period.

One way that insurers can address this is by figuring out ways to add value to policyholders beyond just payments of covered losses. Insurers may create mobile apps that can provide policyholder with risk mitigation information. In addition insurers are beginning to create “rewards” to policyholders who have contributed premiums over time without a loss. One common example is “the disappearing” deductible for policyholders who have a claim after years of payments without claims.

Another characteristic of insurance that can sometimes frustrate insurance consumers, is that most insurance policies are a contract of adhesion for individual insureds. In other words, a prospective insurance client is presented with an insurance policy on a “take it or leave it” choice. As an individual insured, you don’t get to write your own policy. This type of choice is known as a “Hobson’s choice” and the feeling of the purchaser may be similar to those of old Mr. Hobson, the curmudgeonly Cambridge livery stable owner who presented his clients with only one option for the choice of a horse.

The unfortunate effect of this characteristic is that the purchase may not be understood, or particularly desired, but it is reluctantly accepted as better than nothing. Products and services purchased as “better than nothing” (whether it is insurance, cable television service, or a utility company) are not likely to engender customer satisfaction.

One way to address this, is through enhancing the choices and customization of insurance products where customers feel engaged in creating the insurance product purchased. Recent research shows that co-production (where the customer helps to create the final product) is what drives customer satisfaction and loyalty with modern consumers of financial services. So this contract of adhesion characteristic can be changed, and is being changed. Many insurers are now providing greater flexibility to customers to create the policy that best fits their needs, and can be customized based on their own risk.

Finally, historically insurance was closely tied to the indemnity principle, a legal doctrine stating that insureds should be paid (indemnified) for actual losses. This principle hold that a person’s recovery should be limited to an amount sufficient to restore him or her to the (financial) status prior to the loss. The application of the principle is to put the insured back to financial position where he or she was prior to the loss. The intent is to avoid giving an incentive to insureds to cause a loss or take measures to prevent one. Today, the measure of what it means to make a person financially whole is undergoing transition. It’s helpful to look at what was originally considered indemnity by insurers and compare that to what modern day consumers may now expect.

Although this principle of indemnity applies to automobile losses by paying an insured’s totally demolished 3 year old car, with the market value of that age and model car. Historically property claims have based on the “actual cash value” of the loss, determined by the formula of replacement cost of item lost, minus depreciation for the use of the item over time. So if your home burned down you would be paid for a home based on its age, not its current replacement cost. The original intent was to reduce moral hazard or paying new for old.

Over time the principle of indemnity has been eroded by changing policy terms and conditions over time. During the 1970's insurance policies began to introduce provisions for replacement costs, that provided more than an indemnity payment, but instead a full replacement cost, for example the full replacement cost for a new roof *without* depreciation if the insured spent the money to replace the roof. More recently insurers began to introduce replacement cost provisions to automobile insurance for newer vehicles that are totally destroyed in an accident. As these replacement cost insurance provisions began to become more prevalent, customer expectations for payments began to run counter to the traditional principle of indemnity.

As the principle of indemnity has changed over time, it has vexed insurers, customers, and insurance regulators. For example, insurers who are familiar with the traditional view of indemnity, might see a way to compromise by avoiding taking depreciation in automobile losses by making repairs with used (also known as "like kind and quality" parts). So a part from a vehicle would be replaced without depreciation by a part from a salvaged vehicle of the same year. Although insurers might view this as quite reasonable given the original understanding of indemnity, consumers who are used to replacement of new for old, often find this practice controversial.

Insurers may address this risk to reputation by level setting customer expectations with their own expectations. Using the concept of co-producing mentioned in the previous section, a policyholder and an insurer could agree to a method of claim settlement at the time they purchase a policy. This would enable customers to decide to choose a specific claim settlement methods in exchange for appropriate premium discount (or surcharge).

3. THE MOST IMPRACTICAL THING IS A WRONG THEORY

The traditional argument for insurance is that customers are better off with insurance than without it. Given customers personal preferences, expressed in a utility function, customer may find it preferable to pay a premium with certainty and avoid or limit a random loss. This happens because customer's utility function while increasing with wealth, is increasing at a decreasing rate (diminished marginal utility of wealth). On the other hand, the insurance firm is assumed to have a greater risk appetite than the customer, be willing to assume more risk, and additionally, by combining many policies from independent risks, diversify the pool of risks, thus decreasing the amount of risk per unit insured. In fact, a sufficiently large and well-capitalized insurance company can even be assumed to be risk-neutral, thus being indifferent between a fixed endowment of wealth and receiving the same expected value of random wealth. Thus the insurance firm is better equipped to assume the risk than a customer, and a mutually beneficial trade can be made.

But let us note that the argument for insurance provided above is a static argument. It assumes that the transaction of purchase of insurance does not change the behavior of customer and the insurance firm. On the other hand, the core argument for existence of financial instruments from theoretical finance is the one in the Modigliani-Miller Theorem, which in its most abstract form says that, in the absence of taxes, bankruptcy costs and agency costs, a method of financing of a firm is irrelevant for the value of the firm. Insurance is typically presented as a product. It is, of course, a financial product. And that simply means that for the individuals and firms insured it is a method of financing, i.e., a rearrangement of cash flows, both deterministic and contingent. How does insurance create value then? According to Modigliani-Miller Theorem, if value is created for the customer, it must be through tax savings, reduction in the cost of bankruptcy, and through lowering of agency costs. Indeed, insurance in practice is often a tax-reducing contract, directly through provisions of tax law, or indirectly, in the presence of progressive income tax, as under a convex tax functon, the Jensen's Inequality tells us that the tax due in expected income is lower than the expected value of the tax on random income. Furthermore, the very nature of insurance brings about lowering of bankruptcy cost for the insured, at the very least through lowering of probability of bankruptcy. But the affect of insurance on human behavior is, in our view, the most important aspect.

What is the social purpose of insurance? On the level of an individual contract, insurance protects the insured against a random loss. But we would like to ask that question at the social level. What does the insurance industry do for the society as a whole? After all, the negative image the industry has resulted from its purported damage to the society as a whole. In order to answer this question, let us first ask what insurance does for the whole society? What is the effect on the entire society of having an insurance industry, when compared with the same society without an insurance industry? Baranoff (2011) discusses something of an unintended social experiment that actually happened in Australia: the failure of the HIH insurance company in March 2001. HIH was a major insurer of construction companies, in a country with high level of home ownership (close to 70%). Following the HIH demise, home construction in Australia ground to a halt, although only temporarily. Faced with a situation where they could not obtain insurance for construction, or where they could not afford such insurance due to higher premiums asked by the remaining solvent insurance companies, Australia's construction firms did not buy insurance and stopped their construction business for a while. The situation was eventually resolved, but it illustrates well the significance of the insurance industry for the real economy.

A similar story can be told about the American real economy. A hundred years ago, Americans used a joking statement, something like "I will sell you some land in Florida", to indicate how undesirable buying land in Florida was in the early 1900s (and even more so in the 1800s). Now Florida is the state that has more people moving in than any other state in the United States. What caused such change? The reason why Florida was an undesirable location hundred years ago was rather plain to see: It is mostly a swamp, with frequent hurricanes that can destroy real estate completely and pose a deadly threat, especially in the case of homes built without a basement – which, unfortunately, is a necessity in a swamp. But now Americans built homes in Florida, in large numbers. What happened? The development and wide availability of homeowners' insurance happened. Americans now can have a home on a beach, or near a beach and Florida and have it insured. And while there is an intense debate about the price of homeowners' insurance, with those supposedly greedy insurance companies wanting a lot of money in premium payments, insurance is available widely.

And let us also illustrate this phenomenon with the simplest possible example. Imagine a world without automobile insurance. In such an alternative world, would people drive more or less? Obviously, they would drive less. This means that the social purpose of car insurance is to get people to drive more. And, similarly, the social purpose of business insurance is for businesses, such as construction business in Australia, to undertake more activities. The purpose of homeowners insurance is for people to own more homes.

This is the social purpose, the actual net social result of having an insurance industry, as opposed to not having one. In radically simpler words: *The mission of the insurance industry is to get people to do more crazy stuff*! And yes, this is a noble mission. Without risk taking, no innovation would ever take place, and most likely, no industry of any kind would ever take place.

4. CONCLUSIONS

The insurance industry represents itself to the public as a protector. And in pure financial terms that may be true. The insurance industry pays out billions of dollars to restore lives, communities and the health and wealth of individuals and businesses. But the insurance industry does not protect in the sense of preventing random losses. Insurance firms cannot protect their customers from hurricanes, they have no power to stop hurricanes or reduce their ferocity. The "protection" theory of the benefit of insurance is incomplete.

In reality, insurance industry enables rational and expanding risk taking in a society. This allows consumers and economic decision makers to undertake a greater array of economic projects, and expand opportunities available to them. While this does encourage risk-taking, the pricing and risk-management expertise of insurance enterprises directs that risk-taking towards activities with social value. That is the most important social function of the insurance industry, largely ignored in its current image presented to the public. Without that perspective, the reputational challenge the industry faces will not be properly addressed and resolved.

REFERENCES

Etti Baranoff, "Systemic Risk vs. the 10-Year Old Insurance Failure in Australia" Geneva Association' Insurance and Finance newsletter, No. 8, August 2011.

Deloitte, The Insurance Industry -- The Next Generation: The Deloitte Talent in Insurance Survey 2014: Canada in Focus, 2014. Accessed on January 28, 2017 at: https://www2.deloitte.com/content/dam/Deloitte/ca/Documents/financial-services/ca-en-15-2834T-talent-studies-insurance-aoda.PDF

Aaron Doyle and Richard Ericson, “Five Ironies of Insurance.” The Appeal of Insurance, Edited by Geoffrey Clark et al., University of Toronto Press, 2010, pp. 226–247, www.jstor.org/stable/10.3138/9781442685888.15.

Richard Ericson, Aaron Doyle, and Dean Barry, Insurance as Governance, Toronto: University of Toronto Press, 2003.

Aditi Gowri, The Irony of Insurance: Community and Commodity, PhD dissertation, University of Southern California, 1997.

Indeed.com:http://blog.indeed.com/2016/05/05/fortune-500-top-companies-to-work-for/, accessed January 28, 2017.

FINANCIAL AND NON-FINANCIAL RISK: FAMILY FIRMS AND NON-FAMILY FIRMS

Mara Madaleno
University of Aveiro, GOVCOPP – Research Unit in Governance, Competitiviness and Public Policies, DEGEIT – Department of Economics, Management, Industrial Engeneering and Tourism, Campus Universitário de Santiago 3810-193, Aveiro, Portugal
maramadaleno@ua.pt

ABSTRACT

Family firms (FF) tend to be classified as less risky and volatile than non-family firms (NFF). This article aims to examine whether there are differences in risk and volatility between FF and NFF, using a sample of 33 Portuguese listed firms during 2008 and 2016. The fundamental goal of this work is to determine the effect of family control over volatility and risk attending to the effects of investment, of indebtedness and of the firm and financial market characteristics of FF as compared to NFF. For that we have formulated hypothesis which have been tested using panel data estimation and it was possible to take from the analysis important results. Estimations have been made for the total number of companies in the sample, considering different model and variables specifications, measures of financial and accounting performance, different measures of risk and for the two subsamples, considering 22 FF and the 11 NFF for which it was possible to divide the sample. Results seem to indicate that investors consider in their decisions the non-financial risk associated to FF, but that in the case of NFF the most relevant kind of risk is the financial risk, a result that remained consistent even under different estimation specifications. As such, considering the risk into the analysis is very important when we try to understand the relationship between risk and return among FF and NFF.

1 INTRODUCTIÓN

The Portuguese firm structure is composed in 75% by family firms (FF), which are still a very infant research area in terms of risk and not explored in Portugal. The literature states that FF are more long run oriented, more conservative in fiscal terms and more risk averse in their decision making with respect to business investment and adopted strategies (Cassia et al., 2012). It also seems to indicate that FF are more focused in reducing risk than non-family firms (NFF), once that the FF sees the organization as a wealth extension and intends to pass the firm to the next generations. As such, it should keep the firm good functioning to ensure the incoming generations patrimony. This could be one of the reasons for why the literature classifies these companies are more prudent and risk averse with respect to new investments (Erbetta et al., 2013). To FF are associated more conservative strategies as a way to limit risk as compared to NFF where property is more disperse, once that uncertainty may impact wealth, heritage, reputation and visibility (Patel and Chrisman, 2013).

Previous studies about FF focus their attention: in the relationship between family control and company performance (Miralles-Marcello et al., 2014); (b) in the financial market performance (Miralles-Marcello et al., 2013); (c) in performance with respect to innovation (Meroño-Cérdan et al., 2017); (d) in the release of information about social corporative responsibility (Nekhili et al., 2017); (e) in family heterogeneity, control and risk (Lisboa and Miralles-Quirós, 2015); (f) in entrepreneurial decisions (Lins et al., 2013); and, (g) in the relationship between debt policy and company performance (Vieira, 2017; Favara et al., 2017). The goal of this work in face with what already exists is in determining the family control effect over volatility and risk considering the investment and debt effects, firm characteristics and financial market characteristics of FF versus NFF. Portugal is a very interesting case study because FF represent 75% of Portuguese firms (European Family Businesses, 2016) and 50% of the market stock exchange index in Portugal (Miralles-Marcello et al., 2013). Moreover, it has a very small market and therefore more exposed to external risks and to market movements (Lisboa and Miralles-Quirós, 2015).

Understanding volatility dynamics is very important for decision making, derivatives, risk hedging and investments in financial markets. Notwithstanding the conclusions undertaken here may be easily extended to other countries. Also uncertainty with respect to volatility and risk may have important effects in terms of prices and of decisions able to affect portfolio composition (Agarwal et al., 2017). However, it is not fully understood in the literature if FF assume more or less risks (Meroño-Cérdan et al., 2017). What we may deepened from the literature is that to understand company performance, being FF, NFF, SME or big companies it is necessary to dominate the risk variable (Meroño-Cérdan et al., 2017), or at least understand how it behaves. A balanced panel methodology was adopted using specification tests and robustness checks.

The rest of the work develops as follows. Section 2 presents the hypothesis analyzed and the data. Section 3 presents the methodology and results attained while section 4 concludes this work.

2. HYPOTHESIS AND DATA

All the hypothesis raised in this section where based in a deep literature revision, which will be made available upon request. Since there are different visions and conclusions with respect to the relationship between investment and risk in FF, we formulate H1 and H2.

H1: Both FF and NFF are negatively related with risk acceptance.

H2: Risk is different with respect to investment and indebtedness in FF and NFF.

Schäfer et al. (2017) try to see if FF have more financing needs and how this affects process innovation in these firms. Being FF implies a positive relationship between long term debt and negative with short term debt because this is more risky (Díaz-Díaz et al., 2016).

H3: Being FF makes a difference between the relationship control among risk, performance and financial debt.

H4: FF present different growth, debt, risk and performance from that of NFF.

Vieira (2017) finds evidence that for Portuguese listed firms' debt contributes negatively for the company performance and that the relationship between debt and performance does not differs significantly between FF and NFF.

H5: In FF the demanded return by investors is higher, exhibiting a direct relationship between family control and return.

H6: FF have higher risk and return, depending on the kind of risk and its measurement.

Some of the ambiguous results found in the literature may be due to the way this risk is measured. So we follow Gómez-Mejía et al. (2011) and Poletti-Hughes and Williams (2017) to measure firm risk attending to three kinds of risk: performance hazard risk (PHR), to capture non-financial firm goals, venturing risk (VR), for financial goals, and total risk (RT), indicator of risk of asset returns. The rest of the variables used are described at the appendix in table A1. The sample has also been divided into two samples: one of FF and the other of NFF.

3. METHODOLOGY AND RESULTS

To test our hypothesis and determine if there are differences in risk between FF and NFF we have applied the panel regression model defined by:

$$Y_{it} = \alpha_{it} + \beta_1 DEF_i + \beta_k \sum VI_{it} + \beta_j \sum VC_{it} + \varepsilon_{it} \qquad (1)$$

Where Y_{it} represents the set of dependent variables for firm i in year t (PHR, VR, RT), DEF equals 1 for FF and 0 otherwise, VI_{it} represents the set of independent variables (ROA, ROE, VCA, TQ, MB, RET, CM, INV, DCP, DLP, END, CD, CRV, CRRO) and VC_{it} represents a set of control variables (DIM, ID, ID2, IND, SEC, TER, YEARS, CRISIS). For robustness check some variables were excluded and interaction terms used into the several estimations. Additionally, we performed distinct estimations for the set of FF and of NFF based over:

$$\text{FF:} \quad Y_{it} = \alpha_{it} + \beta_k \sum VI_{it} + \beta_j \sum VC_{it} + \varepsilon_{it} \qquad (2)$$

$$\text{NFF:} \quad Y_{it} = \alpha_{it} + \beta_k \sum VI_{it} + \beta_j \sum VC_{it} + \varepsilon_{it} \qquad (3)$$

But now the estimations do not consider the variable DEF.

Table 1. Panel estimation for the total sample; for FF and NFFs

	Dependent								
Indep.	PHR	VR	RT	PHR	VR	RT	PHR	VR	RT
INV	0.3818*	-0.5591	-0.611	0.3818*	-0.5591	-0.611	0.3124	-0.2754	-0.1635
DCP	-0.031	-0.2433**	-0.5043***						
DLP				0.031	0.2433**	0.5043***			
END	0.0972	1.1382	-2.9335*	0.0972	1.1382	-2.9335*	-0.1475	0.4438	-3.2357**
CD							0	-0.0001	-0.0028
RET	-0.3049***	-1.0508***	-1.4364***	-0.3049***	-1.0508***	-1.4364***	-0.3155**	-1.0978***	-1.4403***
TQ	0.0243	0.5393**	2.5434***	0.0243	0.5393**	2.5434***	0.0377	0.3639*	2.5859***
CRV	-0.0031***	0.0001	-0.0005	-0.0031***	0.0001	-0.0005	-0.0036***	0.0001	-0.0006
DIM	0.0264	-1.0770*	-1.9955***	0.0264	-1.0770*	-1.9955***	0.1287	-0.6714***	-2.2091***
ID	0.2929	-1.8201	-0.7491	0.2929	-1.8201	-0.7491	0.7591	2.6576	-0.5458
ID2	-0.0487	1.3669	1.3227	-0.0487	1.3669	1.3227	-0.0212	-0.3315	1.1039
IND	0.1604	0.8035	-0.0364	0.1604	0.8035	-0.0364	0.166	0.7744	-0.1192
R2	0.2872	0.2116	0.3063	0.2872	0.2116	0.3063	0.2906	0.1893	0.2802
Model	RE	FE	FE	RE	FE	FE	FE	RE	FE
pHaus.	0.1137	0.0888	0	0.1137	0.0888	0	0.0674	0.7726	0.0139
	Dependent - FF								
Indep.	PHR	VR	RT	PHR	VR	RT	PHR	VR	RT
INV	0.2616	-0.5266	-0.1958	0.2616	-0.5266	-0.1958	0.2423	-0.5352	-0.2002
DCP1	-0.4985	-0.2574	-0.0769						
DLP1				0.4985	0.2574	0.0769			
END	-1.2011	1.5712	0.015	-1.2011	1.5712	0.015	-1.0598	1.6837	0.0331
CD							0.0001	0.0003	0.0001
RET	-0.4506**	-0.5233	-0.2602	-0.4506**	-0.5233	-0.2602	-0.4549**	-0.5357	-0.2629
TQ	0.17	-0.0496	0.3399***	0.17	-0.0496	0.3399***	0.13	-0.0567	0.3411***
CRV	-0.0035***	0.0003	0.0002	-0.0035***	0.0003	0.0002	-0.0036***	0.0002	0.0002
DIM	0.44	-0.3311	-0.0515	0.44	-0.3311	-0.0515	0.4052	-0.2991	-0.0437
ID	-1.0761	0.3585	0.1303	-1.0761	0.3585	0.1303	-0.8718	0.1249	0.0876
ID2	0.9029	0.0731	0.0144	0.9029	0.0731	0.0144	0.7432	0.1128	0.0218
IND	0.1502	0.7519	0.339	0.1502	0.7519	0.339	0.229	0.8267	0.3603
R2	0.3786	0.0991	0.1218	0.3786	0.0991	0.1218	0.3682	0.0951	0.1198
Model	FE	RE	RE	FE	RE	RE	FE	RE	RE
pHaus.	0.0606	0.4859	0.7787	0.0606	0.4859	0.7787	0.0441	0.5374	0.8564
	Dependent - NFF								
Indep.	PHR	VR	RT	PHR	VR	RT	PHR	VR	RT
INV	-0.3302	0.4193	0.9934	-0.3302	0.4193	0.9934	-0.1931	0.8186	1.1033
DCP	-0.0585*	-0.1675	-0.0754						
DLP				0.0585*	0.1675	0.0754			
END	0.2721	0.9418	-14.1128***	0.2721	0.9418	-14.1128***	0.1061	0.6328	-13.7480***
CD							2.128	1.8927	-24.097
RET	-0.1619	-1.3230**	-0.8146*	-0.1619	-1.3230**	-0.8146*	-0.1521	-1.2853**	-0.8745*
TQ	-0.1514	0.7532	11.7117***	-0.1514	0.7532	11.7117***	-0.0729	0.8769	11.6244***
CRV	-0.7677***	0.3489	0.0192	-0.7677***	0.3489	0.0192	-0.7649***	0.3591	0.0102
DIM	-0.0574	-4.4240*	-6.1919***	-0.0574	-4.4240*	-6.1919***	-0.0826	-4.5923*	-5.9314***
ID	1.9298	-7.0354	92.2508	1.9298	-7.0354	92.2508	2.5567	0.2839	100.2699*
ID2	-0.4001	3.296	-19.309	-0.4001	3.296	-19.309	-0.4974	1.5996	-20.9583
IND	-0.0921	5.0519	-0.2836	-0.0921	5.0519	-0.2836	-0.1417	4.9902***	-0.5446
R2	0.5176	0.3879	0.8677	0.5176	0.3879	0.8677	0.5005	0.3776	0.871
Model	RE	FE	FE	RE	FE	FE	RE	FE	FE
pHaus.	0.4642	0.059	0	0.4642	0.059	0	0.1492	0.0272	0

Note: See variables description in table A1. *,**,*** represents statistically significant at 10%, 5% and 1%, respectively. RE – Random effects; FE – Fixed effects. Dsectors and Dyears variables included. Total number of observations for each estimation: 297 for total sample; 198 for FF; 99 for NFF. CRISIS include but only statistically significant for FF.

The relationship between indebtedness, performance and risk differs among FF and NFF. Indebtedness increases total risk of FF and NFF but this debt level has a differentiated impact depending if we attend to non-financial risk (PHR) or financial risk (VR). Financial performance has a positive and significant impact over total risk in FF and NFF. Results are sensitive to different variables specifications, but there are situations where we can validate general results. FF present growth, debt, risk and performance which are different from that of NFF and seem to impact more financial risk of NFF. Investment does not seems to impact significantly the firm risk independently of the kind of risk considered, such that we may not confirm that risk increases more with investment in FF.

We conclude that is the firm is more risky that takes to the increase in return and value of herself and by the results here obtained TQ coefficients over total risk are higher in NFF, validating the hypothesis that there is a higher positive relationship between risk and performance in NFF. Being a FF increases the control on the relationship between risk, performance and financial debt, at least in what respects accounting performance measure. In FF there is a positive relationship with long term debt and negative with short term debt which favors the literature which indicates that in FF there is a higher debt control and these are less prone to loose "family value". Also FF have the duty to preserve business to future generations which makes that FF use more long term debt since it implies lower risk. Indebtedness impact is not so relevant and significant over performance in FF when compared to NFF. Results also seem to indicate that in FF the return demanded by investors is lower, where there is evidence of a direct relationship between family control and return.

FF assume as important variable for financial return (RET) the non-financial risk (PHR), which does not happens in NFF whose risk which really matters is only financial risk, although in both total risk (RT) is relevant to explain performance. As such, value in FF should be higher than in NFF since the formers assume higher risk, meaning they consider the additional risk source which is the non-financial risk. Being FF implies that performance depends over the risk of family control. In FF there seems to exist a superior relationship between risk and performance but it depends over the type of risk we are analyzing. Investors seem to demand lower market returns to FF which may be explained by the fact that these control more in terms of family management, transmitting higher insurance to investors, which may explain why investors demand lower returns to FF as compared to those demanded to NFF. Results reveal the importance to include both market and accounting measures since results are sensitive to different measurement ways. Risk is an important variable to be included in the distinct behavior analysis of FF and NFF both in terms of investment, indebtedness, total risk and performance.

4. CONCLUSIONS

This paper has presented results able to determine the family control effect over volatility and risk attending to investment effects, indebtedness and company characteristics, and of the financial market, over financial performance of FF as compared to NFF. Results seem to indicate that there are differences in the behavior among FF and NFF, which results are in fact sensible to different variables specifications and that investor's demand different return to FF and NFF, being that the level of risk investors attribute to them differs among the two types of firms studied. It was also possible to verify that we need to distinguish the kind of risk to analyze because results seem to indicate that investors consider in their decisions the non-financial risk associated to FF, but that in the case of NFF the most relevant kind of risk is the financial risk, a result that remained consistent even under different estimation specifications. In summary, results allowed us to conclude that differences between FF and NFF are distinct and may not be generalized. There are clear differences among FF and NFF with respect to risk and return for which variables like debt, management independence, size, age and debt terms contribute. It was also possible to infer that return demanded by investors to FF and NFF is different when we use stock exchange data into the analysis.

ACKNOWLEDGMENTS

This work has been in part supported by the Research Unit GOVCOPP (project POCI-01-0145-FEDER-008540), financed through FEDER funds through COMPETE2020 - Programa Operacional Competitividade e Internacionalização (POCI) – and by national funds through FCT - Fundação para a Ciência e a Tecnologia. Any persistent error or missing are from the authors' entire responsibility.

REFERENCES

Agarwal, V., Arisoy, Y.E. and Naik, N.Y. (2017). "Volatility of aggregate volatility and hedge fund returns". Journal of Financial Economics, 1-20.

Cassia, L., De Massis, A. and Pizzurno, E. (2012). "Strategic innovation and new product development in family firms". International Journal of Entrepreneurial Behavior Research, 18, 2, 198-232.

Díaz-Díaz, N.L., García-Teruel, P.J. and Martínez-Solano, P. (2016). "Debt maturity structure in private firms: Does the family control matter?" Journal of Corporate Finance, 37, 393-411.

Erbetta, F., Menozzi, A., Corbetta, G. and Fraquelli, G. (2013). "Assessing family firm performance using frontier analysis techniques: evidence from Italian manufacturing industries". Journal of Family Business Strategy, 4, 2, 106-117.

European Family Businesses (2016). "Definition of a family business". www.europeanfamilybusinesses.eu/family-businesses/definition; http://www.europeanfamilybusinesses.eu/family-businesses/facts-figures

Favara, G., Morellec, E., Schroth, E. and Valta, P. (2017). "Debt enforcement, investment, and risk taking across countries". Journal of Financial Economics, 123, 1, 22-41.

Gómez-Mejía, L.R., Cruz, C., Berrone, P. and De Castro, J. (2011). "The bind that ties: socioemotional wealth preservation in family firms". The Academy of Management Annals, 5, 1, 653-707.

Lins, K.V., Volpin, P. and Wagner, H.F. (2013). "Does family control matter? International evidence from the 2008–2009 financial crisis". The Review of Financial Studies, 26, 10, 2583–2619.

Lisboa, I. and Miralles-Quirós, M. del M. (2015). "Family firms' heterogeneity and firm risk". Boletín de Estudios Económicos, 70, 214, 139-157.

Meroño-Cerdán, A.L., López-Nicolás, C. and Molina-Castillo, F.J. (2017). "Risk aversion, innovation and performance in family firms". Economics of Innovation and New Technology, 1-15.

Miralles-Marcelo, J. L., Miralles-Quirós, M. del M. and Lisboa, I. (2013). "The stock performance of family firms in the Portuguese market". Applied Financial Economics, 23, 22, 1721-1732.

Miralles-Marcelo, J.L., Miralles-Quirós, M. del M. and Lisboa, I. (2014). "The impact of family control on firm performance: Evidence from Portugal and Spain". Journal of Family Business Strategy, 5, 2, 156-168.

Nekhili, M., Nagati, H., Chtioui, T. and Rebolledo, C. (2017). "Corporate social responsibility disclosure and market value: Family versus nonfamily firms". Journal of Business Research, 77, 41-52.

Patel, R. and Chrisman, J. (2013). "Risk abatement as a strategy for R&D investments in family firms". Strategic Management Journal, doi:10.1002/smj.2119.

Poletti-Hughes, J. and Williams, J. (2017). "The effect of family control on value and risk-taking in Mexico: A socioemotional wealth approach". International Review of Financial Analysis, 1-13.

Schäfer, D., Stephan, A. and Mosquera, J. S. (2017). "Family ownership: does it matter for funding and success of corporate innovations?" Small Business Economics, 48, 4, 931–951.

Vieira, E.S. (2017). "Debt policy and firm performance of family firms: the impact of economic adversity". International Journal of Managerial Finance, 13, 3, 267-286.

APPENDIX

Table A1. Variables description, formulas and references

Variable	Description	Formula	Base authors
PHR	Performance Hazard Risk – Non financial risk	$\ln\left(\frac{VN_{t-1}}{VN_t}\right)$	Gómez-Mejía et al. (2007); Poletti-Hughes and Williams (2017)
VR	Venturing Risk – Financial risk	Annual standard deviation of market model residuals	Gómez-Mejía et al. (2007); Poletti-Hughes and Williams (2017)
RT	Total risk	Return standard deviation x $\times \sqrt{245} \times \frac{VMCP}{A}$	Pathan (2009); Poletti-Hughes and Williams (2017)
RET	Annual asset return	Average $\left[\ln\left(\frac{P_t}{P_{t-\$}}\right)\right] \times 245$	Romero Mª and Ramirez (2017)
TQ	Tobin's Q	$\frac{A - CP + VMCP}{A}$	Poletti-Hughes and Williams (2017); Su et al. (2017)
MB	Market-to-Book	$\frac{VMCP}{CP}$	O'Brien (2003); Poletti-Hughes and Williams (2017); Vieira (2017)
ROA	Return-on-Assets	$\frac{RAJI}{A}$	Li et al. (2015); Vieira (2017)
ROE	Return-on-Equity	$\frac{RL}{CP}$	Umar et al. (2012); Vieira (2017)
VCA	Stocks accounting value	CP/nº stocks	Vieira (2017)
CM	Market capitalization	$\ln(VMCP)_t$	Vieira (2017)
INV	Investment	$\frac{DC_t}{A_{t-1}}$	Favara et al. (2017)
DCP	Short term debt	$\frac{P_{cp}}{P}$	Ramadan (2013); Díaz-Díaz et al. (2016)
DLP	Long term debt	$\frac{P_{lp}}{P}$	Ramadan (2013); Díaz-Díaz et al. (2016)
END	Indebtedness	$\frac{P}{A}$	Salim and Yadav (2012); Díaz-Díaz et al. (2016); Vieira (2017)
CD	Debt cost	$\frac{JD}{P}$	Andres (2008); Caprio et al. (2011)
DEF	Family Firm (FF)	Dummy = 1 if FF	Vieira (2017); Poletti-Hughes and Williams (2017)
CRV	Sales growth	$\frac{VN_t - VN_{t-1}}{VN_{t-1}}$	Díaz-Díaz et al. (2016)

CRRO	Operational results growth	$\frac{RO_t - RO_{t-1}}{RO_{t-1}}$	Su et al. (2017)
DIM	Dimension	$\ln(A)$	Zhou et al. (2017); Sue t al. (2017); Vieira (2017)
ID	Age	Ln (years + 1)	Díaz-Díaz et al. (2016); Su et al. (2017)
ID2	Age squared	(ln(years +1))^2	Poletti-Hughes and Williams (2017)
IND	Independence	Dummy = 1 if IND	Li et al. (2015); Dang (2016); Poletti-Hughes and Williams (2017)
SEC	Secondary sector	Dummy = 1 if SEC	Su et al. (2017); Vieira (2017)
TER	Terciary sector	Dummy = 1 if TER	Su et al. (2017); Vieira (2017)
Years	Year Dummies	Dummy = 1 if 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015	Su et al. (2017); Vieira (2017)
CRISIS	Dummy Crisis	Dummy = 1 if from 2008 to 2013	Vieira (2017); Poletti-Hughes and Williams (2017)
ENDEF	Cross Product	END x DEF	In the spirit of the cross products considered by: Díaz-Díaz et al. (2016); Su et al. (2017); Vieira (2017); Poletti-Hughes and Williams (2017)
TQEF	Cross Product	TQ x DEF	
PHREF	Cross Product	PHR x DEF	
VREF	Cross Product	VR x DEF	
RTEF	Cross Product	RT x DEF	
DCPEF	Cross Product	DCP x DEF	
DLPEF	Cross Product	DLP x DEF	
CRVEF	Cross Product	CRV x DEF	
INVEF	Cross Product	INV x DEF	
RETEF	Cross Product	RET x DEF	

Note: VN respects to business volume (sales of goods + services); VMCP represents the market value of equity or the quotation value in the last day of the year multiplied by the number of stocks available for trade; t represents the year t and t-1 in the last year; A refers to accounting assets; CP represents accounting equity; P respects to accounting liabilities; Pcp – Short term liabilities; Plp – Long run liabilities; RAJI – Result before interest and taxes; RL – Liquid Result; DC represents capital expenses; JD – debt interest; RO – Operational results.

JOINT MODELLING FOR CUSTOMER LAPSES IN THE INSURANCE SECTOR

Alemar E. Padilla-Barreto
Department of Econometrics, Riskcenter-IREA, University of Barcelona
Av. Diagonal 690, 08034 Barcelona, Spain
alemarpadillabarreto@ub.edu

ABSTRACT

In this article, insurance customer lapsing is analized from a joint modelling perspective. Based on the ROC curve performance, it compares the predictive capacity of three churn prediction models: two univariate models, one for each line of business -auto and home-, and the bivariate model for the two different lines of business jointly. The estimation is based on a sample of real customers of a major insurance company in Spain. The results show that, if the information from both business lines simultaneously is combined, the predictive power of the bivariate model is enhanced. On the contrary, if only it is considered univariate models or marginal effects, then the predictive power decreases. The main contribution of this study is bringing to light the importance of taking into account global policyholder information from several products in order to improve decision making processes involving customers in the insurance sector.

1. MOTIVATION

Insurance companies usually focus their economic activity through what are known as lines of business (LoB). Each LoB has specific characteristics, such as, a particular insured object, risk factors, coverages, pricing models, customer strategies, among others. In this sense, given the nature of the type of insurance, companies establish specialized resources at the individual and personalized level to analyze the circumstances and results obtained by each LoB.

Policyholders who hold multiple insurance policies with the same company belong to a specific group of risk, whose risk profiles have not been fully addressed. Moreover, the members of a family share common characteristics and probably have similar risky attitudes, preferences, make joint decisions, so it is not surprising that they behave similarly and the decisions about one of their products is related to some circumstances affecting any of its other products (see Brockett *et al.*, 2008; Guillen *et al.*, 2008 and Guillen *et al.*, 2012 for more details). For example, one policyholder may have contracted an auto insurance policy for his spouse and another for his son/daughter, and at the same time, he may have subscribed a home insurance policy. Therefore, the renewal of policies could be a consensual decision among one or more individuals belonging to the same family unit, which establishes an instantaneous relationship between business lines.

The existing published contributions about customer churn generally concentrate on one-line of business. This means that they analyze only car insurance or only homeowner insurance, but not both. So, they present separate statistical models for every sample (see Bolancé *et al.*, 2016 for a comparative analysis between performance measures to assess customer churn models in motor insurance). Only a small number of contributions address the issue of dependency between business lines (see for instance Brockett *et al.*, 2008; Bermúdez *et al.*, 2013 and Avanzi *et al.*, 2016). Similarly, in the insurance sector, there is no evidence to indicate that the analyzes performed at the pricing, reserving or customer level are being carried out considering the dependence between business lines at both aggregate and customer level.

This application contributes to the existing literature by exploring the influence of some factors on the probability of renewal of a specific type of policyholders, testing the existence of dependence between LoBs and comparing the performance of univariate and bivariate probit models, and their marginal effects by combining information from both lines of business. For this purpose, the area under the ROC curve (AUC) is proposed as the criterion for predictive performance evaluation of the different models.

2. PREDICTIVE MODELS

In this application, two different probit models are proposed for estimating the probabilities of a customer leaving the company. Probit models are generalized linear models, which have been widely recognized for their ability to relate the expected proportion p of an event to some explanatory variables (see McCullagh and Nelder, 1983, for a detailed explanation about generalized linear models).

The comparative analysis is carried out in two phases. First, the outcome of a probit regression model is analyzed individually for each LoB. Second, the information from both LoB is combined into a bivariate probit model and the conditional and marginal probabilities are deducted.

2.1. Bivariate probit model

Since the main interest lies in studying risk profiles of customers who have subscribed both types of policy – motor and home - and whose decisions on the renewal of their policies it is presumed are interrelated, it is proposed the bivariate Probit model which allows to model two response variables, which can be correlated.

2.1.1. Model specification

The bivariate Probit model was first introduced by Ashford and Sowden, 1970 and it is a particular case of a multivariate qualitative response models, defined from the joint probability distribution of two or more discrete dependent variables (see Amemiya, 1985 for more details).

Let $\{y_{im}, x_{im}\}$ and $\{y_{ih}, x_{ih}\}$ denote two different forms associated with the ith client, represented by two binary dependent variables of interest (y_m and y_h) corresponding to the occurrence of a lapse in motor (m) and home (h) LoB, and a collection of known risks factors vectors ($x_{m,}$ $m=1\ldots p$ and x_h, $h=1\ldots q$). Similar to subsection 1.1.1 each of the response variables takes values 1 when the client renews its policy or 0 when the policy is canceled. Here, their joint distribution can be represented by a four-cell probabilities table as show in Table 1, where

$$p_{jk} = Pr(y_{-im} = \mathrm{j}\,, y_{ih} = k) = \Phi\rho\,(x_{im}'\,\beta_m, x_{\mathrm{ih}}'\,\beta_h),$$

$$j, k = \{0,1\} \qquad (4)$$

represents the joint probability function, given by the bivariate normal distribution function with zero means and with covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_{mm} & \sigma_{mh} \\ \sigma_{hm} & \sigma_{hh} \end{pmatrix} = \begin{pmatrix} 1 & \boldsymbol{\rho} \\ \boldsymbol{\rho} & 1 \end{pmatrix}$$

It is to be noted that the dependent variables y_m and y_h should be correlated, through correlation $\boldsymbol{\rho}$, which represents the correlation between the errors in the two Probit models. Otherwise, these probit models could be estimated independently.

Table 1. Joint distribution of the binary random variables

		y_h	
		1	0
y_m	1	p_{11}	p_{10}
	0	p_{01}	p_{00}

3. RESULTS

At first glance intuitively, it is suppose that both LoBs are correlated. However, quantitative evidences are sought. A first approach, in order to find evidences about this relationship, was to identify the proportion of policies underwritten by the clients according to the two LoB. Table 1 shows the overall percentage of policies underwritten in one or both lines of business.

Table 2. Summary of clients according to the number of policies underwritten

		# of motor policies			
	%	0	1	2	3
# of home policies	0	-	62.47%	4.58%	0.21%
	1	19.52%	9.04%	1.27%	0.09%
	2	1.64%	1.03%	0.14%	0.01%

Grouped insurance is not a common selling practice in Spain, however for the same company, approximately 11,5% of the insured have underwritten motor and home policies simultaneously.

In order to ensure that there is a correlation between the lapsing in the two LoBs, first the bivariate probit model is estimated and its correlation coefficient ρ is examined. Table 3 presents model estimates. The resulting estimated ρ is equal to -0.252 that is significant at 99,9% confidence level, this result suggests that it does make sense to model both lines of business simultaneously. In this sense, if the existence of correlation is not taken into account, the parameters used to make inference will be poorly calculated.

Table 3. Probit model results

		LoB			
		Home		Motor	
	Variables	Coefficients	Significance	Coefficients	Significance
Common factors	Intercept	1.615	***	2.374	***
	Male	-0.052	-	0.027	-
	Age	0.002	-	-0.002	-
	Last premium paid	-0.002	-	-0.001	***
	Premium paid over exposure	-0.001	***	-0.015	***
	Other policies	-1.327	***	-1.526	***
	Policies in force by mediator	0.004	**	0.002	*
	Agent	0.056	-	0.217	***
	Supplements	-0.128	***	-0.062	***
	Change in premium	0.001	*	0.001	*
	Bonus	0	-	0.003	***
	Surcharge	0	-	-0.003	***
	Last bonus	0.003	-	-0.004	***
	Last surcharge	-0.001	-	0.002	***
	Way to pay - A	0.203	-	-0.242	*
	Way to pay - S	-0.026	-	-0.308	**
	Seniority of the policy	0.015	**	0.008	*
	Cancellation ratio by mediator	-0.004	-	-0.324	*
Home	Type of home A	-0.551	-		
	Type of home B	-0.34	-		
	Type of home C	0.036	-		
	Type of home D	-0.277	-		
	Continent	0.0000	-		
	Content	0.004	***		
Motor	Guarantees - 1			-0.457	***
	Guarantees - 2			-0.462	***
	Guarantees - 3			-0.135	-
	Seniority of the license			0.007	-
	Power			0.003	***
	Weight/power			-0.001	-
	Number of seats - 2			-0.195	-
	Number of seats - 4			-0.138	-
	Number of seats - 5			-0.033	-
	Type of fuel - D			-0.109	-
	Type of fuel - G			-0.172	-
	Second driver - No			-0.075	-

Source: Own calculations.

*** indicates significance at the 1% level, ** (5%) and * (10%), similarly.

Table 3 shows the corresponding parameter estimates for two LoBs. At first glance the highest concentration of significant variables is found in the variables related to the insured and the policy. Most effects are significant in the case of motor insurance. Premium paid over exposure, other policies, policies in force by mediator, supplements, change in premium and seniority of the policy, besides having a significant effect in both cases, they are characterized by having the same sign, which supposes that they contribute or not to the propensity to renew the policies of each LoB in the same direction.

It is concluded that Other policies has a significant effect in explaining the renewal decision of customers for both types of policies. Supplements could increase the value of the premium paid by customers, hence the negative sign of the estimations. In the motor insurance case, evidences of a generalized negative effect with respect to the type of guarantees contracted are observed, which suggests that independent of the contracted guarantees, there is a counterproductive effect on the chances of renewal.

4. CONCLUSIONS

Multivariate Probit models represent an alternative for insurance companies interested in modeling the lapse of clients who have contracted more than one type of risk insured in different LoB. The simplest way to analyze the clients' propensity for desertion is to assume independence between LoBs, however, this assumption would lead us to incorrect estimates of relativities and hence of the probabilities since the existing correlations would not be considered. In turn, Probit models are useful to examine a segment of customers that are analyzed so far by the portfolio to which they belongs instead of as the individuals that they are.

This work contributes to the understanding the importance of handling bundled risks in the context of insurance markets.

To identify and to analyze sources of dependencies between risks groups should become a common practice in insurance companies, due to the advantages it provides at the business level. First, the company would be able to analyze clients globally rather than as individuals who belong to a portfolio. Second they would have the possibility to analyze unknown interrelations.

Further research should be carried out, taking into account those customers with more than one insurance contract in each line of business. It is suggested to include more variables more variables related to the risk object and customer characteristics, which would help to explain the customer's renewal intentions.

ACKNOWLEDGMENTS

I thank the Spanish Ministry of Economy FEDER ECO2016-76203-C2-2-P and AGAUR.

REFERENCES

Brockett, P., Golden, L., Guillen, M., Nielsen, J., Parner, J. and Perez-Marin, A. (2008). "Survival Analysis of a Household Portfolio of Insurance Policies: How Much Time Do You Have to Stop Total Customer Defection?". The Journal of Risk and Insurance, 75(3), 713-737. Retrieved from http://www.jstor.org.sire.ub.edu/stable/25145302.

Guillen, M., Nielsen, J.P., Scheike, T.H. and Pérez-Marín, A. M. (2012). "Time-varying effects in the analysis of customer loyalty: A case study in insurance". Expert Systems with Applications, 39(3), 3551-3558.

Guillen, M., Nielsen, J. P. and Pérez-Marín, A. M. (2008). "The need to monitor customer loyalty and business risk in the European insurance industry". The Geneva Papers on Risk and Insurance Issues and Practice, 33(2), 207-218.

Bolancé, C., Guillen, M. and Padilla-Barreto, A. E. (2016). "Predicting probability of customer churn in insurance". In Modeling and Simulation in Engineering, Economics and Management, 82-91. Springer International Publishing.

Bermúdez, L., Ferri, A. and Guillen, M. (2013). "A correlation sensitivity analysis of non-life underwriting risk in solvency capital requirement estimation". ASTIN Bulletin, 43(1), 21–37. doi: 10.1017/asb.2012.1.

Avanzi, B., Taylor, G. and Wong, B. (2016). “Correlations between insurance lines of business: An illusion or a real phenomenon? some methodological considerations”. Astin Bulletin, 46(02), 225-263.

McCullagh, P. and Nelder, J. A. (1983). “Generalized linear models”. Monographs on statistics and applied probability. London: Chapman and Hall.

Ashford, J. R. and Sowden, R. R. (1970). “Multi-variate probit analysis”. Biometrics, 535-546.

Amemiya, T. (1985). “Advanced econometrics”. Harvard university press.

¿ES ADECUADO CONSTRUIR NUESTRA TABLA DE MORTALIDAD A PARTIR DE LA DEL CONJUNTO DEL SECTOR? RIESGOS Y SOLUCIONES

Jose M. Pavía
Universitat de Valencia, Economía Aplicada, España
pavia@uv.es

Josep Lledó
Universidad de Alcalá y Reale Seguros, España
Josep.lledo@uah.es

Francisco G. Morillas
Universitat de Valencia, Economía Aplicada, España
Francisco.Morillas @uv.es

ABSTRACT

Hasta donde conocen los autores, las empresas aseguradoras españolas (y una parte importante de europeas) utilizan unas tablas de mortalidad basadas en datos del conjunto del sector. Cada compañía construye su propia tabla aplicando un factor o porcentaje propio a la tabla general. Este procedimiento no está basado en un fundamento biométrico y es equivalente a asumir ciertas hipótesis restrictivas, como que la tabla de mortalidad libre de riesgo de la compañía tiene el mismo comportamiento 'edad a edad' que la tabla de mortalidad general. La nueva normativa contable internacional (IFRS 17), sin embargo, establecerá que deberá observarse la propia experiencia para el cálculo, por ejemplo, de las provisiones técnicas. El objetivo de este trabajo es doble. Por un lado, se explican los riesgos que asumen las compañías al utilizar una tabla de mortalidad no ajustada a su estructura de riesgo. Por otro lado, inspirados en Benjamin y Pollard (1986) y Lledó, Pavía y Morillas (2017) proponemos un nuevo estimador de cohorte (extendido) para construir tablas de mortalidad. Ejemplificamos el proceso con una base de datos real de una compañía que opera en el canal de banca-seguros.

1. INTRODUCCIÓN

La medición de la mortalidad y la longevidad es un tema siempre de actualidad, que ha recibido gran atención en la literatura (e.g., Lee y Carter, 1992; Lee y Miller, 2001; Cairns et al., 2008; Biffis y Blake, 2009; Cairns et al., 2011; Barrieu et al., 2012; Börger et al., 2014; Enchev et al., 2016; o, Lledó et al., 2017). Las estimaciones de las probabilidades de supervivencia y fallecimiento, recogidas en la tabla de mortalidad, son utilizadas por las compañías aseguradoras para los procesos de Reserving y Pricing, los cuales impactan en las rentabilidades a largo plazo de las aseguradoras.

En los últimos años el sector asegurador ha vivido importantes cambios normativos. Por un lado, Solvencia II impone la separación de dos componentes básicos en el cálculo de las provisiones técnicas: el best-estimate (BE) (CEIOPS, pp. 25-50) y el risk margin (RM) (CEIOPS, pp. 54-67). Por otro lado, la nueva normativa contable internacional (*International Financial Reporting Standard* 17, IFRS 17), vigente a partir de 2020, establecerá que las hipótesis operativas, como por ejemplo la mortalidad, utilizadas para el cálculo de las provisiones técnicas, deberán construirse utilizando la propia experiencia para reflejar los distintos riesgos vigentes en las carteras de seguros.

Sin embargo, en la actualidad, hasta donde los autores conocen, más del 50% del mercado asegurador español utiliza unas tablas de mortalidad que no necesariamente reflejan el comportamiento de sus propias estructuras de riesgo. Un gran número de compañías aseguradoras utilizan como tabla de mortalidad BE un porcentaje, o factor, de los valores de la tabla base del sector (PERMF 2000P), donde el factor suele calcularse considerando (E1) las defunciones y el número de expuestos al riesgo o (E2) las sumas aseguradas. Ello implica que los cálculos se realizan en conjunto, para toda la cartera, con independencia de la edad.

El objetivo de este trabajo es doble. Por un lado, se detallan los actuales estimadores y se describen las limitaciones y riesgos que se asumen al utilizar una tabla de mortalidad general al construir una cartera de seguros específica. Por otro lado, se desarrolla un estimador de cohorte aplicable a una población asegurada. Este nuevo estimador permita separar el componente el BE del RM e incorporar el momento exacto de entrada y salida de asegurados. Este estimador se compara con el método clásico en el cálculo de provisiones técnicas para una cartera real.

2. MORTALIDAD. SOLVENCIA II Y IFRS 17

La nueva normativa Solvencia II establece normas más precisas relacionadas con el cálculo de la solvencia de las compañías y con la evaluación de riesgos en el sector asegurador. Bajo Solvencia II, el cálculo de provisiones técnicas se fundamenta en separar el riesgo de mortalidad en dos partes: BE y RM. En esta misma línea, la nueva normativa contable internacional (International Financial Reporting Standard 17, en adelante, IFRS 17, vigente a partir de 2020) establecerá un nuevo enfoque en el estudio de las hipótesis operativas (mortalidad, longevidad, gastos…) utilizadas para el cálculo de las provisiones técnicas. IFRS 17 considera que las hipótesis deberán construirse sobre la base de la propia experiencia y tendrán como objeto reflejar los distintos riesgos vigentes en la cartera.

Es decir, bajo Solvencia II y IFRS 17, una adecuada gestión de riesgos requiere disponer una tabla de mortalidad propia para cada compañía, construida sobre la base de su propia experiencia. Aunque los productos de seguros de vida comercializados por las distintas compañías sean similares, la mortalidad y la composición de cada cartera de seguros difieren entre compañías debido, entre otras cuestiones, a: (i) el canal de comercialización (internet, agencias, corredores o banca-seguros), (ii) las políticas restrictivas o plausibles de suscripción (cuestionario médico, tele selección, pruebas médicas), (iii) los años de experiencia en el ramo de vida o (iv) la política de reaseguro.

En la actualidad, sin embargo, las compañías aseguradoras no parecen utilizar metodologías muy precisas para separar el BE del RM (Solvencia II) o generar hipótesis basadas en información interna (IFRS 17), lo cual no es adecuado y se puede convertir en una mala aplicación de la normativa en vigor. Por ejemplo, un gran número de compañías aseguradoras construyen sus tablas BE aplicando un porcentaje ($\mathbf{fac}$) a la tabla de mortalidad base (PERMF 2000P): $\mathbf{q_{x,t}^{BE}} = \mathbf{fac_t} * \mathbf{q_x^T}$. El procedimiento para obtener este $\mathbf{fac_t}$ suele depender de los niveles de información disponibles en cada momento $\mathbf{t}$, utilizándose: (E1) el ratio entre fallecidos observados y esperados, $\mathbf{fac} = \sum_{\mathbf{x}}^{\omega} \mathbf{d_x^r} / \sum_{\mathbf{x}}^{\omega} \mathbf{q_x^T} * \mathbf{l_x^r}$; o (E2) las sumas aseguradas, $\mathbf{fac} = (\mathbf{SA_x^{fal}})/(\mathbf{SA_x^{exp}})$ [1].

[1] Dónde: d_x^r denota el número de fallecidos a la edad cumplida x, registrados por la compañía aseguradora r; q_x^T es la probabilidad en la tabla de mortalidad base que una persona de edad x no alcance la edad $x+1$; l_x^r es el total de personas expuestas al riesgo al inicio del período t para cada edad x en la compañía aseguradora r; SA_x^{fal} y SA_x^{exp} son las sumas aseguradas a la edad x de los fallecidos y expuestos al riesgo, respectivamente.

Las aproximaciones (E1) y (E2) proporcionan estimaciones que presentan limitaciones para poder considerarse tablas de mortalidad BE. Entre ellas, (i) asumir que la tasa de mortalidad de la población asegurada de la compañía tiene el mismo comportamiento "edad a edad" que la correspondiente tasa de la tabla base, (ii) no recoger los efectos de determinadas políticas comerciales; o (iii) ser insensible a incrementos de fallecidos o salidas en periodos cortos para ciertas edades. Esta operativa además no permite separar el BE presente en la propia cartera de seguros de los recargos de seguridad implícitos que la tabla base: "*Allowance for uncertainty does not suggest that additional margins should be included within the best estimate*" (CEIOPS, 2010, pp. 25).

La utilización de estas soluciones supone, por tanto, un riesgo inherente al negocio asegurador. Una incorrecta medición de la mortalidad podría suponer una infravaloración (sobrevaloración) de las probabilidades de fallecimiento, con el correspondiente impacto en una mayor (menor) dotación en concepto de provisiones técnicas y sus implicaciones fiscales, contables y financieras.

3. METODOLOGÍA, DATOS Y SOFTWARE

La legislación española contempla la posibilidad de construir tablas de mortalidad sobre la propia experiencia siguiendo la disposición adicional quinta del Real Decreto 1060/2015, de 20 de noviembre, de Ordenación, Supervisión y Solvencia de las entidades aseguradoras y reaseguradoras (ROSSEAR, 2015). Así, con el objetivo de superar las limitaciones señaladas en la sección anterior, en este trabajo, siguiendo la senda marcada por Benjamin y Pollard (1986) y Lledó et al. (2017), desarrollamos un nuevo estimador de cohorte que permite obtener las probabilidades brutas de fallecimiento, q_x, de una población asegurada. En concreto, asumiendo distribución uniforme, se compara el número de defunciones registradas en los años t y $t+1$ (para una determinada cohorte) con los expuestos al riesgo (corregidos) de esa cohorte que alcanzan la edad exacta x a lo largo del año t. En concreto, el estimador propuesto viene dado por la ecuación (E3).

$$q_x = \frac{D_{x:t-x}^{t} + D_{x:t-x}^{t+1}}{l_x^t - R_{x:t-x}^{t,t+1} + \sum_{j=1}^{r_{x:t-x}^{t}} r_{x:j}^{t} + \sum_{j=1}^{r_{x:t-x}^{t+1}} r_{x:j}^{t+1} + \sum_{j=1}^{NN_{x:t-x}^{t}} nn_{x,j}^{t} + \sum_{j=1}^{NN_{x:t-x}^{t+1}} nn_{x,j}^{t+1}} \qquad \text{(E3)}$$

El numerador contabiliza el número de personas que fallecen en los años t y $t+1$ con edad x de la cohorte nacida en el año $t-x$ ($D^{t}_{x:t-x} + D^{t+1}_{x:t-x}$). El denominador contiene la población asegurada perteneciente a la cohorte nacida en el año $t-x$ expuesta al riesgo de fallecer, l^{t}_{x}, ajustada de entradas y salidas (por causa distinta a fallecimiento) en la cartera. A l^{t}_{x}, que representa el número de personas que alcanzan la edad exacta x a lo largo del año t dentro de la cartera[2], hay que sumar (restar) el tiempo que los nuevos asegurados están (dejan de estar) en riesgo de fallecer. Esto se puede conseguir: (i) sumando el número de años que aportan las pólizas de nuevo negocio con edad x, de la generación nacida en $t-x$, $\sum_{j=1}^{NN^{t}_{x:t-x}} nn^{t}_{x,j}$ and $\sum_{j=1}^{NN^{t+1}_{x:t-x}} nn^{t+1}_{x,j}$; (ii) menos el total de pólizas que han causado baja (rescate) con edad x, en el año t de la generación nacida en $t-x$, $R^{t,t+1}_{x:t-x}$; y (iii) sumando el tiempo que estas pólizas han aportado a la población en riesgo $\sum_{j=1}^{r^{t}_{x:t-x}} r^{t}_{x:j}$ y $\sum_{j=1}^{r^{t+1}_{x:t-x}} r^{t+1}_{x:j}$.

Con el objetivo de comparar en una situación real el estimador (E3) con las soluciones (E1) y (E2), hemos utilizado los microdatos pertenecientes a los años 2015 (n = 329,011) y 2016 (n = 325,831) de una población asegurada entre 65 y 100 años correspondientes a una cartera de rentas vitalicias comercializadas a través del canal de banca-seguros de una compañía de seguros. Los flujos de entrada y salida de dicha cartera de seguros (nuevo negocio, rescates y fallecimientos) han sido también considerados. Asimismo, como es habitual, las probabilidades de fallecimiento brutas han sido estimadas también suavizadas, para lo cual hemos utilizado un kernel gaussiano no paramétrico (ver, e.g., Ayuso et al., 2007) con ventana igual a 2. Todos los cálculos han sido implementados en el software R, versión 3.4.1 (R Core Team 2017).

4. RESULTADOS Y CONCLUSIONES

Las tablas de mortalidad han sido calculadas diferenciando por género dada sus diferentes patrones de mortalidad. Para las edades estudiadas, la Figura 1 muestra la comparación entre las probabilidades de fallecimiento de la tabla base (PERMF2000P) y los estimadores E1, E3 y E3-graduado. Como se esperaba, para ambos géneros, la estimación E1 es paralela a la tabla base. Las estimaciones E3 y E3-graduados muestran un

[2] Esta cantidad puede ser calculada a partir de contabilizar el flujo de asegurados que en el año t cumplen exactamente x años mientras forman parte de la cartera.

comportamiento más libre. Siguen el patrón de los siniestros de la cartera. En general, para las edades entre los 65 y 80 años las probabilidades de fallecimiento de la cartera son más bajas que las del estimador E1. A partir de los 85 años, el comportamiento es distinto.

Las compañías aseguradoras periódicamente calculan contablemente la reserva matemática correspondiente a las obligaciones futuras con sus asegurados. Dependiendo de la tabla utilizada las provisiones serán diferentes. La Figura 2 muestra los valores de las provisiones matemáticas que se obtienen para una renta vitalicia anual de 5.000€ utilizando los estimadores E1 y E3-graduado. Como se observa, las provisiones tienden a divergir a partir de los 10 años de la venta del producto, siendo mayor las diferencias para hombres que para mujeres. Por ejemplo, a los 15 años, se obtiene con el estimador E1 un BE de 37,937€ (53,675€) para hombres (mujeres), más de un 7% (3%) superior a los 35,317€ (51,854€) que se obtiene con el estimador E3-graduado. La menor mortalidad que muestra el estimador E3-graduado para las edades entre 65 y 80 años origina un menor nivel de provisión. Las mayores discrepancias en términos relativos y absolutos se dan entre los 10 y 25 años del comienzo del comienzo de la renta.

Figura 1. Estimaciones de los logaritmos de las probabilidades de fallecimiento con estimadores E1, E3 y graduación de E3. Panel derecho: hombres 2015. Panel derecho: mujeres 2015

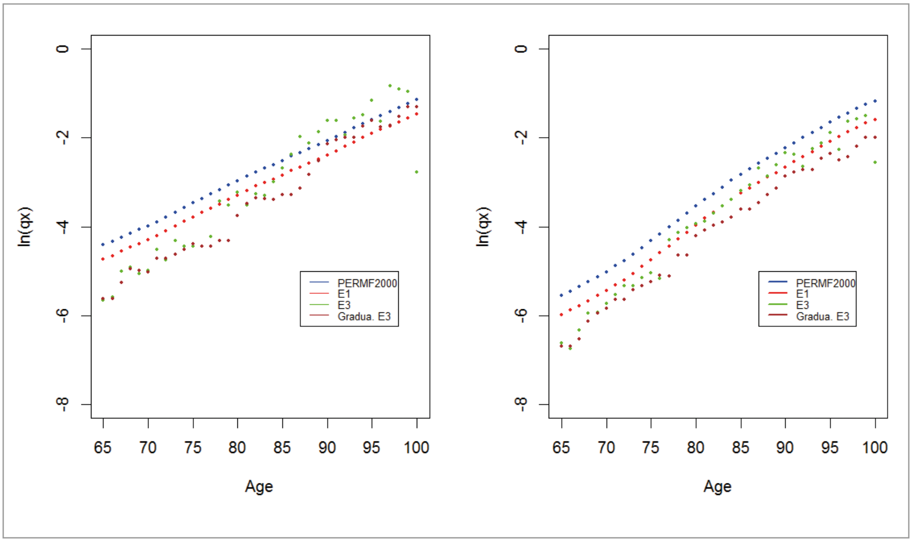
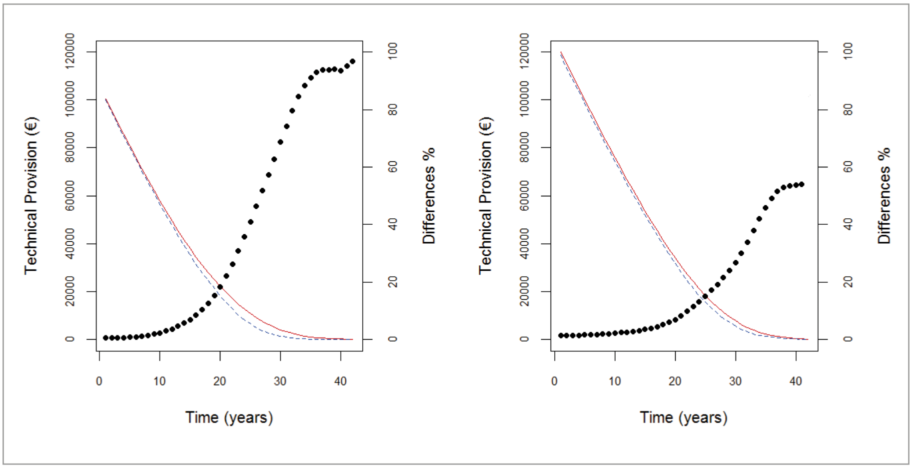


Figura 2. Provisión best-estimate de una renta vitalicia anual de 5.000€ utilizando E1 (línea continua) y la graduación de E3 (línea discontinua). Escala derecha: discrepancias relativas (línea discontinua): $\left|q_x^{E1} - q_x^{\text{grad E3}}\right| / q_x^{\text{grad E3}}$. Panel izquierdo: hombres. Panel derecho: mujeres.



Los resultados anteriores tienen importantes implicaciones. Una variación de la mortalidad en los niveles obtenidos podría afectar a una proporción importante de compañías aseguradoras, con efectos económicos significativos. Según los datos de ICEA, en 2015, las aseguradoras españolas tenían en el pasivo del balance contable un total de 69,959.911 millones de euros en concepto de reservas.

La metodología presentada se caracteriza por su sencillez y podría ser usada internamente en compañías aseguradoras tanto nacionales como internacionales. Asimismo, a diferencia de las metodologías actuales, permite construir una tabla de mortalidad sin recargos de seguridad implícitos: criterio indispensable bajo las normativas reguladoras de Solvencia II y IFRS17. En conclusión, el coste de incorporar la nueva metodología propuesta en este trabajo es mínimo, siendo en cambio elevado el riesgo potencial de continuar con las actuales prácticas.

REFERENCES

Ayuso, M., Corrales, H., Guillen, M., Perez-Marin, A. M., and Rojo, J. L. (2007). "Estadistica Actuarial Vida". Barcelona: Publicacions Universitat de Barcelona.

Barrieu, P., Bensusan, H., El Koroui, N., Hillairet C., Loisel S., Ravanelli C., and Salhi Y. (2012). "Understanding, modelling and managing longevity risk: key issues and main challenges". Scandinavian Actuarial Journal, 2012(3), 203-231.

Benjamin, B., and Pollard, J. (1986). "The Analisys of mortality and other Actaurial Statistics". London: Heinemann.

Börger, M., Fleischer, D., and Kuksin, N. (2014). "Modeling the mortality trend under modern solvency regimes". Astin Bulletin, 44, 1-38.

Biffis, E., and Blake, D. (2009). Mortality-linked securities and derivatives. Pensions Institute. Discussion Paper, PI 0901, Pensions Institute, London.

Cairns, A., Blake, D., and Dowd, K. (2008). "Modelling and management of mortality risk: A Review". Scandinavian Actuarial Journal, (2-3), 79-113.

Cairns, A., Blake, D., Dowd, K., Coughlan, G., and Khalaf-Allah, M. (2011). Bayesian stochastic mortality modelling for two populations, ASTIN Bulletin, 41, 29-59.

CEIOPS (2010). QIS5 technical specifications. Technical report, European commission – internal market and services DG.

Enchev, V., Klinow, T., and Cairns, A. (2016). "Multi-population mortality models: fitting, forecasting and comparisons". Scandinavian Actuarial Journal, 2013(4), 319-342.

IFRS 17 (2017). "IFRS 17 Insurance Contracts". International Accounting Standards Board.

Lee, R., and Carter, L. (1992). "Modelling and forecasting U.S. mortality". Journal of the American Statistical Association, 87, 659-671.

Lee, R., and Miller, T. (2001). "Evaluating the performance of the Lee-Carter method for forecasting mortality". Demography, 38, 537-549.

Li, N., and Lee, R. (2005). "Coherent mortality forecast for a group of populations: an extension of the Lee-carter method". Demography, 3, 575-594.

Lledó, J., Pavía, J.M., and Morillas, F. (2017). "Assessing Implicit Hypotheses in Life Table Construction". Scandinavian Actuarial Journal, 2017 (6), 495-518.

R Core Team. 2017. R: A Language and Environment for Statistical Computing. Vienna: R Foundation for Statistical Computing. http://www.R-project.org/

ROSSEAR (2015). Real Decreto 1060/2015, de 20 de noviembre, de ordenación, supervisión y solvencia de las entidades aseguradoras y reaseguradoras, Boletín Oficial del Estado, 288, 113617-113816.

INCORPORATING TELEMATICS INFORMATION IN MOTOR INSURANCE RATEMAKING: THE ROLE OF MILEAGE AND DRIVING HABITS

Ana María Pérez-Marín
Universidad de Barcelona, Departamento de Econometría y Economía Aplicada
Riskcenter, España
amperez@ub.edu

ABSTRACT

Nowadays, the calculation of premium rates based on driver behaviour is a great opportunity for the insurance industry. In this paper, we show which is the effect of incorporating data collected from a GPS device regarding driving patterns in motor insurance ratemaking. We use a classical Poisson regression model that is updated with telematics information. We use real data from usage-based insurance policies. We conclude that the distance travelled by the driver and driving habits have a significant effect on the expected number of accidents and, therefore, on the cost of insurance coverage.

1. INTRODUCCIÓN

Predictive models traditionally used in insurance ratemaking include variables related to the driver and vehicle. Information about driving habits are not considered directly, as driving patterns and intensity are difficult to measure objectively. Nevertheless, the distance driven is an exposure variable that should be taken into consideration in motor insurance premiums calculations. Nowadays, the technology available makes it possible to collect mileage information automatically. In the field of transportation research, many authors have stressed the significance of GPS-based technology (Isaacson et al., 2016) and it seems reasonable to think that future ratemaking models will incorporate these technological advances. Here, we propose a two-step method that combines traditional motor insurance rating factors with new information obtained from telemetric data collection for defining a new ratemaking system.

Pay-as-you-drive insurance (PAYD) is the basic way of adopting a mileage-based pricing scheme. Under this system drivers' speed profiles, the type of roads they most frequently take, and the time of day they are usually on the roads are considered in the rating system (Paefgen et al., 2013, 2014). These policies are often only sold to young drivers, but recent papers suggest that young policyholders are a heterogeneous risk group (Ayuso et al., 2014). Moreover, Ayuso et al. (2016) show that gender differences in accident rates are mainly attributable to differences in the intensity of vehicle use, so no gender discrimination is necessary if telematics provides enough information about driving patterns.

2. METHODS

Let Y_i, $i = 1,..., n$ be the number of claims reported by insured i during a fixed time period of one year, where n represents the number of policyholders. We denote by $x_i = (xi_1,..., x_{ik})$ the vector of k exogenous variables that measure the individual characteristics or the risk factors that we believe that have some impact on the number of claims. We assume that given x_i, the dependent variable Yi follows a Poisson distribution with parameter λ_i, which is a function of the linear combination of parameters and regressors. Namely,

$$E(Y_i \mid x_i) = exp\ (\beta_0 + \beta_{1xi1} + \cdots + \beta_{k\,xik}) \quad (1)$$

Here $(\beta_0, \dots, \beta_k)$ are the unknown parameters to be estimated. An offset can be included in the model when exposure to risk varies. Let us denote T_i the exposure factor for policy holder i, then the model that incorporates this factor is:

$$E(Y_i \mid x_i, T_i) = T_i\ exp\ (\beta_0 + \beta_{1xi1} + \cdots + \beta_{k\,xik}).$$

Alternatively, here we propose a two-step procedure that includes classical actuarial risk factors and also telematics information. In the first step, we use a classical Poisson model, such as (1), to obtain a prediction of the expected number of claims for every policy i, that we call $\widehat{Y}_i$, given the information on the initial classical actuarial characteristics xi. In the second step, we assume that additional information collected by a GPS system becomes available. $\widehat{Y}_i^{UBI}$ is the prediction from usage-based insurance that is obtained as in the second step. Let us specify

$$\boldsymbol{E}(\hat{\boldsymbol{Y}}_i^{\text{UBI}} \mid \boldsymbol{z}_i, \boldsymbol{Y}_i) = \hat{\boldsymbol{Y}}_i\, \boldsymbol{exp}(\boldsymbol{\eta}_0 + \boldsymbol{\eta}_1\, \boldsymbol{z}_{i1} + \cdots + \boldsymbol{\eta}_k\, \boldsymbol{z}_{ik}). \quad (2)$$

Here, $z_i = (z_{i1},..., z_{il})$ denote the vector of l variables that are collected from the telematics device on driving behavior and $(\gamma_1, \ldots, \gamma_l)$ are the unknown parameters that are used in the specification of the model to include information on driving behavior. Therefore, we use $(\hat{Y}_i)$ as an offset in the second model.

3. DATA

We have information on risk exposure and number of claims for 3,841 insured drivers, with car insurance coverage throughout 2009 and 2010, that is, individuals exposed to the risk for two full years. We will use the information of 2009 as the training dataset and the information of 2010 as the validation dataset. So, in a first step we are going to model the number of claims only by using traditional risk factors (and the dataset of 2009) by using model (1). In a second step we will use the log of the prediction of the number of claims obtained in the first step as an offset in a second model were telematics information are introduced as explanatory variables, also with the dataset of 2009 (model (2)). Then, we will apply these two models to the dataset of 2010 to get two predictions of the number of claims (one by using model (1) and another by using model (2)), and we will compare the results.

The variables included in the modelling are shown in Table 1. Table 2, presents the descriptive statistics for the continuous variables, and highlight differences between drivers with no claims and those with claims.

Table 1. Variable description

Traditional ratemaking factors	
Age	Age of the insured driver
Sex	Sex of the driver (1 if male, 0 if female)
Age Driving License	Number of years in possession of a driving license
Vehicle Age	Age of the insured vehicle
Power	Power of the insured vehicle
Parking	1 if the car is parked in a garaje overnight, 0 otherwise
New telematics rating factors	
Km per year (000s)	Total kilometers travelled per year (in thousands)
Km per year at night (%)	Percentage of kilometers travelled at night during the year
Km per year over speed limit (%)	Percentage of kilometers travelled during the year above the limit
Urban km per year (%)	Percentage of kilometers travelled in urban areas during the year

Table 2. Descriptive statistics for insureds with and without claimsn

	All sample (N=3841)		No claims (N=3116)		With claims (N=725)	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Age	25.01	2.44	25.11	2.44	24.57	2.38
Age driving license	4.84	2.46	4.97	2.49	4.27	2.24
Vehicle Age	6.63	3.77	6.68	3.78	6.42	3.73
Power	95.93	25.53	95.85	25.99	96.27	23.45
Km per year (000s)	13.87	8.06	13.58	7.85	15.09	8.78
Km at night (%)	6.58	5.78	6.37	5.72	7.45	5.99
Km over speed (%)	9.40	8.94	9.40	9.04	9.40	8.51
Km urban (%)	26.33	13.96	25.76	13.77	28.78	14.50

4. RESULTS

Column (1) in Table 4 presents the Poisson model estimates for all claim types using traditional ratemaking factors (the so-called model (1)). On the other hand, column (2) in Table 4 presents the Poisson model estimates using telematics with log of prediction of non-telematics as offset (the so-called model (2)). Both models have been estimated by using data from 2009 only. Regarding the classical variables (column (1)), we observe that the number of years in possession of the driving license is the only variable that has a significant effect in the number of claims, the more driving experience the driver has, the lower the number of accidents. Column (2) shows that the inclusion of variables related to mileage and driver behaviour give better results than when only the traditional variables are included (the AIC value is lower). Therefore, the inclusion of telematics information is relevant. Moreover, all the parameters that include an offset with the log of prediction of the non-telematics model are statistically significant, indicating that all the telematics variables are relevant. The percentage of kilometres per year over the speed limit, the percentage of urban kilometres per year and, even, the total number of kilometres per year (all of which present a p-value lower than 0.02) show a direct relationship with the number of claims reported to the insurance company.

Table 3 shows the results of applying model (1) and (2) to the validation dataset of 2010. We compare the real frequency of claims in 2010 and the estimated number of claims obtained when applying models (1) and (2) to validation dataset of 2010. We clearly see that model (2) provides better results compared to model (1). Table 4 shows the corresponding Pearson Chi squared statistics for both models when applied to the validation dataset of 2010, and we see that the statistic is much lower for model (2) compared to model (1). This is an evidence of the relevance of including telematics information in insurance automobile ratemaking.

Table 2. Poisson regression parameter estimates for two models: in column (1) only non-telematics explanatory variables are used and, in column (2) the log of the prediction of the non-telematics model is introduced as an offset in the Poisson model with all telematics-related variables. Both models have been estimated by using data from 2009.

	Non – telemàtics (1)		Telematics with offset (2)	
	Coefficient	p-value	Coefficient	p-value
Intercept	-0.5320	0.1989	-1.7279	<.0001
Age	-0.0219	0.1963		
Sex	0.0883	0.1942		
Age Driving License	-0.1039	<.0001		
Vehicle Age	-0.0167	0.0701		
Power	0.0016	0.2528		
Parking	0.1239	0.0767		
Log of km per year (000s)			0.3941	<.0001
Km per year at night (%)			0.0152	0.0027
Km per year over speed limit (%)			0.0089	0.0190
Urban km per year (%)			0.0196	<.0001
AIC	4940.8594		4834.4286	
BIC	4984.6320		4865.6947	

Table 3. Real and expected number of claims when applying model (1) and (2) to the validation data set of 2010

Total number of claims	Real frequency	Estimated frequency Non – telematics	Estimated frequency Telematics with offset
0	3184	2989.92	3030.50
1	487	739.53	698.55
2	133	100.038	98.60
3	27	9.71	11.12
4	8	0.75	1.11
5	1	0.05	0.10

Table 4. Pearson Chi2 statistic for models (1) and (2) when applied to the validation datasate of 2010

	Non-telematics	Telematics with offset
Pearson Chi2	229.20	156.82

5. CONCLUSIONS

The inclusion of factors related to driver behaviour shows that the group of young drivers are quite heterogeneous. The same happens regarding driving experience, as we conclude that the expected number of claims decreases as driving experience increases. However, the rest of traditional variables do not have a significant effect. On the other hand, the model that includes the log of the prediction of the number of claims as an offset and telematics explanatory variables provides a better fit, and let us conclude that there is a significant influence of the annual distance but also with the percentage of kilometres driven per year over the speed limit and the percentage of urban and nighttime kilometres driven per year. Therefore, we conclude that the use of telematics provides very valuable information in insurance ratemaking.

ACKNOWLEDGMENTS

We want to acknowledge the support received by the Spanish Ministry of Economy and Competitiveness (Grant ECO2016-76203-C02-02-P).

REFERENCES

Ayuso, M., Guillen, M. and Perez-Marin, A.M. (2014). "Time and distance to first accident and driving patterns of young drivers with pay-as-you-drive insurance". Accident Analysis and Prevention 73, 125-131.

Ayuso, M., Guillen, M. and Perez-Marin, A. M. (2016). "Telematics and gender discrimination: some usage-based evidence on whether men's risk of accidents differs from women's". Risks 4, 2, 1-10.

Paefgen, J., Staake, T. and Fleisch, E. (2014). "Multivariate exposure modelling of accident risk: Insights from Pay-as-you-drive insurance data". Transportation Research Part A: Policy and Practice 61, 27-40.

Paefgen, J., Staake, T. and Thiesse, F. (2013). "Evaluation and aggregation of pay-as-you-drive insurance rate factors: a classification analysis approach". Decision Support Systems 56, 192-201.

Isaacson, M., Shoval, N., Wahl H.W. and Oswald, F., Auslander, G. (2016). Compliance and data quality in GPS-based studies. Transportation 43, 1, 25-36.

RISK ON FINANCIAL REPORTING: INSIGHTS FROM THE RESEARCH

Yolanda Pérez Pérez
Facultad de Ciencias Económicas y Empresariales
Universidad Complutense de Madrid, Spain,
yolape01@ucm.es

María del Mar Camacho-Miñano
Facultad de Ciencias Económicas y Empresariales
Universidad Complutense de Madrid, Spain
marcamacho@ccee.ucm.es

María Jesús Segovia Vargas
Facultad de Ciencias Económicas y Empresariales
Universidad Complutense de Madrid, Spain
mjsegovia@ccee.ucm.es

ABSTRACT

This paper examines the importance of risk on financial reporting through an analysis of the business environment as well as the literature review. The impact of risk on financial information is a trendy topic for many firms' stakeholders: for investors as relevance for their decision making process, for companies as preserving their value in terms of going concern and reputation; for regulators as responsible of oversight the well market operation and for external auditors as providers of the most level of assurance over financial reporting. Our study carries out a revision of the literature in depth to know what areas are more interesting to study and cover and which ones are already developed in the financial reporting research. The results highlight that the most interesting areas of research are those linking the risk in financial reporting either with performance as well as with audit opinion. Thus, our results confirm a room for future research in the light of new regulation in the accounting and auditing standards.

1. INTRODUCTION

The impact of risk on financial information is a trending topic for many firms' stakeholders: for investors as relevance for their decision-making process, for companies as preserving their value in terms of going concern and reputation; for regulators as responsible of oversight the well market operation and for external auditors as providers of the most level of assurance over financial reporting (Sunder, 2015). Investors recognize that there is a remarkable gap between financial information used for their decision-making processes and its effectiveness due to factors like the lack of enough detail in qualitative information (PWC, 2014). As a consequence of this asymmetry, some financial scandals have taken place in the last decade reaching millionaire losses and other social drawbacks. This has highlighted the importance of risk of financial information and its focus on the regulator. The origin of risk regulation comes from the US, especially through the Stock Exchange Commission (SEC) and the Public Company Oversight Board (PCAOB), especially since 2002 due to the Enron scandal.

In Spain context, the regulation about the internal control over financial reporting has been also reinforced since 2010 by the stock market regulator, the Spanish National Securities Market Commission (CNMV). In addition, for some time now, investors have been demanding an audit report that provides more information to assist them in decision making, as well as greater transparency regarding the auditor's responsibilities and the essential aspects of the audit.

The International Auditing and Assurance Standards Board (IAASB), aware of the need for a change in focus in the audit reports, has approved a new international standards on audit reports that incorporate major changes in the information contained in the reports, which has been integrated into the regulatory framework of the audit in Spain (IFAC, 2013). One of the major changes is the obligation for listed companies to describe in the audit report the key audit issues (audit matters) and the audit work performed in relation to those issues (KPMG, 2016).

In this study, our objective is to highlight the gaps in the literature review in order to know what areas are more interesting to study and cover and which ones are already developed in financial reporting research.

2. METHODOLOGY OF THIS STUDY

The term "risk" is a concept widely investigated in the past. There is an extensive research from a financial perspective, but not from an accounting one. On the other hand, there are also lots of articles related to financial reporting with a special focus on its quality. However, most of the prior literature has been focused on the concept of risk and financial reporting without setting a connection between both concepts and not from an accounting perspective. In the following review of research, the association between risk and financial reporting is addressed.

The database "Web of Science", provided by Thomson Reuters, gave 331 results when we searched for two keywords, "audit" and "financial reporting", under "Topic", as of October 22th, 2017. It shows that such a topic is of relevant interest to the international academic community.

Since the year 2008, the pattern of publications related to the risk in financial reporting has consistently increased, showing a singular rise in 2015 and 2016. Another interesting fact is the number of references to articles on "risk in financial reporting". The trend is growing since 2008 but it almost doubled from 2013 to 2016. In 2016 the number of "risk in financial reporting" citations reached almost 700 references.

After a thorough review of all articles, we exclude 273 papers because their main objective focuses on either one of the two concepts, but not on the connection between them. Thus, 58 articles remain for our study. With the purpose of systematizing and organizing the literature, we decide to split up the papers according to their subject into five areas of research, according to the main topic of each paper: disclosure, quality, performance, regulation and auditing.

3. RISK AND FINANCIAL REPORTING IN LITERATURE REVIEW

The five areas of research identifies are Financial Reporting and: i) Risk and Disclosure (10 %), ii) Risk and Quality (12 %), iii) Risk and Performance (19%), iv) Risk and Regulation (12 %) and v) Risk and Auditing (47%).

i) Financial Reporting, Risk, and Disclosure

Risk and uncertainty are inherent in the environment and functions of accounting. (Sunder, 2015). Nowadays, accounting information user profile became more sophisticated and the financial reports face new challenges in accomplishing process to meet users' needs (Nichita et al., 2015). Some studies encourage firms to provide an open and informative discussion of their risks in their financial reports (Ryan, 2012).

The recent financial crisis has brought the need for companies to effectively manage their risks. However, little is known about the link between risk management and the financial reporting process which can affect financial ratios to an extent that misleads users (Cohen et al., 2017). According to prior literature in this area, whether and to what extent, accounting should try to protect users from their decision-making imperfections remains an open question (Sunder, 2015).

ii) Financial Reporting, Risk, and Quality

There is a positive association between investors' demands for firm-specific information and financial reporting quality (Cohen, 2008). Traditional financial reporting typically does not provide good predictions of future financial performance (Kristensen & Westlund, 2003). Policy makers may want to encourage the adoption of reporting standards that enhance the transparency of financial reports. They may also want to pursue ways that improve the quality of assurance services and strengthen the penalties associated with failure in the assurance services provided by audit firms (Khalil et al., 2015).

iii) Financial Reporting, Risk, and performance

Internal control quality has an economically significant effect on internal management reports and also in financial reporting fraud by top managers (Donelson et al., 2017). Research in financial reporting needs to consider reporting of risks and research needs to come up with clear and concise disclosure methods and standards across firms that quantify exposures to underlying volatility and risk factors (Magnan & Markarian, 2011). Fraudulent financial reporting should lead to an increase in the cost of equity capital as a firm's future cash flows become less certain (Nicholls, 2016).

An important insight of this area is that financial reporting did not fulfill its task reporting on risk ahead of the recent crisis; risks were hidden (Singleton-Green, 2012).

iv) Financial Reporting, Risk, and Regulation

The articles in this area are related to corporate governance and accounting standards, especially focus on the Anglo-Saxon markets. Audit committees are seen as having sufficient expertise and power to fulfill their responsibilities with members playing important roles in overseeing internal controls, focusing on reporting quality, identifying risks, asking challenging questions, and overseeing the whistleblowing process (Cohen et al., 2010).

The 2007 crisis has brought into sharp focus the reality that the regulation of corporate reporting is just one piece of a larger regulatory configuration, and that forces are at play that would subjugate accounting standard setting to broader regulatory demands (Bushman & Landsman, 2010). The use of a common set of standards such as International Financial Reporting Standards (IFRS) aims, in broad terms, to promote the comparability and transparency of financial statements and to improve the quality of financial reporting (Brown et al., 2014).

v) Financial Reporting, Risk, and auditing

Present-day audit reports communicate some information about financial reporting quality. Issuing a Standard Audit Report reduces confidence and could eventually erode the audit such as assurance function (Asare & Wright, 2016), while audit reports containing explanatory language brings more information to users of financial reporting and also more probability of restatement (Czerney et al., 2014). Considering the different types of audit opinions, firms with audit qualifications show higher information asymmetry levels than those with unqualified opinions and there is a stronger effect on the level of informational asymmetry in the case of going concern qualifications (Abad et al., 2017).

The auditor's opinion on the Internal Control over Financial Reporting (ICFR) provides financial statement users with value-relevant information. In particular, an adverse audit opinion on ICOFR relative to an unqualified opinion is significantly associated with investors assessing a higher risk of financial statement misstatement, higher risk of a future financial statement restatement, higher information asymmetry, lower financial statement transparency, higher risk premium, higher cost of capital, lower sustainability of earnings, and lower earnings predictability (Lopez et al., 2009).

4. CONCLUSIONS

This paper has presented enough arguments to conclude that risk on financial reporting is a trending topic and there is room for new research. First, there is a positive association between investors' demands for firm-specific information and financial reporting quality (Cohen, 2008). Recent public policy initiatives seek greater transparency in financial reporting (Caplan, Dennis et al., 2016). Second, since the year 2008, the trend of publications related to the risk of financial reporting has consistently increased. In short, this topic has had a huge importance for researchers until now and the trend shows that this would reach even more importance. After a deep analysis of literature review, we have identified some gaps that we would want to address in our proposal of study.

On the one hand, the articles provided by Web of Science database are based on the information until the fiscal year 2013-2014, so recent changes in the business and regulatory environment has not been considered. In particular, since 2015, the IASSB has carried out a reform of the Auditing Standards on Reports with a new approach aimed at improving and increasing the information in the audit report so that users can take advantage of it. In general, we expect that most countries, including those that are part of the EU, will apply the new requirements in the fiscal years closed in 2017. Other countries, such as the United Kingdom, have already made the transition beforehand and the response of interest groups has been very positive.

On the second hand, the most part of the articles are related to US markets, so there is little research in other markets, especially in UE and particularly in Spain.

For all these reasons, we propose a study based on two areas of research: an empirical analysis about the risk on financial reporting and its impact in the performance of the companies and a qualitative analysis of the risk on financial reporting and the new audit report after the Audit EU Reform.

Additionally, we think that regulators, analysts, auditors, managers and investors could benefit from the results of this study, as evidence about how risk on financial reporting could be considered as an important factor to be taken into consideration firstly in order to determine the company's performance and secondly to analyze the impact in the audit report after the IAASB's reforms.

REFERENCES

Abad, D., Sanchez-Ballesta, J.P. & Yague, J (2017). Audit opinions and information asymmetry in the stock market. Accounting and Finance, 57(2), 565-595.

Asare, S.K. & Wright, A.M. (2016).Inferring Remediation and Operational Risk from Material Weakness Disclosures. Behavioral Research in Accounting, 29(1), 1-17.

Brown, P., Preiato, J. & Tarca, A. (2014). Measuring Country Differences in Enforcement of Accounting Standards: An Audit and Enforcement Proxy. Journal of Business Finance and Accounting, 41(1-2), 1-52.

Bushman, R.; Landsman & Wayne R. (2010).The pros and cons of regulating corporate reporting: a critical review of the arguments. Accounting and Business Research, 40 (3), 259-273.

Caplan, D. & Dutta, S.K. (2016). Journal of Accounting Literature (36), 1-27.

Cohen, D.A. (2008).Does Information Risk Really Matter? An Analysis of the Determinants and Economic Consequences of Financial Reporting Quality. Asia-Pacific Journal of Accounting and Economics, 15(2), 69-90.

Cohen, J., Krishnamoorthy, G. & Wright, A. (2010). Corporate Governance in the Post-Sarbanes-Oxley Era: Auditors' Experiences. Contemporary Accounting Research, 27 (3), 751-801.

Czerney, K.; Schmidt, J.J. & Thompson, A. M. (2014). Does Auditor Explanatory Language in Unqualified Audit Reports Indicate Increased Financial Misstatement Risk? Accounting Review, 89(2), 2115-2149.

Donelson, D. C.; Ege, M.S. & McInnis, J.M.(2015).Internal Control Weaknesses and Financial Reporting Fraud. Auditing-A Journal of Practice and Theory, 36 (3),45-69.

IFAC (2013). The Evolving Role of Auditors and Auditor Reporting.

Khalil, S., Saffar, W. & Trabelsi, S. (2015). Disclosure Standards, Auditing Infrastructure, and Bribery Mitigation. Journal of Business Ethics, 132 (2), 379-399.

KPMG (2016). Nuevo informe de auditoría: mayor transparencia e información más relevante.

Kristensen, K & Westlund, AH. (2003).Valid and reliable measurements for sustainable non-financial reporting. Total Quality Management and Business Excellence, 14 (2), 161-170.

Magnan, M. & Markarian, G. (2011).Accounting, Governance and the Crisis: Is Risk the Missing Link?. European Accounting Review, 20 (2), 215-231.

Nichita,M&Turlea,C(2015).Approach regarding a framework for risk reporting in order to enhance the related good practices. Amifeatru Economic17(40)1108-1121.

Nicholls, C. (2016). The impact of SEC investigations and accounting and auditing enforcement releases on firms' cost of equity capital. Review of Quantitative Finance and Accounting 47 (1), 57-82.

PWC (2014).Corporate performance: What do investors want to know? Innovate your way to clearer financial reporting.

Ryan, S. (2012). Risk reporting quality: implications of academic research for financial reporting policy. Accounting and Business Research, 295-324.

Sunder, S. (2015). Risk in Accounting. Abacus, 51(4), 536-548.

CAMELS MODEL: DETECTION AND TREATMENT OF COLLINEARITY

Ainara Rodríguez Sánchez
Universidad de Granada, Programa de Doctorado en Ciencias Económicas y Empresariales Técnicas Cuantitativas Avanzadas en el Ámbito Económico y Empresarial, España, arsanchez@ugr.es

Claudia García García
Universidad de Granada, Programa de Doctorado en Ciencias Económicas y Empresariales Técnicas Cuantitativas Avanzadas en el Ámbito Económico y Empresarial, España, garciaclaudia@ugr.es

Catalina B. García García
Universidad de Granada, Departamento de Métodos Cuantitativos para la Economía de la Empresa, España, cbgarcia@ugr.es

Román Salmerón Gómez
Universidad de Granada, Departamento de Métodos Cuantitativos para la Economía de la Empresa, España, romansg@ugr.es

ABSTRACT

Nowadays, the importance of risk analysis of financial institutions has increased, due to the recent financial and economic crisis. To evaluate this risk, there is a known methodology called CAMELS. The variables of this model are commonly used as explanatory factors in regression model leading to collinearity since these variables may be related between them. In spite of this fact, collinearity is usually disregarded and the model is estimated by ordinary least squares obtaining instable estimations. This paper presents a literature review about the detection and treatment of this problem in the different scientific works that use the variables of CAMELS model as explanatory variables in regression model.

1. INTRODUCCIÓN

Tras la crisis financiera y económica, el papel de las entidades de crédito se ha visto acentuado. No solo por su actividad de intermediación financiera sino, por los efectos que tiene sobre la economía real, especialmente en fases de recesión, como transmisoras de la restricción del crédito hacia empresas y familias, contribuyendo al agravamiento de la situación económica en general. Por estos motivos, crece el interés por evaluar la eficiencia de estas entidades (su solvencia y capacidad para afrontar momentos de tensión económica a fin de evitar que aparezcan crisis bancarias).

Aparecen ciertas dudas con el modelo de Basilea II (conjunto de directrices elaboradas con el fin de aconsejar a nivel internacional regulaciones bancarias), debido a que existe coexistencia temporal con los inicios de la última crisis financiera. Esta situación ha motivado la creación una nueva normativa denominada Basilea III. Una de las principales metodologías que permite evaluar el riesgo de las instituciones financieras es el modelo CAMEL que fue instaurado por los Estados Unidos en 1979 y que consiste en un sistema uniforme de calificación de las instituciones en función de distintas variables que conforman sus siglas: Capital, calidad del Activo, administración, rentabilidad y liquidez. Posteriormente, se incorporó la variable sensibilidad pasando a conocerse como modelo CAMELS.

Alternativamente, algunos autores han utilizado las variables del modelo CAMELS como factores explicativos en un modelo de regresión. Sin embargo, es evidente que las variables del modelo CAMELS podrían estar relacionadas entre sí. Cortina (2009) advierte que el comportamiento de unas variables puede repercutir en otras, ya que son interdependientes. Por tanto, si se utilizaran como variables explicativas en un modelo econométrico se podría presentar un posible problema de multicolinealidad.

El objetivo de este trabajo, es realizar una revisión bibliográfica sobre la detección y tratamiento de colinealidad en los distintos trabajos científicos que usan las variables del modelo CAMELS como variables explicativas en un modelo de regresión. El trabajo se estructura como sigue: en la sección 2 se define el posible problema de multicolinealidad y sus consecuencias, en la sección 3 se revisa los distintos trabajos que utilizan las variables del modelo CAMELS como variables explicativas centrándonos en la detección y tratamiento de la colinealidad. Finalmente, en la sección 4 se destacan las principales conclusiones.

2. COLINEALIDAD: CONCEPTO Y CONSECUENCIAS

La regresión lineal es una herramienta estadística ampliamente utilizada para explicar una variable dependiente o explicada (Y) a partir de un conjunto de variables independientes o explicativas (X). Johnston et al. (2001) afirma que a menudo dichas variables independientes se acercan a la dependencia lineal, en cuyo caso podemos calcular el vector de estimadores mínimos cuadrados ordinarios (MCO), aunque es probable que sus elementos tengan errores estándar muy elevados conduciendo a los siguientes inconvenientes: tendencia en los contrastes de significación individual a no se rechazar la hipótesis nula, mientras que al realizar contrastes conjuntos sí, la varianza estimada de los estimadores son muy elevadas y coeficientes muy sensibles ante pequeños cambios en los datos. Gujarati (2003) afirma que si el objetivo del análisis de regresión es el de predecir, entonces la multicolinealidad no es un problema serio. Sin embargo, si el objetivo de la regresión es establecer relaciones causales entre las variables, la colinealidad debe ser detectada y tratada debido a la inestabilidad de los estimadores obtenidos por MCO.

Para su detección, el método más extendido es el Factor Inflación de la Varianza (FIV) que se calcula como

$$\mathbf{VIF}_k = \frac{\mathbf{1}}{\mathbf{1-}R_k^{2}}, \qquad (1)$$

donde $\mathbf{R}_{\mathbf{k}}^{\mathbf{2}}$ es el coeficiente determinación de la regresión de la variable k en función del resto de variables explicativas del modelo. Generalmente se considera que existen problemas importantes de multicolinealidad en el modelo si el valor del VIF es mayor a 10, Marquardt (1970). Algunos autores bajan el umbral a 4. Otra medida es el número de condición (NC) este valor está basado en los autovalores de la matriz $X^t X$,

$$\mathbf{K(X)} = \sqrt{\frac{\mu_{max}}{\mu_{min}}} \qquad (2)$$

El valor del NC es mayor a uno, y Belsley et al (1980) nos dicen que valores de este número menores a 20 implican baja multicolinealidad, valores entre 20 y 30 suponen colinealidad moderada, y si el NC es mayor a 30 entonces estaremos ante problemas de multicolinealidad graves. Berk (1977) mostró que el NC es una cota superior del máximo FIV. Se podría pensar que el FIV no detecta cierta información sobre la colinealidad que el NC si capta.

Una vez que se detecta la existencia de colinealidad, una de las soluciones más expandidas es la eliminación de alguna de las variables causante. Sin embargo, esta solución que puede ser adecuada en algunos casos, puede, en otros casos, limitar el potencial del trabajo a realizar. La transformación de los datos es también una solución muy aplicada. Otra opción, es utilizar métodos de estimación alternativos a MCO, como el estimador cresta, el estimador alzado, la regresión con variables ortogonales o componentes principales.

3. REVISION

Es evidente que si un banco es rentable, ha debido presentar una alta calidad en sus activos, liquidez, y solidez para hacer frente a los posibles riesgos. Además, si existe solidez en la entidad financiera, existirá una menor sensibilidad a los riesgos. Finalmente si se existen estas características en una institución financiera, significará que se ha realizado una correcta administración. Por todo ello, existen motivos para pensar que las variables del modelo CAMELS se encuentran relacionadas entre sí, pudiéndose producir un posible problema de multicolinealidad si se utilizan conjuntamente como variables explicativas en un modelo de regresión. Sin embargo, se ha realizado una revisión de numerosos artículos que utilizan las variables CAMELS como variables explicativas en un modelo de regresión encontrándose que en su mayoría no se realiza análisis alguno sobre la posible existencia de colinealidad. A continuación, se presentan de forma cronológica los artículos revisados que si tratan de alguna forma la colinealidad.

Gasbarro et al (2002) analiza los percentiles de puntuación CAMEL para 52 bancos indonesios en función de las variables del modelo CAMEL. Este trabajo reconoce la existencia de colinealidad aunque no muestra resultados al respecto. Simplemente dice que la colinealidad no afecta al estudio dado que el propósito del modelo CAMEL es predecir como en un futuro van a afectar estos factores a la eficiencia de las entidades de crédito, para que este resultado pueda ser utilizado por los reguladores, y la colinealidad se espera que prevalezca en un futuro. Sin embargo, los propios autores resaltan que de manera inesperada la liquidez no resulta significativa y sugiere que puede deberse a que la colinealidad con otra variable está disminuyendo su relevancia. Por lo que los autores, se contradicen diciendo que la colinealidad debe ser reconocida pero que no afecta mientras que la consideran la causa de que una de las variables no resulte significativa.

Nurazi et al. (2005) presentan una regresión logística para analizar la quiebra bancaria a partir de 13 variables que representan las distintas categorías del modelo CAMEL. Para analizar la colinealidad, presentan la matriz de correlaciones y reconocen la existencia de correlación entre las dos variables que se consideran para medir la categoría capital pero no consideran que sea un problema ya que no consideran la opción de usar ambas variables conjuntamente.

Karminsky et al. (2012) analizan los datos de bancos rusos desde 1998 a 2011 para predecir la quiebra bancaria a partir de las variables del modelo CAMEL. Parten de 18 variables explicativas pero eliminan las que tienen alta correlación para evitar problemas de colinealidad. De igual forma, utilizan una escala logarítmica. Okoth et al. (2013) analizan la rentabilidad de bancos en Kenia desde 2001 a 2010 utilizando las variables del modelo CAMEL. Muestran en la tabla 4 de su artículo el coeficiente de determinación para cada una de las variables con respecto al resto de regresores. Se observa que existen dos $\mathbf{R}_{\mathbf{k}}^{\mathbf{2}}$ con valores superiores a 0,75 (0,83 y 0,86) que conducirían a valores del FIV aplicando la expresión (1) superiores a 4 (5,88 y 7,14, respectivamente). Sin embargo, los FIV que aparecen en la tabla son 3,21 y 3,84, respectivamente, y en base a ellos los autores concluyen que no existen problemas de colinealidad. Hemos comprobado que estos valores se obtienen haciendo el cuadrado al coeficiente de determinación por lo que la aplicación de la expresión (1) se ha hecho de manera incorrecta, entendemos que de forma no intencionada.

Pison et al. (2014) analiza la rentabilidad de los recursos propios de 19 bancos comerciales españoles desde 2006 a 2012 utilizando las variables CAMEL como variables explicativas. Para el análisis de la colinealidad muestra la matriz de correlaciones y el FIV resultando siempre valores inferiores a 0,75 y 4, por lo que se concluye que no existe colinealidad.

Mekonnen et al. (2015) utilizan las variables del modelo CAMEL para analizar la rentabilidad de los activos y la rentabilidad de los recursos propios en bancos de Etiopia, concluyendo que no existen problemas de colinealidad basándose exclusivamente en valores del FIV mayores que 10 y sin presentar tabla alguna que recoja dichos datos.

Salhuteru and Wattimena (2015) analizan la influencia de las variables CAMELS en la gestión de ingresos utilizando la regresión múltiple. Analizan la existencia de colinealidad mediante el VIF obteniendo valores menores que 10 pero en algunos casos mayores que 4, por ejemplo el caso de RORA(Swasta) cuyo VIF toma el valor 9,487 por lo que las conclusiones de este trabajo podrían estar influenciadas por la existencia de colinealidad ya que se estima mediante MCO.

Tovar (2015) usa una muestra de 37 bancos 14 de los cuales emitieron deuda secundaria durante el período de diciembre de 2008 a septiembre de 2012 y usa las variables CAMEL para analizar una serie de hipótesis. Este trabajo muestra la matriz de correlaciones y reconoce la existencia de relación fuerte entre las variables por lo que al diseñar cada uno de los modelos de regresión las variables son elegidas cuidadosamente para evitar colinealidad aunque no presenta ninguna medida que corrobore su inexistencia.

Yuksel et al. (2015) analizan las calificaciones crediticias de 20 bancos de Turquía para los años 2004-2014 a partir de 21 variables basadas en el modelo CAMELS y reconocen la existencia de colinealidad que corrigen eliminando variables explicativas de la regresión. La conclusión que obtienen es que los componentes de calidad del activo, administración y sensibilidad afectan a las calificaciones crediticias mientas que las variables capital y rentabilidad, no afectan. Estas conclusiones podrían verse alteradas con un tratamiento adecuado de la colinealidad.

Hasan et al. (2016) analiza la diferencia de intercambio por defecto de crédito de 161 bancos globales en 23 países usando como variables explicativas las del modelo CAMELS junto con otras estructurales. Inicialmente, los autores dicen que seleccionan las variables que tienen mayor número de observaciones para mitigar la colinealidad, además calculan el FIV obteniendo valores menores que 10 aunque no muestran los datos y no se puede saber si están por encima de 4. Posteriormente, introducen una variable dicotómica “Crisis” y los términos de interacción para investigar el impacto de la crisis financiera sobre los determinantes de los diferenciales. Las variables estructurales y los indicadores CAMELS interactúan con Crisis y para evitar problemas de multicolinealidad, le restan la media a cada variable antes de construir los términos de interacción. Wanke et al. (2016) analiza la eficiencia de 128 bancos de países de la OECD desde 2004 a 2013 usando las variables del modelo CAMELS y utilizando la metodología TOPSIS. Para ello, realiza en primer lugar un análisis de componentes principales y análisis de componentes independientes para reducir el número de variables y eliminar la colinealidad. De esta manera, reconoce la existencia de colinealidad aunque no muestra resultados de ninguna medida de detección.

4. CONCLUSIONES

Las variables del modelo CAMELS son utilizadas en numerosos trabajos como variables explicativas dentro de un modelo de regresión y aunque la existencia de colinealidad entre ella es evidente, en la mayoría de los trabajo no se analiza. En la revisión bibliográfica realizada se observa que no consideran el problema de multicolinealidad como prioritario, debido a que se dice que el modelo CAMELS se utilizará para predecir. Sin embargo, algunos trabajos reconocer que afecta a los resultados.

Se ha encontrado que en la mayoría de trabajos utilizan el FIV para detectar el posible problema de multicolinealidad, sin embargo, no se han encontrado referencias que usen el NC. Este hecho es relevante ya que el NC puede captar colinealidad no detectada por el FIV. En cuanto al tratamiento de la colinealidad, algunos trabajos eliminando las variables causantes, lo que puede eliminar también parte del interés del trabajo a realizar. Otros aplican transformaciones sobre los datos: aplicando una escala logarítmica o centrando los datos lo que puede no resultar valido, Aiken et al. (1991). El único método alternativo de estimación que se ha encontrado es el de componentes principales utilizado como paso previo para la metodología TOPSIS. No se han encontrado referencias que apliquen otras metodologías de estimación alternativas como el cresta, el alzado, etc. Como futura línea de investigación se pretende aplicar estas metodologías en la estimación del modelo CAMELS.

ACKNOWLEDGMENTS

To my tutors, Catalina García y Román Salmerón for his great help and collaboration in each moment the consultation and support in this research work.

REFERENCES

Aiken. L, West. S, Reno. R. (1991). "Multiple Regression: Testing and Interpreting Interactions". Business & Economics: Sage.

Berk, K.N. (1977). "Tolerance and condition in refression computations". Journal of the American Statistical Association, 71, 863-866.

Besley. D.A, Kuh. E, Welsch. R.E. (1980). Regression diagnostics. New York, Estados Unidos: J. Wiley & Sons.

Cortina, R.E. (2009). "Metodología de asignación de cupos de emisor con base al modelo Camel aplicado a Entidades Financieras". Contribuciones a la Economía, 10.

Gasbarro. D, Sadguna. IGM, Zumwalt. JK. (2002). "The Changing Relationship Between CAMEL Ratings and Bank Soundness during the Indonesian Banking Crisis". Review of Quantitative Finance and Accounting, 19, 247-260.

Gujarati. D. (2003). Basic Econometrics (4° edición). New York, Estados Unidos: Mc Graw Hill.

Hasan. I, Liu. L, Gayan. Z. (2016). "The Determinants of Global Bank Credit- Default- Swap Spreads". J Finance Serv Res, 50, 275-309.

Johnston. J., Dinardo. J. (2001). Métodos de Econometría. Barcelona, España: Vicens-Vives.

Karminsky. A, Kostrov. A, Murzenkov. T. (2012). "Comparison of default probability models: russian experience". National Research University Higher School of Economics.

Marquardt. DW. (1970). "Generalized Inverses, Ridge Regression, Biased Linear Estimation, and Nonlinear Estimation". Technometrics, 12 (3), 591-611.

Mekonnen. Y, Kedir. H, Shibru. M. (2015). "Soundness of Ethiopian Banks". International Journal of Finance & Banking Studies, 4 (2)

Nurazi. R, Evans. M. (2005). "An Indonesian Study of the Use of CAMELS Ratios as Predictors of Bank Failure". Journal of Economic and Social Policy, 10 (1).

Okoth. V, Berhanu. G. (2013). "Determinants of Financial Performance of Commercial Banks in Kenya". International Journal of Economic and Financial Issues, 3 (1), 237-252.

Pison. I, Cibrán. P, Ntoung. L. (2014). "Financial performance after the Spanish banking reforms: a comparative study of 19 commercial banks". Risk governance & control: financial markets & intitutions, 4 (2).

Salhuteru. F, Wattimena. F. (2015). “Bank performance with CAMELS Ratios towards earnings management practices In State Banks and Private Banks”. Advances in Social Sciences Research Journal, 2(3), 301-314.

Tovar. E.D. (2015). “Market Discipline Through subordinated debt in Mexican banks”. Revista de Economía Aplicada, 68, 61-80.

Yuksel. S, Dincer. H, Hacioglu. U. (2015). “CAMELS-based Determinants for the Credit Rating of Turkish Deposit Banks”. International Journal of Finance & Banking Studies, 4(4).

Wanke. P, Azad. A.K, Barros. C.P. (2016). “Efficiency factors in OECD banks: A ten- year analysis”. Expert Systems with Applications, 64, 208-227.

APPLICATIONS OF THE GLM CONWAY-MAXWELL-POISSON REGRESSION MODEL ON THE ANALYSIS OF FINANCIAL AND ACTUARIAL DATASETS

Antonio J. Sáez-Castillo
Universidad de Jaén, Department of Statistical and Operational Research, Spain
ajsaez@ujaen.es

Antonio Conde-Sánchez
Universidad de Jaén, Department of Statistical and Operational Research, Spain
aconde@ujaen.es

ABSTRACT

The GLM Conway-Maxwell-Poisson regression model is adapted in order to admit covariates in the dispersion parameter. It permits to distinguish over- and under-dispersed subsets depending on the different covariates values, which may provide interesting applications in different fields. Two examples illustrate that methodology in financial and actuarial datasets.

1. BACKGROUND

The Conway-Maxwell-Poisson family of distribution (from now on, CMP) has been commonly described in terms of a location and a dispersion parameters, θ and ν, respectively, and includes under-dispersed and over-dispersed distributions when $\nu>1$ and $\nu<1$, respectively.

Recently, Huang (2017) has described a new parametrization in terms of the mean of the distribution, μ, instead of the location parameter. He demonstrates that this $\mathrm{CMP}(\mu, \nu)$ distribution is a member of the two-parameter exponential family, which makes it immediately adaptable to regression modelling via the generalized linear model (GLM, McCullagh and Nelder, 1989) framework.

This model can be estimated by MLE simply optimizing its corresponding log-likelihood functions constrained to the equation that defines the mean. Huang (2017) has also found interesting theoretical results which provide common inference results.

In this work, we consider the introduction of covariates in the dispersion parameter, permitting to identify over- or under-dispersion within the dataset from the different value of the covariates. For that, we have implemented R scripts to obtain empirical results with covariates in μ and ν.

2. APPLICATIONS

The possibility to distinguish over- and under-dispersion in a dataset is especially interesting if we consider the interpretation of both phenomena:

- Under-dispersion is commonly associated to a negative contagion effect, that is, a count event decreases the probability of a new count event.

- Over-dispersion may be associated to a positive contagion effect or to individual heterogeneity in the dataset.

What we analyze in this work is two examples in the field of financial and insurance where, in principle, under-dispersion is expected, while a more profound analysis with the GLM CMP regression model permits to identify a subset of individuals affected by over-dispersion, probably due to individual heterogeneity.

2.1. Application to financial data. Takeover bids

The response variable of the dataset is the number of bids received by 126 U.S. firms that were targets of tender offers from 1978 to 1985. The dataset was published originally from Jaggia and Thosar (1993) and used as examples in Cameron and Trivedi (2013) and Saez-Castillo and Conde-Sanchez (2013), between others. These authors emphasized the under-dispersion of the global dataset and proposed theoretical models which took into account that characteristic.

Now we have found that, regardless of the global under-dispersion, there exists, for example, a subset of 23 companies which must be fitted as over-dispersed with a GLM CMP: this is exactly the subset of firms which suffered a real restructuring along the observation period. That result emphasizes that these companies may be affected by a positive contagion or, what is perhaps more probably, are affected by an important individual heterogeneity.

2.2. Application to actuarial data. Insurance claims

Data consist of 64 policyholders of an insurance company who were exposed to risk, where the response variable is the numbers of car insurance claims made by those policyholders in the third quarter of 1973 (Baxter et al., 1980).

A first analysis with a CMP GLM with constant dispersion parameter leads to an under-dispersed model, which supposes to admit that a claim implies a decrease in the probability of a new claim. But again, if we fit a new model with covariates in the dispersion parameter we can obtain important differences in the variability structure of the dataset.

A model including as covariates the size of city and the type of car (both categorical) identifies 24 policyholders with an over-dispersed distribution instead an under-dispersed one. In this group, for example, there are a quite higher frequency of small cars in major cities than in the entire population.

3. CONCLUSIONS

The GLM CMP regression model for count data developed by Huang (2017) supposes a much more interpretative statistical framework to fit real data than the traditional CMP model, in which the covariates are introduced in the location parameter.

The possibility to introduce covariates also in the dispersion parameter offers a wide range of possibilities in a more profound analysis of datasets with a regression analysis, since it permits to discriminate different stochastic generating data mechanisms in terms of covariates.

REFERENCES

Baxter, L. A., Coutts, S. M. and Ross, G. A. F. (1980). "Applications of linear models in motor insurance". In Proceedings of the 21st International Congress of Actuaries, Zurich, 11–29.

Cameron, A.C. and Trivedi, P.K. (2013). Regression Analysis of Count Data. Cambridge University Press, second edition.

Huang, A. (2017). "Mean-parametrized Conway–Maxwell–Poisson regression models for dispersed counts". Statistical Modelling, 17 (6), 359-380.

Jaggia, S. and Thosar, S. (1993). "Multiple Bids as a Consequence of Target Management Resistance". Review of Quantitative Finance and Accounting, 447-457.

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, Second Edition, Chapman & Hall/CRC Monographs on Statistics & Applied Probability.

Saez-Castillo, A.J. and Conde-Sanchez, A. (2013). "A hyper-Poisson regression model for overdispersed and underdispersed count data". Computational Statistics and Data Analysis, 61, 148-157.

UN ANÁLISIS DE SENSIBILIDAD BAYESIANA APLICADA A DISTINTOS PRINCIPIOS DE PRIMAS

Marta Sánchez-Sánchez
Universidad de Cádiz
Dpto. Estadística e Investigación Operativa
marta.sanchez@uca.es

Miguel Ángel Sordo
Universidad de Cádiz
Dpto. Estadística e Investigación Operativa
mangel.sordo@uca.es

Alfonso Suárez-Llorens
Universidad de Cádiz
Dpto. Estadística e Investigación Operativa
alfonso.suarez@uca.es

Emilio Gómez-Déniz
Universidad de las Palmas de Gran Canaria
Dpto. Métodos Cuantitativos
emilio.gomez-deniz@ulpgc.ese

ABSTRACT

En este artículo discutimos una nueva metodología que nos permite obtener una clase de distribuciones a priori basadas en funciones de distorsión que facilita el cálculo de cotas superiores e inferiores para las primas Bayes. Esta nueva metodología se basa en las propiedades de los principios de primas de preservación de ordenaciones estocásticas y de las funciones de distorsión que usemos para cuantificar la incertidumbre asociada a una distribución a priori específica. Aplicamos el método al conocido modelo gamma-Poisson

1. INTRODUCCIÓN

Dado X una variable aleatoria que representa el riesgo en actuariales, el principio de cálculo de la prima asociado al riesgo X es una función $H[X]$ que proyecta el valor aleatorio X en un número real, es decir, la prima. Esta prima se calcula para compensar a la aseguradora por asumir el riesgo de un pago. Los actuarios describen en la literatura un conjunto de propiedades deseables para los principios de prima: principio de independencia, carga de riesgo, carga de riesgo no justificada, estafa, invarianza de escala, aditividad, subaditividad, superaditividad, monotonicidad, preservación de ordenaciones estocásticas, etc. Entre los principios de primas clásicos se encuentran el principio de prima neta, del valor esperado, de utilidad Exponencial, de Esscher, Holandesa, de varianza, de desviación típica, de Wang, etc. Para más información véase Kamps (1998).

A partir de la modelación aleatoria del riesgo, se observa frecuentemente que el riesgo X suele depender de un parámetro θ; el cual caracteriza los riesgos individuales dentro de una misma clase o perfil de riesgo. Por tanto, tiene una función de densidad, $f(x|\theta)$, donde el parámetro pertenece a un espacio paramétrico Θ. En general, los actuarios consideran que el valor del parámetro es cuantificable a partir de la información pasada, asumiendo una distribución a priori del parámetro y resolviendo el problema a través de un enfoque bayesiano, véase por ejemplo Eichenauer et al. (1988), Heilmann (1989), Klugman et al. (1998), Makov et al. (1996), entre otros.

A partir del enfoque anterior, denotaremos la distribución a priori del parámetro como π, cuyo soporte es Θ, conocida como la función estructura en el mundo actuarial. Entonces, para un valor fijo del parámetro, la variable aleatoria condicionada, $[X|\pi=\theta]$, denotada X_θ, simboliza el riesgo aleatorio dado un valor particular del parámetro. A través del teorema de Bayes y la experiencia proporcionada por una muestra, $x=(x_1,\ldots,x_n)$, de la siniestralidad, se obtiene la distribución a posteriori del parámetro, π_x, cuya densidad viene dada por la expresión $\pi_x(\theta)= l(\theta\,|\,x)\pi(\theta)/m(x)$, donde $m(x)$ y $l(\theta\,|\,x)$ corresponden a la densidad marginal y a la función de verosimilitud, respectivamente. Bajo este punto de vista, la prima H que proyecta el valor del riesgo condicionado X_θ en un número real $H[X_\theta]$ hereda la dependencia del parámetro y se conoce como prima de Riesgo basada en H, denotado por$H[X_\theta]=P_{R,H})(\theta)$. Desde un punto de vista Bayesiano, se observa fácilmente que la prima de Riesgo corresponde a un nuevo riesgo aleatorio, debido a que es la resultante de la transformación de la creencia, $P_{R,H}(\pi)$. Por tanto, siguiendo un mismo razonamiento, la prima de la prima de riesgo aleatoria, conocida como prima colectiva, denotada por $H^* [P_{R,H} (\pi)]=P_{C,H,H^*} (\pi)$, cuantifica el riesgo a priori asociado a un colectivo modelado por una creencia a priori. Análogamente, una vez obtenida la distribución a posteriori, y siguiendo un enfoque idéntico, se calcula la prima de Bayes o individual, denotada por $H^* [P_{R,H} (\pi_x)]=P_{B,H,H^*}) (\pi_x)$, la cual cuantifica el riesgo una vez incorporada la experiencia muestral individual. Es importante tener en cuenta que H y H^* no tienen porque seguir el mismo principio de prima.

A partir del desarrollo anterior, la principal crítica y desventaja del modelo Bayesiano se encuentra en la dificultad de modelar el conocimiento a priori. La distribución a priori es difícil de determinar y en muchas ocasiones es criticada por haber sido elegida en función de la "tratabilidad" matemática, más que por la propia adecuación al problema, véase Gray y Pitts (2012). Muchos estudios se basan en usar clases de distribuciones a priori, Γ , que pivotan sobre una información base a priori, π, y en estudiar el rango de alguna característica de estas clases acorde a la incertidumbre del problema. Estas clases permiten afrontar las críticas en cuanto a la parcialidad y a la arbitrariedad en la elección de π. Algunos ejemplos clásicos de clases de distribuciones a priori son las familias paramétricas, las clases contaminadas, las bandas de densidad, las densidades con algún percentil especifico, las bandas de distribución, etc. Adicionalmente, y por la misma razón, la elección de los principios de prima puede ser criticada por su arbitrariedad.

El propósito principal de este trabajo es abordar el problema anterior a partir de una clase de distribuciones a priori, banda distorsionada, recientemente publicada en Arias et al. (2016). Esta clase permite cuantificar la incertidumbre sobre la función de estructura y los principios de primas. Finalmente mostramos un ejemplo que complementa los resultados obtenidos por Gómez-Déniz et al, (1999) para el modelo gamma-Poisson.

2. CLASE DISTORSIONADA

Las propiedades de la clase distorsionada dependen de los siguientes ordenes estocásticos. Sea X e Y dos variables aleatorias con función de distribución F y G, y con funciones de densidad (o densidades discretas) f_X y f_Y con soportes $supp(fX)$ y $supp(f_Y)$, respectivamente.

X es menor que Y en el orden estocástico (también conocido como primer dominancia de primer orden), denotado por $X \leq_{st} Y$, si $E[\Phi(X)] \leq E[\Phi(Y)]$ para toda función no decreciente $\Phi: R \rightarrow R$ tal que existe el valor esperado.

X es menor que Y en el orden razón de verosimilitud, denotado por $X \leq_{lr} Y$, si $f_Y(t)/f_X(t)$ es creciente en la unión de los soportes de X e Y (como convenio a/0 igual a ∞ para todo $a>0$).

La siguiente implicación es bien conocida en la literatura, ver Muller y Stoyan (2002), Shaked y Shanthikumar (2007) y Whitt (1985), para más detalles:

$$X \leq_{lr} Y \Longrightarrow X \leq_{st} Y. \qquad (1)$$

La banda distorsionada ha sido definida recientemente en Arias et al. (2016) y cumple los requisitos recomendados en Berger (1994) para la definición de clases de distribuciones a priori. Primero, la elección y la interpretación son sencillas. Segundo, usando diferentes métricas podemos cuantificar la incertidumbre a priori. Finalmente, el rango de la clase se calcula fácilmente a partir de las distribuciones de los extremos que definen la clase. En cuanto a la elección y la interpretación, esta clase se basa en el concepto de función de distorsión introducido en actuariales en Denneberg (1990), Wang (1995) y Wang (1996). Dicho concepto permite evaluar la incertidumbre asociada a una determinada función de distribución. Focalizando en nuestro problema, si $F_\pi(\theta)$ denota la función de distribución de la función estructura π, una función de distorsión h, i.e., una función continua no decreciente $h:[0,1] \rightarrow [0,1]$ tal que $h(0)=0$ y $h(1)=1$ distorsiona $F_\pi(\theta)$ a través de la expresión:

$$F_{\pi h}(x) = h \circ F_\pi(x) = h[F_\pi(x)],$$

representando la incertidumbre en el conocimiento a priori a través de una perturbación. Obsérvese que $F_{\pi h}$ es una función de distribución asociada a una nueva variable aleatoria $X_{\pi h}$ simbolizando el conocimiento a priori distorsionado.

Como podemos ver en Denuit et al. (2005) existen diversos criterios para elegir h a la hora de distorsionar el conocimiento a priori. En Arias et al. (2016) consideran funciones de distorsión cóncavas o convexas porque representan un cambio en magnitud y variabilidad en la incertidumbre y además permiten comparar la distribución original con la distorsionada. El siguiente lema (ver Lema 1 en Arias et al. (2016)) es clave en el estudio de las propiedades de la clase distorsionada.

Lema 1: Sea π una información específica a priori con función de distribución F_π (absolutamente continua o discreta) y sea h una función de distorsión convexa (cóncava) en [0,1]. Entonces $\pi \leq_{lr} (\geq_{lr}) \pi_h$.

A partir de todo la anterior, dada una función de estructura π y dadas h_1 y h_2 dos distorsiones cóncava y convexa, respectivamente, se define la clase de distribuciones a priori, banda distorsionada, asociada a π, como:

$$\Gamma_{h1,h2,\pi)} = \{\pi' : \pi_{h1} \leq_{lr} \pi' \leq_{lr} \pi_{h2}\}. \quad (2)$$

Obviamente, por el Lema 1, π está incluida en la clase. Por tanto, la banda distorsionada es un entorno de π donde las cotas inferior y superior vienen dadas por las distribuciones distorsionadas de π a partir de h_1 y h_2, respectivamente. Además, las distribuciones a posteriori heredan el ordenamiento en razón de verosimilitud, i.e., para todo $\pi' \in \Gamma_{h1,h2,\pi}$, se obtiene que

$$\pi_{h1,x} \leq_{lr} \pi'_x \leq_{lr} \pi_{h2,x}. \quad (3)$$

Por tanto, las distribuciones a posteriori de las cotas superior e inferior de la banda distorsionada son también las cotas superior e inferior de todas las distribuciones a posteriori de la banda.

Con respecto a cuantificar el tamaño de la incertidumbre a priori, es frecuenta usar una métrica de probabilidad, ver Basu y DasGupta (1995). En Arias et al (2016) se consideran las métricas de Kolmogorov y Kantorovich para medir la distancia entre una función de distribución y su versión distorsionada. Para nuestro propósito, recordamos la definición de la métrica de Kantorovich:

$$KW(X,Y) = \int_{-\infty}^{\infty} |F_X(x) - F_Y(x)| dx). \quad (4)$$

Si dos variables aleatorias están ordenadas en el $\leq_{lr}$, se comprueba en Arias et al (2016) que la distancia de Kantorevich se corresponde con la diferencia de los valores esperados. Por tanto, la distancia $KW(\pi_{h_1},\pi_{h_2})$, permite cuantificar el “tamaño” de la incertidumbre a partir de la diferencia de sus esperanzas. Además, es fácil de ver que $KW(\pi_{h_1}),\pi_{h_2}) = KW(\pi,\pi_{h_2}) + KW(\pi_{h_1}),\pi)$, lo que nos permite decidir cuál de los extremos contribuye con mayor incertidumbre.

3. APLICACIÓN: MODELO GAMMA – POISSON

Supongamos que el número de reclamaciones (el riesgo) sigue una distribución Poisson con parámetro $\theta>0$, $X_\theta \sim P(\theta)$, y sea π una distribución gamma con parámetro de forma $a>0$ y parámetro de escala $b>0$, $\pi \sim G(a,b)$ y con función de densidad

$$\pi(\theta)= \frac{b^a}{\Gamma(a)}\theta^{a-1} e^{-b\theta}.$$

Es de sobra conocido, familias conjugadas, que la distribución a posteriori es una distribución gamma con parámetros $\pi_x \sim G(n\bar{x}+a,b+n)$.

Respecto a los principios de prima, consideramos primero una prima H tal que $H[P(\theta)]=P_{R,H}(\theta)$, sea creciente en el parámetro θ. Esta última suposición no es irreal debido a que el parámetro de la Poisson corresponde al número medio de reclamaciones. Segundo, consideramos un principio de prima H^* que preserve la ordenación estocástica, i.e., si $X\leq_{st} Y$, entonces $H(X)\leq H(Y)$, en Gómez-Déniz y Sarabia (2008) se muestra un listado de principios de prima que satisfacen dicha propiedad. Entonces, a partir de la definición de la clase distorsionada dada en (2), usando la propiedad de preservación del orden en razón de verosimilitud dada en (3) y la cadena de implicaciones dada en (1), se obtiene fácilmente que todas las primas colectivas y bayesianas quedarían acotadas entre las correspondientes asociadas a las distribuciones distorsionas a priori y posteriori, respectivamente. Por ejemplo, en nuestro caso, es fácil de ver que dadas H y H^* como las primas netas, se cumplen los dos requisitos anteriores. De forma general, esto también ocurrirá en distribuciones de la familia exponencial paramétrica, donde siempre es posible encontrar una reparametrización adecuada donde el parámetro represente el valor medio, valor Jewell (1974). En la Tabla 1 mostramos los valores de las primas colectivas y bayesianas para distintas combinaciones de primas H y H^* que satisfacen las propiedades anteriormente descritas.

Tabla 1. Valores para las primas dependiendo del principio de prima

H–H*	Neta - Neta	Esscher - Neta	Utilidad exponencial -Neta
Prima Colectiva	$\frac{a}{b}$	$e^{\beta}\frac{a}{b}$	$(e^{\beta-1})\frac{a}{b^2}$
Prima Bayes	$\frac{a+n\,\bar{x}}{b+n}$	$e^{\beta}\frac{a+n\,\bar{x}}{b+n}$	$(e^{\beta-1})\frac{a+n\,\bar{x}}{b+n}$

Entonces, usando el argumento anteriormente descrito tenemos que para toda distribución a priori en la banda distorsionada, $\pi' \in \Gamma_{h1,h2,\pi'}$, se verifica que:

$$\boldsymbol{P}_{C,H,H^*}(\boldsymbol{\pi}_{h1}) \leq \boldsymbol{P}_{C,H,H^*}(\boldsymbol{\pi}') \leq \boldsymbol{P}_{C,H,H^*}(\boldsymbol{\pi}_{h2}),$$

$$\boldsymbol{P}_{B,H,H^*}(\boldsymbol{\pi}_{h1,x}) \leq \boldsymbol{P}_{C,H,H^*}(\boldsymbol{\pi}'_{x}) \leq \boldsymbol{P}_{C,H,H^*}(\boldsymbol{\pi}_{h2,x}).$$

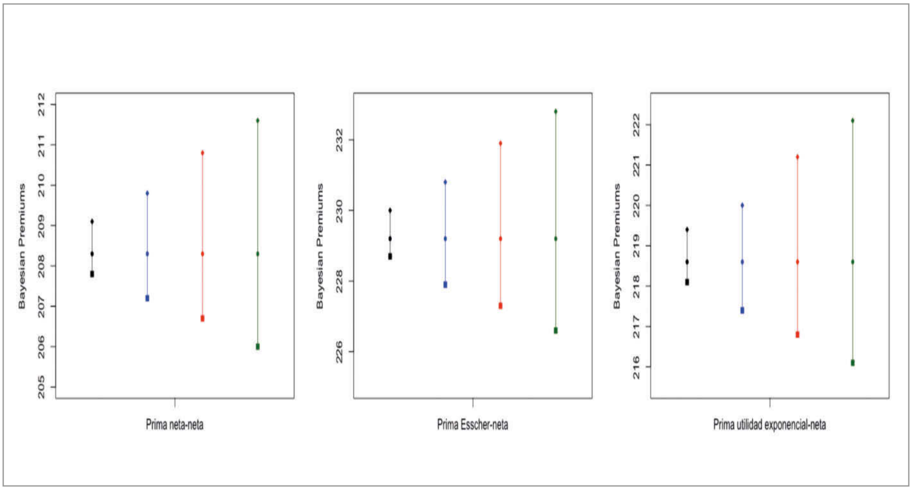
Las desigualdades anteriores nos permiten estudiar la robustez del problema a partir de una distorsión de la función estructura gamma. Además, las cotas inferiores y superiores pueden interpretarse en sistema de tarificación Bonus-Malos. A partir de ahora, usaremos los datos proporcionados en Gómez-Déniz et al (1999). En primer lugar, supondremos fijo el valor esperado de la cantidad de reclamaciones, 100 u.m. y los parámetros de forma y escala de la función estructura son 5 y 2, i.e. $\pi \sim G(5,2)$, modelando a priori 2,5 siniestros en término medio. Tomaremos el tamaño muestral n=10 y consideraremos dos ejemplos dependiendo del promedio muestral de siniestros, $\bar{x} = 2$ y 5.

Por otra parte, introducimos la incertidumbre sobre la función estructura a partir de las clásicas distorsiones cóncavas y convexas, h1 y h2, respectivamente, dadas por las funciones potencias y definidas como

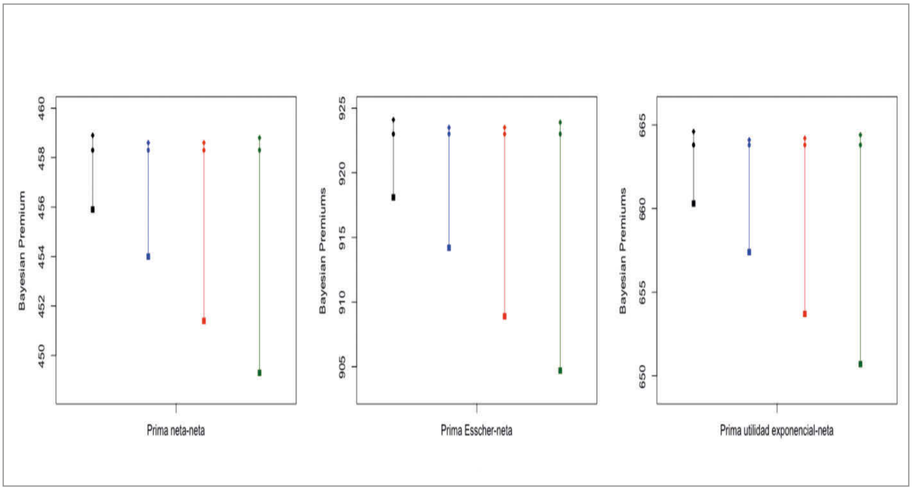
$$h_1(x) = 1-(1-x)^{\alpha} \text{ y } h_2(x) = x^{\alpha},\ x \in [0,1].$$

Para observar varios escenarios tomaremos distintos valores del parámetro de distorsión $\boldsymbol{\alpha}$ ($\boldsymbol{\alpha} = \mathbf{1.05, 1.1, 1.15 \text{ y } 1.2}$). Finalmente, fijamos $\boldsymbol{\beta}\mathbf{=0.0953}$, el parámetro de aversión al riesgo presente en los principios de prima Esscher y utilidad exponencial, para que presente diferencias significativas con los valores de la prima neta. Las Gráficas 1 y 2 muestran los resultados:

Gráfica 1. Rango de las primas Bayes basadas en los distintos principios de primas de la Tabla 1 con $\bar{x}=2$ con $\alpha=1.05$ (línea negra), $\alpha=1.1$ (línea azul), $\alpha=1.15$ (línea roja) y $\alpha=1.2$ (línea verde) y $n=10$ para el modelo gamma-Poisson. El cuadrado representa $\pi_{h_1,x}$), el circulo representa π_x y el rombo representa $\pi_{h_2,x}$



Gráfica 2: Rango de las primas Bayes basadas en los distintos principios de primas de la Tabla 1 con $\bar{x}=5$ con $\alpha=1.05$ (línea negra), $\alpha=1.1$ (línea azul), $\alpha=1.15$ (línea roja) y $\alpha=1.2$ (línea verde) y $n=10$ para el modelo gamma-gamma. El cuadrado representa $\pi_{h_1,x}$, el círculo representa π_x y el rombo representa $\pi_{h_2,x}$.



4. CONCLUSIONES

Observamos que el efecto de la distorsión es similar para todas las combinaciones de primas consideradas, concluyendo cierta robustez en la elección de las primas. De forma general, se observa que el rango de las primas Bayesianas es mayor cuando la incertidumbre en la distribución a priori base es mayor, es decir, a medida que aumenta α. Por otra parte, otras simulaciones muestran que el rango de los extremos de la clase decrece cuando el valor medio muestral del número de reclamaciones es cercano al esperado a priori, i.e., 2,5. Finalmente, es interesante observar como en todos los casos la distorsión cóncava contribuye más a la incertidumbre cuando el valor medio muestral del número de reclamaciones es menor que 2,5, mientras que para valores mayores es la distorsión convexa la que más afecta a la incertidumbre.

Al no contar con una expresión cerrada para las funciones de distribución distorsionadas, tampoco tenemos una expresión cerrada para la métrica de Kantorovich. Sin embargo, usando métodos numéricos de aproximación, mostramos en la Tabla 2 el cálculo de la expresión (4) para distintos valores de α.

Tabla 2. Métrica KW dependiendo de los valores del parámetro de distorsión

Métrica KW	α=1.05	α=1.1	α=1.15	α=1.2
$\mathbf{KW}(\pi_{h_2}, \pi_{h_1})$	0.03406	0.06697	0.09875	0.12945
$\mathbf{KW}(\pi_{h_{2,x}}, \pi_{h_{1,x}})$	0.01577	0.02494	0.04352	0.05654

Como es esperado, el tamaño de la incertidumbre disminuye a posteriori. En futuros trabajos se pretende extender esta teoría para que pueda ser aplicada a un mayor número de principios de primas.

ACKNOWLEDGMENTS

Esta investigación ha sido financiada por el Ministerio de Economía y Competitividad (España) con el proyecto MTM2017-89577-P y por los fondos propios de la Universidad de Cádiz.

REFERENCES

Arias-Nicolás, J.P.; Ruggeri, F.; Suárez-Llorens, A. (1988). “A gamma-minimax result in credibility theory”. Bayesian Analysis, 11,4, 1107-1136.

Basu, S.; DasGupta, A. (1995). “Robust Bayesian analysis with distribution bands”. Statistics & Decisions, 13, 333-349.

Berger, J. (1994). “An overview of robust Bayesian analysis (with discussion)”. Test, 3, 5-124.

Denneberg, D. (1990). “Premium calculation: Why standard deviation should be replaced by absolute deviation”. ASTIN Bulletin, 20, 181-190.

Denuit, M.; Dhaene, J.; Goovaerts, M.J.; Kaas, R. (2005). “Actuarial Theory for Dependent Risks”. John Wiley & Sons, New York.

Eichenauer, J.; Lehn, J.; Rettig, S. (1988). “New classes of priors based on stochastic orders and distortion functions”. Insurance: Mathematics and Economics, 7, 49-57.

Gómez-Déniz, E.; Hernández, A.; Vázquez-Polo, F.J. (1999). “The Esscher premium principle in risk theory: a Bayesian sensitivity study”. Insurance: Mathematics and Economics, 25, 387-395.

Gómez-Déniz, E.; Sarabia, J.M. (2008). “Teoría de la credibilidad: Desarrollo y aplicciones en primas de seguros y riesgos operacionales”. Fundación Mapfre.

Gray, R.J.; Pitts, S.M. (2012). “Risk Modelling in General Insurance”. Cambridge University Press.

Heilmann, W. (1989). “Decision theoretic foundations of credibility theory”. Insurance: Mathematics and Economics, 8, 1, 75-95.

Jewell, W.S. (1974). “Credible means are exact Bayesian for exponential families”. ASTIN Bulletin, 8, 1, 77-90.

Kamps, U. (1998) "On a class of premium principles including the Esscher premium". Scandinavian Actuarial Journal, 1, 75-80.

Klugman, S.; Panjer, H.; Willmot, G. (1998). "Loss Model from Data to Decisions". John Wiley & Sons, New York.

Makov, U.E.; Smith, A.; Liu, Y. et al. (1996). "Bayesian methods in actuarial science". The Statistician, 45, 503-515.

Muller, M.; Stoyan, D. (2002). "Comparison methods for stochastic models and risks". John Wiley & Sons, New York.

Shaked, M.; Shanthikumar, J.G. (2007). "Stochastic orders". Series: Springer Series in Statistics, Springer.

Wang, S. (1995). "Insurance pricing and increased limits ratemaking by proportional hazards transforms". Insurance: Mathematics and Economics, 17, 43-54.

Wang, S. (1996). "Premium calculation by transforming the layer premium density". ASTIN Bulletin, 26, 71-92.

MODELLING DEPENDENT RISKS WITH HEAVY-TAIL MARGINALS

José María Sarabia
Universidad de Cantabria, Departamento de Economía
jose.sarabia@unican.es

Montserrat Guillen
Universidad de Barcelona, Departamento de Econometría, Riskcenter-IREA, España
mguillen@ub.edu

Faustino Prieto
Universidad de Cantabria, Departamento de Economía
faustino.prieto@unican.es

Vanesa Jordá
Universidad de Cantabria, Departamento de Economía
jordav@unican.es

ABSTRACT

In this paper we present a general class of dependent risks, where the individual risks are represented according to second kind beta distributions. The second kind beta distribution, characterized by three parameters, is considered a suitable model in risk management because it has heavy tails and many of the risk measures are available in a closed form. First, we define a vector of dependent risk, where the marginals are second kind beta distributions and the dependence arises in the form of a common factor. One of the advantages of the multivariate model is that many of its features (risk measures, risk aggregation, etc.) can be easily obtained by simulation. We study the basic properties of the new dependent risks model, including the joint density function, marginal and conditional distributions. We also discuss the aggregated random variable for which we obtain analytic formulas for the probability density function. After describing the estimation of

the model by maximum likelihood, we provide initial estimates for the parameters based on the method of moments. We illustrate the performance of the model with an application with a data set containing information about losses from individual insurance policies.

1. INTRODUCTION

In risk management, Value-at-Risk (VaR) and Tail Value-at-Risk (TVaR) are among the most widely used measures. Practitioners have commonly assumed that losses are normally distributed because these indicators have simple formulas under the normality framework. The data, however, suggest that the distribution of losses presents heavier tails. To circumvent this limitation, the t-Student distribution has been often considered to represent the distribution of losses, which also allows for straightforward calculations of risk indices. The essential objection to elliptical distributions is the symmetry forced upon the marginals. Insurance losses conventionally present a heavy right tail, hence we should consider a model which is able to accommodate positive skewness and Pareto tail behavior.

Risk aggregation is a key aspect in actuarial sciences which still represents a challenge in this field. Aggregation includes every function that combines a multivariate random vector into a single random variable. Although the sum of the losses is the most common type of aggregation, closed formulas for the convolution of random variables are generally intractable, especially when non-elliptical distributions are considered, so approximation algorithms need to be used to evaluate the operational risk in those cases (Wang, 1998). Although the aggregated risk could be easily obtained for independent losses, this assumption is in most cases too restrictive. Dependence is generally introduced via copulas (see, e.g., Arbenz et al.(2012), Bølviken and Guillén (2017), Coqueret (2014), Côté and Genest (2015), Gijbels and Herrmann (2014)) and multivariate distributions including the multivariate Pareto distribution (Sarabia et al., 2016), multivariate mixtures of Exponential distributions (Sarabia et al., 2017) and the multivariate second kind beta distribution (Guillén et al., 2013).

In this paper we present a general class of dependent risks, where the individual risks are represented according to second kind beta distribution, which is able to accommodate a variety of shapes and is characterized by a Pareto heavy-tail behavior. The dependence is introduced in the form of a common factor. We discuss the aggregated random variable for which we obtain analytic formulas for the probability density function. Before presenting an empirical application with real data, we describe the estimation procedure by maximum likelihood and provide initial estimates for the parameters based on the method of the moments.

2. METHODOLOGY

In this section we present a general class of dependent risks where the marginal are represented by the beta distribution of the second kind. A random variable X is said to be a second kind beta distribution, represented by $X \sim B2(p, q, \lambda)$, if its probability density function (PDF) can be written as:

$$f(x; p,q,\lambda) = \frac{x^{p-1}}{\lambda^p B(p,q)\ (1+x/\lambda)^{p+q}}\ ,\ x > 0\,, \tag{1}$$

where p and q are positive shape parameters and λ is a positive scale parameter.

The cumulative distribution function (CDF) of the second kind beta distribution is given by,

$$F(x; p,q,\lambda) = IB\left(\frac{x}{\lambda+x}, p,q\right),$$

where $IB(x; p, q)$ represents the incomplete beta function: $IB(x; p,q) = \frac{\int_0^x t^{p-1}(1-t)^{q-1}\,dt}{B(p,q)}$ and $B(p,q)$ is the Beta function.

This model includes, as particular cases, distributions that have been widely used to model insurance losses. Setting $p = 1$ in (1) the resulting model corresponds to the Pareto II distribution (Arnold, 2015; Asimit et al., 2013). When q is set equal to one, we obtain the inverse Lomax distribution. Besides its flexibility, an additional advantage of this distribution is that closed expressions are available for many of the most popular risk measures.

Let X be a random variable with CDF $FX(x)$, the value at risk at the α level is defined by,

$$VaR[X;\alpha] = inf\{x \in \mathrm{R}, F_X(\mathrm{x}) \geq \alpha\}, 0 < \alpha < 1.$$

For the second kind beta distribution the VaR can be computed using a simple expression. Let $X \sim B2(p, q, \lambda)$ be a second kind beta distribution, the VaR at the α level is given by,

$$VaR[X;\alpha;p,q,\lambda] = \lambda \frac{IB^{-1}(\alpha;p,q)}{1-IB^{-1}(\alpha;p,q)},$$

where $IB^{-1}(\alpha;p,q)$ is the inverse of the incomplete beta function.

The VaR is not coherent as a risk index because it does not satisfy the subadditivity property. The expected shortfall or Tail Value at Risk (TVaR) overcomes this potential limitation and it is considered a more suitable measure to evaluate the operational risk (Acerbi and Tasche, 2002). The TVaR is the conditional tail expectation $TVaR[X;\alpha]=E[X|X>VaR[X;\alpha]]$.

On the other hand, if X is distributed according to a second kind beta distribution, $\mathrm{X} \sim B2(p, q, \lambda)$, the TVaR can be computed as follows,

$$TVaR[X;\alpha;p,q,\lambda] = E[X] \frac{IB^{-1}\left(\frac{\alpha/\lambda}{1+\alpha/\lambda}; p+1, q^{-1}\right)}{1-IB^{-1}\left(\frac{\alpha/\lambda}{1+\alpha/\lambda}; p,q\right)},$$

where $E[X] = \frac{\lambda p}{q-1}$ and $q > 1$.

Now, we introduce a general class of multivariate dependent distributions with second kind beta marginals.

Definition. Let $G_0, G_1, \ldots, G_d$ be mutually independent gamma random variables with distributions, such that $G_0 \sim G_a(q_0)$ and $G_i \sim G_a(p_i)$, i = 1, 2, …, d. The multivariate second kind beta distribution based on a common factor is defined as,

$$(X_1, X_2, \ldots, X_d)^T = \left(\lambda_1 \frac{G_1}{G_0}, \lambda_2 \frac{G_2}{G_0}, \ldots, \lambda_d \frac{G_d}{G_0}\right)^T \tag{2}$$

with $q_0, p_i, \lambda_i > 0, i = 1, 2, \ldots, d$.

One of the advantages of the multivariate model defined by (2) is that many of its features as risk measures, risk aggregation, etc. can be easily obtained by simulation.

By construction the marginal distributions are distributed according to a second kind beta distribution. Hence, X_i is a second kind beta distribution with shape parameters p_i and q_0 and scale parameter λ_i. It should be noted that the random variable G0 introduces the dependence in the model. The joint PDF can be written as,

$$f(x_1,\ldots,x_d) = C\frac{\prod_{i=1}^{d} x_i^{p_i-1}/\lambda_i^{p_i}}{(1+\sum_{i=1}^{d} x_i/\lambda i)^{q_0+\sum_{i=1}^{d} p_i}},\, x_1,\ldots,x_d > 0, \qquad (3)$$

where $C = \frac{\Gamma(q_0+\sum_{i=1}^{d} p_i)}{\Gamma(q_0)\prod_{i=1}^{d}\Gamma(p_i)}$.

An extension of previous distribution can be obtained using the methodology proposed by Sarabia et al. (2014), using the class of multivariate distributions with beta-generated marginals.

2.1. Distribution of the aggregated risks

The distribution of the aggregated risks in (2) has been obtained for some special cases (See Guillén et al., 2013). In the following theorem we obtain a general expression for the distribution of the sum.

Theorem. Let $\boldsymbol{X}$ be a random vector defined by the stochastic representation (2), and consider the distribution of the aggregated risks $S_d = X_1 + \ldots + X_d$. Then, the probability density function of S_d is given by,

$$f_{S_d}(x;\alpha,\lambda) = C\sum_{k=0}^{\infty}\delta_k\frac{x^{\rho+k-1}/\lambda_{(1)}^{\rho+k}}{B(\rho+k,q_0)\left(1+\frac{x}{\lambda_{(1)}}\right)^{\rho+k+q_0}},\quad x>0,$$

and 0 elsewhere, where $\rho = \sum_{i=1}^{d}\alpha_i = min\{\lambda_i\}$. δk are defined by,

$$\delta_k = \frac{1}{k+1}\sum_{i=1}^{k+1} i\gamma_i\,\delta_{k+i-1},\ k = 0,1,2,\ldots$$

and

$$\gamma_k = \sum_{i=1}^{k} p_i\left(1-\frac{\lambda_{(1)}}{\lambda i}\right)\frac{1}{k}',\qquad k = 1,2,\ldots$$

Proof. The proof is based on Moschopoulos (1985).

If $\lambda_1 = \lambda_2 = \ldots = \lambda_d = \lambda$ the distribution of the convolution $S_d = X_1 + \ldots + X_d$ is a second kind beta distribution $S_d \sim B2(p_1 + p_2 + \ldots + p_d, q, \lambda)$ (Guillén et al., 2013). Then, the VaR and the TVaR are available in a simple closed form:

$$VaR[S_X;\ \alpha;\ p, q_0, \lambda] \ = \frac{IB^{-1}\ (\alpha;\ p_1 + p_2 + \cdots + pd,\ q_0)}{1 - IB^{-1}\ (\alpha;\ p_1 + p_2 + \cdots + p_d, q_0)},$$

and

$$TVaR[X;\ \alpha;\ \boldsymbol{p}, q_0, \lambda] \ = E[X] \ \frac{IB^{-1}\ \left(\frac{\alpha/\lambda}{1+\alpha/\lambda} p_1 + p_2 + \cdots + pd, +1, q_0 - 1\right)}{1 - IB^{-1}\left(\frac{\alpha/\lambda}{1+\alpha/\lambda}\ p_1 + p_2 + \cdots + pd, +1, q_0\right)},$$

3. ESTIMATION

Let $\boldsymbol{X_1}, \boldsymbol{X2}, \ldots, \boldsymbol{X_n}$ a random sample of size n from (3). The maximum likelihood estimators of the parameters are given by,

$$(\hat{q}_0, \hat{\boldsymbol{p}}, \hat{\boldsymbol{\lambda}})^T = argmax \sum_{j=1}^{n} log\, f(x_{1j}, x_{2j}, \ldots, x_{dj};\ q_0, p, \lambda),$$

where we have a regular situation. The initial estimators can be obtained by the method of moments. Using the mean, the mode and the variance we obtain the following expressions

$$\lambda_i = \frac{\mu_i^2\ (\mu_i - m_i) - (3m_i - \mu_i)\ \sigma_i^2}{\sigma_i^2 - \mu_i^2 + \mu_i\ m_i} \qquad i = 1,2,\ldots,d, \tag{4}$$

$$q_0 = \frac{1}{d} \sum_{i=1}^{d} \frac{\mu_i + m_i + \lambda_i}{\mu_i - m_i}, \tag{5}$$

$$p_i = \frac{\mu_i}{\lambda_i} \left(\frac{2m_i + \lambda_i}{\mu_i - m_i}\right) \qquad i = 1,2,\ldots, \tag{6}$$

where μ_i, σ_i^2, m_i, are, respectively, the mean, the variance and the mode of the i-th random variable.

4 APPLICATIONS

In this section, we present one application with real data on losses from individual insurance policyholders. The data set contains information about 482 policy holders that have been insured for a maximum of 10 years. They all have two types of policies: a motor insurance coverage and a homeowners coverage. All these cases had claims in the two insurance policies, and they claimed three types of costs: bodily injuries (BI), property damage (PD), and some home insurance (HO) losses. We summarize the aggregated overall loss, but we do not take into account that exposure to risk may have been lower than ten years. We do not correct for inflation, hence these losses are expressed in current monetary units (thousand Euros).

Table 1 provides summary statistics for the three types of losses. The average loss is positive in all cases because the amounts are recorded as positive losses. The three types of costs present large maximum values, which suggests that the distributions present positive skewness and a heavy right tail. To provide a picture of the dependence in the data, Figure 1 shows pair scatterplots on the three types of costs. The data exhibits positive dependence between different types of looses that seems to be particularly strong in the lower tail.

Table 1. Losses on motor insurance (property damage -PD- and bodily injury-BI-) and on homeowners insurance -HO- for a sample of insureds (2007-2016) in thousand Euros

	Minimum	Mean	Median	Maximum	Std. dev.	Skewness	Kurtosis
Motor Insurance							
PD	0.0327	2.5175	1.7640	36.5106	2.8494	5.3195	50.8564
BI	0.0010	7.1370	3.5310	316.3180	22.2857	11.4290	145.2503
Home Insurance							
HO	0.0055	0.5250	0.2093	9.5789	1.0362	5.0511	33.2907

The parameters of the multivariate model in (1) are estimated by maximum likelihood, using Eqs. (4) – (6) to obtain a set of initial estimates. Although sample estimates of the mean and the variance can be computed straightforwardly, the mode cannot be obtained directly from the sample because all the observations present the same frequency. Thus, to compute the mode of each random variable, we grouped each sample into a given number of evenly spaced intervals. The modal class interval is characterized by the highest frequency and hence, the mode was the midpoint of that interval. We computed the initial values for the parameters breaking the data into a different number of intervals to analyze the sensitivity of the estimates to the number of intervals defined.

Figura 1. Pair scatterplots of Losses on motor insurance (third party liability -PD- and bodily injury-BI) and on homeowners insurance -HO for a sample of insureds (2007-2016), in thousand Euros

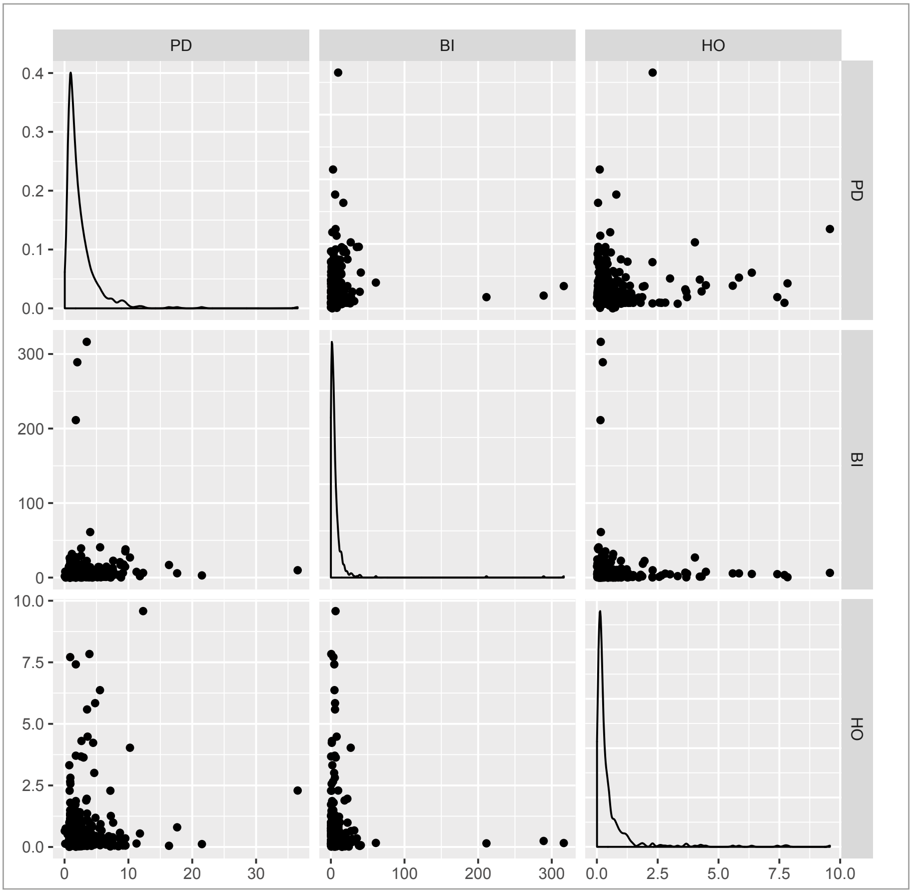


Table 2. Mode and initial estimators obtained from different numbers of intervals used

Number of class intervals	100	500	1000	5000	10000	50000	100000
m_1	0.57988	0.47044	**0.47044**	0.45950	0.45768	0.45768	0.45749
m_2	1.58259	0.31732	**0.15916**	0.09590	0.52292	0.26038	1.50193
m_3	0.10122	0.07250	**0.07250**	0.07059	0.07011	0.06982	0.06978
$\hat{\lambda}_1$	5.7639	7.4398	**7.4398**	7.6248	7.6559	7.6559	7.6590
$\hat{\lambda}_2$	3.2174	7.6357	**8.2006**	8.4273	6.9056	7.8387	3.4938
$\hat{\lambda}_3$	0.4169	0.5446	**0.5446**	0.5533	0.5555	0.5568	0.5570
$\hat{q}_0$	3.0613	3.2769	**3.2796**	3.3013	3.2976	3.3023	3.2818
$\hat{p}_1$	1.5607	1.3853	**1.3853**	1.3707	1.3683	1.3683	1.3681
$\hat{p}_2$	2.5490	1.1335	**1.0625**	1.0367	1.2425	1.1068	2.3555
$\hat{p}_3$	1.8405	1.4691	**1.4691**	1.4501	1.4455	1.4427	1.4422
log l	-2637.12	-2507.83	**-2503.04**	-2504.77	-2520.03	-2509.67	-2630.13

Table 3. Parameter estimates obtained by maximum likelihood (standard error in parenthesis).

$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$	$\hat{q}_0$	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	log l
2.6544 (0.4497)	19.7747 (2.8596)	1.4107 (0.2138)	3.9020 (0.3899)	2.7785 (0.2597)	0.9435 (0.0602)	1.0275 (0.0704)	-2317.78

Table 2 presents the mode of the sample, the initial estimators and the value of the log-likelihood function evaluated at those initial estimators for different number of intervals. We chose the initial estimators with larger value of the log-likelihood function, which corresponds to the estimation based on the definition of 1000 intervals for calculating the mode. Then, maximum likelihood estimates of the parameters were computed by numerical methods, using the R software package optimx and taking those initial values as the seed values. Table 3 shows the parameter estimates obtained and their standard errors.

We evaluate VaR and the TVaR by obtaining 10000 estimates from Monte Carlo simulations with a sample size of 10^6 assuming that the losses are represented according to the multivariate PDF in (3) using the stochastic representation in (2). For each sample, we obtain those risk measures at the 95%, 99% and 99.9% confidence level. We calculate the average value of those 10000 samples, the standard deviation and the 95% confidence level to evaluate the estimation errors of the risk measures (Table 4).

Table 4. Estimates of the VaR and the TVaR under the multivariate second kind beta distribution

	VaR	S.D.	95% CI	TVaR	S.D.	95% CI
95%	27.3059	0.1669	[26.9794, 27.6363]	43.4238	0.3973	[42.6555, 44.1998]
99%	51.1782	0.5701	[50.0718, 52.3080]	75.5307	1.3525	[72.9512, 78.2360]
99.9%	107.914	3.2508	[101.8315, 114.6159]	151.784	7.7371	[138.2546, 168.0159]

5. CONCLUSIONS

In this paper we propose a flexible parametric approach to represent the joint distribution of insurance losses. The multivariate dependent distribution is constructed using a stochastic representation with a common factor, which introduces the dependence in the model. The marginal distributions have heavy tails of the type second kind beta. One of the advantages of this model is that the VaR and TVaR risk measures of the marginals can be calculated in a closed-form. We have also obtained closed expression for the aggregated distribution. As well, many of the main risk measures can also be obtained easily by Monte Carlo simulation. We present an application with three dependent risks.

ACKNOWLEDGMENTS

The authors thanks to the Ministerio de Economía y Competitividad (projects ECO2016-476203-C2-1-P and ECO2016-476203-C2-2-P) for partial support of this work.

REFERENCES

Arbenz, P., Hummel, C. and Mainik, G. (2012) Copula based hierarchical risk aggregation through sample reordering. Insurance: Mathematics and Economics, 51, 122–133.

Arnold, B.C. (2015) Pareto Distributions. Second Edition, CRC and Chapman and Hall.

Asimit, A., Vernic, R. and Zitikis, R. (2013) Evaluating risk measures and capital allocations based on multi-losses driven by a heavy-tailed background risk: the multivariate Pareto-II model. Risks 1, 14–33.

Bølviken, E. and Guillen, M. (2017) Risk aggregation in Solvency II through recursive log-normals. Insurance: Mathematics and Economics, 73, 20-26.

Coqueret, G. (2014) Second order risk aggregation with the Bernstein copula. Insurance: Mathematics and Economics, 58, 150-158.

Côté, M.P. and Genest, C. (2015) A copula-based risk aggregation model. The Canadian Journal of Statistics, 43, 60-81.

Gijbels, I. and Herrmann, K. (2014. On the distribution of sums of random variables with copula-induced dependence. Insurance: Mathematics and Economics, 59, 27-44.

Guillen, M., Sarabia, J. M. and Prieto, F. (2013) Simple risk measure calculations for sums of positive random variables. Insurance: Mathematics and Economics, 53(1), 273-280.

Moschopoulos, P.G. (1985) The Distribution of the Sum of Independent Gamma Random Variables. Annals of the Institute of Statistical Mathematics, 37, 541-544.

Sarabia, J.M., Prieto, F. and Jordá, V. (2014) Bivariate beta-generated distributions with applications to well-being data. Journal of Statistical Distributions and Applications, 1, 1-15.

Sarabia, J.M., Gómez-Déniz, E., Prieto, F. and Jordá, V. (2016) Risk aggregation in multivariate dependent Pareto distributions. Insurance: Mathematics and Economics, 71, 154-163.

Sarabia, J. M., Gómez-Déniz, E., Prieto, F. and Jordá, V. (2017) Aggregation of Dependent Risks in Mixtures of Exponential Distributions and Extensions. preprint arXiv:1705.00289.

Wang, S.S. (1998) Aggregation of correlated risks portfolios: models and algorithms. Proceedings of the Casualty Actuarial Society 85 (163), 848–937.

ENVIRONMENTAL RISK: CLUSTERING COUNTRIES FROM A DISTRIBUTIONAL PERSPECTIVE OF THE MAIN POLLUTANTS

Carmen Trueba
University of Cantabria, Department of Economics, Spain , truebac@unican.es

Lorena Remuzgo
University of Cantabria, Department of Economics, Spain, remuzgol@unican.es

ABSTRACT

In this study, we analysed the evolution of the polarization in the global distribution of the main greenhouse gases: the carbon dioxide, the methane, the nitrous oxide and the fluorinated gases. For this purpose, we apply the multidimensional polarization indices proposed by Gigliariano and Mosler (2009) which are based on the interregional and intraregional components of the multidimensional generalised entropy measures (Maasoumi, 1986; Maasoumi and Nickelsburg, 1988). The analysis has been carried out for the time period 1990-2014, considering an endogenous grouping of countries (Aghevli and Mehran, 1981; Davies and Shorrocks, 1989) and allowing that the groups of countries can be constituted in a different manner in each year.

1. INTRODUCTION

Scientist studies give evidence, from the top of the atmosphere to the depths of the oceans, that the planet is warming, being its main cause the human activity carried out since the mid-nineteenth century. Such increment of the average global temperature is principally due to the burning of fossil fuels.

In this context, the main greenhouse gases (GHGs) are the carbon dioxide (CO_2), the methane (CH_4), the nitrous oxide (N_2O) and the fluorinated gases (F-gases), also known as long-term gases. CO_2 is the most important anthropogenic GHG in the atmosphere and its emissions principally come from the combustion of fossil fuels, the production of cement, the deforestation and other land use change. Around 40 percent of CH_4 is emitted by natural sources –such as wetlands and termites–, while the rest comes from anthropogenic sources –ruminants, rice agriculture, fossil fuel exploitation, landfills and biomass burning. Whereas approximately 60 percent of N_2O emissions are originated by natural sources –oceans and soils–, the remaining 40 percent emerge from anthropogenic sources –biomass burning, fertilizer use and industrial processes (World Meteorological Organization, 2017). Emissions of the main F-gases (hydrofluorocarbons, perfluorocarbons, sulphur hexafluoride and nitrogen trifluoride) are essentially generated in industrial processes.

Given that evidence of a decrease in the polarization in GHG emissions could encourage countries to establish a stronger multilateral climate change agreement, in this paper we study such phenomenon from a multidimensional point of view. For this purpose, we apply the multidimensional polarization indices proposed by Gigliariano and Mosler (2009) which are based on the interregional and intraregional components of the multidimensional generalised entropy measures (Maasoumi, 1986; Maasoumi and Nickelsburg, 1988). In particular, we consider the emissions of the main GHGs: CO_2, CH_4, N_2O and F-gases from 1990 to 2014. It should be noted that the groups of countries are constructed endogenously according to the method proposed by Aghevli and Mehran (1981) and Davies and Shorrocks (1989), allowing that the groups of countries can be constituted in a different manner in each year.

The rest of the paper is organised as follows. The multivariate statistical polarization indices used in this paper are detailed in Section 2. Next, the main results of the analysis are exposed in section 3.

2. METHODOLOGY

In order to construct multivariate polarization indices, we proceed in two stages. In the first phase, the multivariate inequality indices based on the concept of generalised entropy (Maasoumi, 1986; Maasoumi and Nickelsburg, 1988) are obtained.

Let consider the emissions of the main GHGs –CO2, CH4, N2O and F-gases– emitted by N countries. The multivariate inequality indices are given by:

$$GEM_{\gamma}(\boldsymbol{X}) = \frac{1}{\gamma(1+\gamma)} \frac{1}{N} \sum_{i=1}^{N} \left[\left(\frac{S_i}{\bar{S}} \right)^{1-\gamma} - 1 \right], \gamma \neq -1{,}0$$

where the γ parameter symbolises the importance attributed to the emission transfers that may occur in the different parts of the distribution. In such a way, as γ increases, the most polluting countries receive more weight in the index.

We have two special cases when γ is set to -1 –more weight is assigned to the least contaminant countries– and 0 –all countries receive the same importance–. In these scenarios the indices are defined, respectively, as:

$$GEM_{-1}(X) = \frac{1}{N} \sum_{i=1}^{N} \log \left(\frac{\bar{S}}{S_i} \right)$$

and

$$GEM_{-1}(X) = \frac{1}{N} \sum_{i=1}^{N} \log \left(\frac{S_i}{\bar{S}} \right)$$

In all cases, we use a generalised mean of order minus β to sum the different variables:

$$S_i = \left(\sum_{j=1}^{K} \delta_j x_{ij}^{-\beta} \right)^{-1/\beta}, \quad i = 1, \ldots, N,$$

where $\bar{S}$ is the arithmetic mean of the values s_i.

Additionally, the δ_j (j = 1, …, K, $0 \leq \delta_j \leq 1$) parameter denotes the weight assigned to each gas in the overall index, and the β ($-1 \leq \beta \leq \infty$) parameter represents the elasticity of substitution among the different gases.

As all the emissions are expressed in million tonnes of CO2-equivalent ($MtCO_2e$), it is possible to compare directly the effect of all of them. Therefore, it is considered that $\beta = -1$, that is, there is perfect substitution among pollutants. The importance attached to each contaminant is related to the share of the atmospheric concentration of each gas in the year 2011, also measured in CO_2-equivalent using the 100-year GWPs published in the IPCC (1996)[1]. Finally, the γ parameter has been fixed to 1.5 giving more weight to the most polluting countries.

---

1 In particular, the δ parameter takes the value of 0.7391, 0.0954, 0.1623 and 0.0032 for the CO_2, CH_4, N_2O and F-gases, respectively.

Multivariate inequality measures used (GEM_γ, GEM-1 and GEM_0) can be additively decomposable by population sub-groups, allowing the analysis of the level of inequality between and within the different groups considered. Thus, the GEM_γ index can be additively decomposed in the following way:

$$GEM_\gamma(\boldsymbol{X}) = B_\gamma(\boldsymbol{X}) + W_\gamma(\boldsymbol{X})$$

where $B_\gamma(\boldsymbol{X})$ denotes the between-group inequality component:

$$B_\gamma(\boldsymbol{X}) = f\left(\sum_{g=1}^{g} \frac{N_g}{N} h\left(\bar{s}^g, \bar{s}\right)\right)$$

and $W_\gamma(\mathbf{X})$ is the within-group inequality component:

$$W_\gamma(\mathbf{X}) = \sum_{g=1}^{G} w_g f\left(\frac{1}{N_g}\sum_{i\in g} h\left(s^i, \bar{s}^g\right)\right)$$

where N_g is the number of countries which are part of the group g; wg is the weight associated with the group g and, finally, G is the number of groups considered in the analysis. Additionally, f and h functions are continuous functions, being f strictly increasing.

The components of these indices for the different values of the γ parameter, are shown in Table 1, where si and $\bar{s}^g$ are the generalised mean of order minus β and the arithmetic mean of the values si over the countries in group g.

Table 1. Elements of the between- and within-group inequality components

Gamma	$f(y)$	$h(t;t)$	$w_g, g = 1,\ldots G$
$\gamma \neq 0,-1$	$\frac{y}{\gamma(1+\gamma)}$	$\left(\frac{t}{\bar{t}}\right)^{1+\gamma} - 1$	$\frac{N_g}{N}\left(\frac{\bar{S}^g}{\bar{S}}\right)^{1+\gamma}$
$\gamma = -1$	y	$\log\left(\frac{t}{\bar{t}}\right)$	$\frac{N_g}{N}$
$\gamma = 0$	y	$\frac{t}{\bar{t}}\log\left(\frac{t}{\bar{t}}\right)$	$\frac{N_g \bar{S}^g}{N \bar{S}}$

Using the previous decomposition, the multidimensional polarization indices developed by Gigliariano and Mosler (2009) are calculated in a second stage. Thus, keeping the previous notation, three different specifications for the polarization indices are considered[2]:

$$P_1(\mathbf{X}) = \varphi\left(\frac{B(\mathbf{X})}{W(\mathbf{X}) + c}\right) \cdot S(\mathbf{X}),$$

$$P_2(\mathbf{X}) = \varphi\left(B(\mathbf{X}) - W(\mathbf{X})\right) \cdot S(\mathbf{X})$$

$$P_3(\mathbf{X}) = \varphi\left(\frac{B(\mathbf{X})}{B(\mathbf{X}) + W(\mathbf{X}) + c}\right) \cdot S(\mathbf{X}),$$

taking into account that,

$$S(\mathbf{X}) = \left[\left(\prod_{g=1}^{G}\left(\frac{N_g}{N}\right)^{-\frac{N_g}{N}}\right) - 1\right] \cdot \frac{1}{G-1}, \; g = 1, \ldots, G,$$

and $\varphi(\mathbf{X}) = \mathbf{X}$ given that $\varphi(\mathbf{X})$ must be a continuous and strictly increasing function.

3. DATA AND RESULTS

The data have been extracted from the Climate Analysis Indicators Tool database (CAIT, 2015), developed and updated by the World Resources Institute. In order to construct the groups of countries, it has been considered the level of emissions released into the atmosphere by each country in each year. In particular, this analysis is carried out considering the sample divided into six groups because six is the minimum number of groups that allows the between-group inequality component to explain, at least, 70 percent of total inequality in all the years (see Table 2).

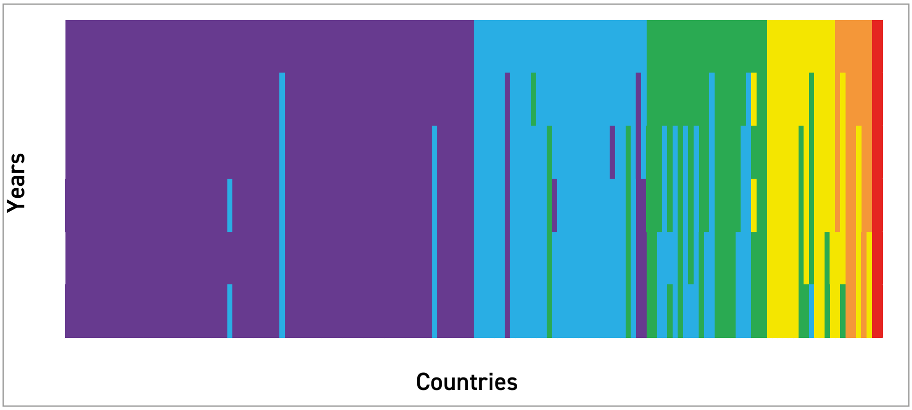
Table 2. Total inequality explained by the grouped distributions

Year	1990	1995	2000	2005	2010	2014
B(X) / GEM(X)	81.97	91.21	91.39	99.33	92.66	83.22

2 The parameter c has to be positive and depends on the values of B(X) and W(X). In this case, the value 0.1 has been considered appropriate

Figure 1 illustrates which countries belong to each group after applying the endogenous method of grouping. Such classification of countries is also detailed in Table 3.

Figure 1. Endogenous classification of countries resulting from the polarization analysis



It should be highlighted that the 50 percent of the countries belong to the first group. By contrast, only two countries –China and the United States– form the last group.

Table 3. Groups of countries determined endogenously by polarisation analysis

Group 1	Afghanistan, Albania, Antigua & Barbuda, Bahamas (The), Barbados, Belize, Benin, Bhutan, Botswana, Brunei, Burkina Faso, Burundi, Cambodia, Cape Verde, Central African Republic, Chad, Comoros, Congo (Dem. Rep.)[1], Congo (Rep.), Cook Islands, Costa Rica, Cote d'Ivoire, Cyprus, Djibouti, Dominica, El Salvador, Equatorial Guinea, Fiji, Gabon, Gambia (The), Ghana, Grenada, Guatemala[2], Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Iceland, Jamaica, Kiribati, Laos, Lesotho, Liberia, Madagascar, Malawi, Mali, Malta, Mauritania, Mauritius, Mozambique, Namibia, Nauru, Nepal, Nicaragua, Niger, Niue, Panama, Papua New Guinea, Paraguay, Rwanda, Saint Kitts & Nevis, Saint Lucia, Saint Vincent & Grenadines, Samoa, Sao Tome & Principe, Senegal, Seychelles, Sierra Leone, Solomon Islands, Sri Lanka[3], Suriname, Swaziland, Togo, Tonga, Uganda, Uruguay, Vanuatu, Zambia[4].
Group 2	Angola, Bahrain, Bangladesh, Bolivia, Bulgaria[5], Cameroon, Chile, Cuba, Denmark6, Dominican Republic, Ecuador, Finland[7], Hungary8, Ireland, Israel[9], Jordan, Kenya, Lebanon[10], Libya[11], Mongolia[12], Morocco, Myanmar, New Zealand, Norway, Oman, Peru, Qatar, Singapore, Sudan, Sweden[13], Switzerland[14], Syria, Tanzania[15], Trinidad & Tobago, Tunisia, Yemen, Zimbabwe[16].
Group 3	Algeria, Argentina, Austria[17], Colombia, Egypt, Greece, Iraq, Korea (Dem. Rep)[18], Kuwait[19], Malaysia, Netherlands[20], Nigeria, Pakistan, Philippines, Portugal[21], Romania[22], Thailand[23], United Arab Emirates, Venezuela, Vietnam[24].

Group 4	Australia, Brazil, France[25], Indonesia, Iran, Italy[26], Korea (Rep.), Mexico, Poland[27], Saudi Arabia, South Africa, Spain[28], Turkey
Group 5	Canada[29], Germany, India, Japan, United Kingdom[30].
Group 6	China, United States.

[1] Group 2 in 1990; [2] Group 2 in 2014; [3] Group 2 in 2000, 2005 and 2014; [4] Group 2 in 1990; [5] Group 3 in 1990 and 1995; [6] Group 3 in 1990 and 1995; [7] Group 3 in 1990 and 1995; [8] Group 3 in 1990 and 1995; [9] Group 3 in 1995; [10] Group 1 in 1990; [11] Group 3 in 1990; [12] Group 1 in 1990; [13] Group 3 in 1990 and 1995; [14] Group 3 in 1990; [15] Group 1 in 2000; [16] Group 1 in 2005, 2010 and 2014; [17] Group 2 in 2010 and 2014; [18] Group 2 in 2010 and 2014; [19] Group 2 in 1990 and 1995; [20] Group 4 in 1990 and 1995; [21] Group 2 in 2010 and 2014; [22] Group 4 in 1990 and Group 2 in 2014; [23] Group 4 in 1995 and 2005; [24] Group 1 in 1990 and 1995; [25] Group 5 in 1990 and Group 3 in 2014; [26] Group 5 in 1990 and 1995; [27] Group 3 in 2014; [28] Gorup 3 in 2010 and 2014; [29] Group 4 in 2010 and 2014; [30] Group 4 in 2010 and 2014.

Table 4. Polarization in per capita GHG emissions from 1990 to 2011

Year	P1	P2	P3
1990	2.5792	10.0857	0.4728
1995	5.6578	12.1759	0.5171
2000	5.5163	11.6726	0.4942
2005	52.2280	14.8923	0.5390
2010	6.2164	13.9252	0.4743
2014	2.5380	13.6443	0.4312

The three indices showed a moderately increasing pattern from 1990 to 1995. During the next five years the polarization suffered a slightly decrease. In the year 2005, it is observed the maximum level of polarization and since then such level has been reduced. In the case of P1 and P3 indices, the level of polarization in the year 2014 is even below the one observed in the year 1990.

ACKNOWLEDGMENTS

The authors thank the Ministerio de Economía y Competitividad (Project ECO2016-76203-C2-1-PE) for the partial support of this work.

REFERENCES

Aghevli, B. B., and Mehran, F. (1981). "Optimal grouping of income distribution data". Journal of the American Statistical Association, 76, 22-26.

CAIT (2015). "Climate Data Explorer". World Resources Institute, Washington, DC, [Available at: http://cait.wri.org].

Davies, J. B., and Shorrocks, A. F. (1989). "Optimal grouping of income and wealth data". Journal of Econometrics, 42, 97-108.

Gigliarano, C., and Mosler, K. (2009). "Constructing indices of multivariate polarization". Journal of Economic Inequality, 7, 435-460.

IPCC (1996). "Climate Change 1995: Economic and Social Dimensions of Climate Change". Contribution of Working Group III to the Second Assessment Report of the Intergovernmental Panel on Climate Change [Bruce, J. P., Lee, H. and Haites, E. F. (eds.)]. Cambridge and New York: Cambridge University Press, 891 pp.

Maasoumi, E. (1986). "The Measurement and Decomposition of Multivariate Inequality". Econometrica, 54, 991-997.

Maasoumi, E., and Nickelsburg, G. (1988). "Multivariate Measures of Well-Being and an Analysis of Inequality in the Michigan Data". Journal of Business and Economic Statistics, 6, 327-334.

World Meteorological Organization, (2017). "Greenhouse Gas Bulletin - N° 13: The State of Greenhouse Gases in the Atmosphere Based on Global Observations through 2016". [Available at: https://www.wmo.int].

AN EXPERIENCE BASED PREMIUM RATE DISCOUNTS SYSTEM IN CROP INSURANCE USING TWEEDIE'S REGRESSIONS

Jose-Luis Vilar-Zanón
Universidad Complutense de Madrid
Departamento de Economía Financiera y Métodos de Decisión, España
jlvilarz@ucm.es

Antonio Heras
Universidad Complutense de Madrid
Departamento de Economía Financiera y Métodos de Decisión, España
aheras@ccee.ucm.es

ABSTRACT

We develop an experience based premium rate discount system in crop insurance able to cope with adverse years when high losses happen within some return period. Technically, it consists of the application of Tweedie's model and regressions embedded in a mean discount model. We also develop our system using censored regression (Tobit model) and compare results. The proposed rating system should be able to cope with those years that destabilize the technical result caused by chronic premiums insufficiency. We use data taken from the Spanish "table grape" line of business to exemplify the methodology.

Keywords: Tweedie regression, Crop insurance, Statistical learning.

ACKNOWLEDGMENTS

This work has been supported by AGROSEGURO S.A. under an UCM project Nº 437-2016

SIMULTANEOUS RISK EVENTS MODELING BY THE BATCH MARKOV-MODULATED POISSON PROCESS

Yoel G. Yera
Universidad Carlos III de Madrid. Departamento de Estadística. España
yoelgustavo.yera@uc3m.es

Rosa E. Lillo
Universidad Carlos III de Madrid. Departamento de Estadística
UC3M-BS Institute of Financial Big Data. España
rosaelvira.lillo@uc3m.es

Pepa Ramírez-Cobo
Universidad de Cádiz
Departamento de Estadística e Investigación Operativa. España
pepa.ramirez@uca.es

ABSTRACT

The Batch Markovian Arrival Process (BMAP) is a general class of point processes suitable for the modeling of dependent and correlated batch events (as arrivals, failures or risk events). BMAPs have been widely considered in the literature from a theoretical viewpoint. However, less works are devoted to study statistical inference which is of crucial importance in operational risk contexts, often characterized by dependent and simultaneous failures. In this work, we consider the estimation for a wide subclass of BMAPs, namely, the Batch Markov-Modulated Poisson processes (BMMPP) which generalize the well-known Markov-Modulated Poisson process. A matching moments technique, supported by a theoretical result that characterizes the process in terms of its moments, is considered. Numerical results with both simulated and real datasets related to operational risk will be presented to illustrate the performance of the novel approach.

1. INTRODUCCIÓN

Operational risk is in conjunction with market and credit risk what is called financial risk, which is defined as the adverse impact on performance due to different sources of uncertainty. Unlike market and credit risk, operational risk is not the result of taking a risk position and is defined as the potential loss due to: deficiencies in controls, errors in processing and storage operations, adverse administrative and judicial rulings, fraud, theft, external factors or the occurrence of extreme events. On the other hand, it does not take into account losses from changes in the political, economic and social environment. Many of the losses related to operational risk are expected (for example, "cash differences" recorded almost daily) and the entity a priori knows what really expect to lose. However, the occurrence of extreme events or other unexpected factors may lead to actual losses that exceed the expected, and dire situations for the entity, which could even lead to bankruptcy. In fact, the occurrence of severe losses has led many organizations attempt to model operational risk to determine their own capital. Moreover, since the terrorist attacks or the financial scandals of the last years the quantification of operational risk is increasingly important and it is clear that a correct estimate of operational risk is indispensable for the stability of the entity (see Carrillo and Suárez (2006), Pakhchanyan (2016) and Panjer (2006).

To evaluate the operational risk, both variables the frequency of the risk occurrence and the severity (or the impact of risk events on the companies' results) are usually considered. In the models of loss distributions, the frequency and severity are usually assumed mutually independent random variables. It is also supposed that the sequence of severities is formed by independent and identically distributed random variables. This hypothesis of independence facilitates greatly the calculations, but in certain contexts (insurance, reliability, finances) may be unrealistic. Next, a series of figures show 225 observations of consecutive severities and frequencies from some minority banking financial institution. The observations were taken from 30/12/93 to 29/06/2007 and it is considered that there is a single type of risk event. In Figures 1-2, it can be seen that the correlation is non-negligible. Therefore, assuming the independence and identical distribution of the variables would not be appropriate for these data.

Figure 1. Traces of consecutive severities (top) and the frequencies associated with each severity (bottom)

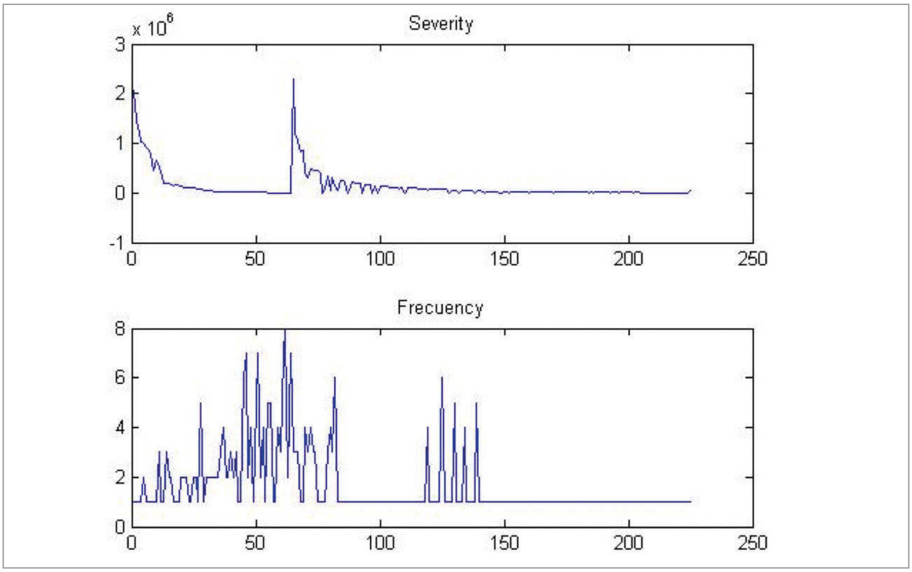
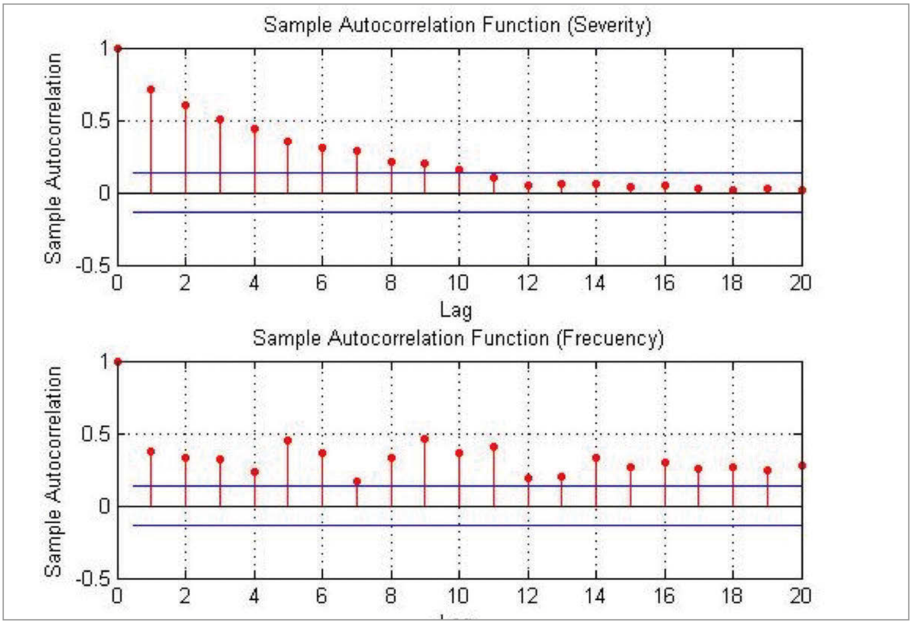


Figure 2. Empirical autocorrelation functions of both variables



The last two figures evidence that the classic point processes are not valid for modeling certain situations of risk. Also, they point out the need of studying more complex point processes that, among other things, relax the hypothesis of independence. In fact, there exist recent works in the literature dealing with such an issue, see for example Lindskog and McNeil (2003), Joshua and Schuermann, (2004) or Cope and Antonini (2008).

In this work, we propose a fitting approach for modeling simultaneous (and possibly) correlated failure times via the Batch Markov-Modulated Poisson processes (BMMPP), a general subclass of the Batch Markovian Arrival Process (BMAP). BMMMPs constitute a large class of point processes that allow for non-exponential and dependent times between the occurrence of (possibly correlated) batches. In addition, the BMMPP allows for an analytic tractability of the process allowing the calculation of important quantities and probabilities of interest, such as the probability and mean number of risk events in an interval (0, t]. This suggests that the BMMPP has a considerable versatility and the possible range of applications is equal to those seen for the BMAP, which has been widely considered in a number of real-life contexts, as queueing, teletraffic, reliability or insurance, where batch dependent events are commonly observed. For a recent account of the literature on BMAPs applications, we refer the reader to Bookbinder et al. (2011), Falin (2010); Gómez-Corral and Economou (2007); Kim et al. (2010), Kim and Kim (2010), Buchholz and Kriege (2017), Sikdar and Samanta (2016), Ghosh and Banik (2017) and Singh et al. (2016).

In this work we present a quite reasonable estimation of a BMMPP for the real dataset depicted in the figures 1-2 using a matching moments technique, supported by a theoretical result that characterizes the process in terms of its moments.

ACKNOWLEDGMENTS

This work has been partially supported by research grants and projects MTM2015-65915-R and ECO2015-66593-P (Ministerio de Economía y Competitividad, Spain) and P11-FQM-7603, FQM-329 (Junta de Andalucía, Spain) and Fundación BBVA.

REFERENCES

Asmussen S. and Koole G. (1993) "Marked point processes as limits of Markovian arrival streams". J Appl Probab 30:365-372.

Bookbinder J., Cai Q. and He Q. M. (2011) "Shipment consolidation by private carrier: the discrete time and discrete quantity case". Stoch Models 27:664-686.

Buchholz, P. and Kriege, J. (2017) "Fitting correlated arrival and service times and related queueing performance". Queueing Systems, 85(3-4), 337-359.

Carrillo, S. and Suárez, A. (2006). "Medición efectiva del riesgo operacional". Estabilidad Financiera, 11, 61-90.

Falin, G.I. (2010) "A single-server batch arrival queue with returning customers". Eur J Oper Res 201:786-790.

Gómez-Corral A. and Economou A. (2007) The batch Markovian arrival process subject to renewal generated geometric catastrophes. Stoch Models 23:211-233.

Ghosh, S. and Banik, A. (2017) "An algorithmic analysis of the BMAP/MSP/1 generalized processor-sharing queue". Comput Oper Res, 79, 1–11,

Kim, B. and Kim, J. (2010) "Queue size distribution in a discrete time D – BMAP/G/1 retrial queue". Comput Oper Res 37:1220-1227.

Kim, C.S., Klimenok, V.I., Mushko, V. and Dudin, A. (2010) "The BMAP/PH/N retrialqueueing system operating in Markovian random environment". Comput Oper Res 37:1228-1237.

Pakhchanyan, S. (2016). "Operational Risk Management in Financial Institutions: A Literature Review." International Journal of Financial Studies, 4(4), 20.

Panjer, H. H. (2006). "Operational risk: modelling analytics" (Vol. 620). John Wiley & Sons

Singh, G., Gupta, U. C., and Chaudhry, M. L. (2016). "Detailed computational analysis of queueing-time distributions of the BMAP/G/1 queue using roots." J Appl Probab, 53(4), 1078-1097.

Sikdar, K. and Samanta, S. (2016) "Analysis of a finite buffer variable batch service queue with batch Markovian arrival process and server's vacation." OPSEARCH, 53, 553–583.

COLECCIÓN "CUADERNOS DE LA FUNDACIÓN"

Para cualquier información sobre nuestras publicaciones consulte:
www.fundacionmapfre.org

223. Contributions to Risk Analysis: RISK 2018

222. Individual Cancer Mortality Prediction. 2017

221. Defensa jurídica y dolo del asegurado en el seguro de responsabilidad civil. 2017

220. El proceso precontractual en el contrato de seguro: nuevo marco jurídico. 2017

219. Teoría de cópulas. Introducción y aplicaciones a Solvencia II. 2017

218. Cualificación profesional del actuario. Estudio internacional comparado. 2016

217. El seguro de responsabilidad civil derivada de la navegación de buques. 2016

216. El impacto de las últimas reformas en materia de jubilación: envejecimiento activo, sostenibilidad financiera y planes de pensiones. 2016

215. Previsión complementaria empresarial: estudio comparado internacional 2016

214. Normas sobre protección de los derechos de los consumidores en el contrato de seguro en Chile. 2016

213. *Gamificación*: un modelo de fomento y gestión de comportamientos deseados en las relaciones entre individuos y organizaciones. 2015

212. Modelo de gestión integral para el sector atunero. 2015

211. Opções embutidas em planos *unit-linkeds* brasileiros: avaliação sob a medida de probabilidade real. 2015

210. El enfoque de Solvencia II para las pensiones ocupacionales españolas. 2015

209. El seguro privado de obras de arte. 2015

208. Definición y medición de la cultura aseguradora. Aplicación al caso español. 2015

207. Tipos de interés para valorar las provisiones técnicas de seguros. 2015

206. Teledetección aplicada a la elaboración de mapas de peligrosidad de granizo en tiempo real y mapas de daños en cultivos e infraestructuras. 2015

205. Current Topics on Risk Analysis: ICRA6 and Risk 2015 Conference. 2015

204. Determinantes do Premio de Default de (Res)seguradores. 2014

203. Generación de escenarios económicos para la medición de riesgos de mercado en Solvencia II a través de modelos de series temporales. 2014

202. Valoración de los inmuebles del patrimonio histórico y los riesgos sísmicos en el contrato de seguro: el caso de Lorca. 2014

201. Inteligencia computacional en la gestión del riesgo asegurador: opera-dores de agregación OWA en proceso de tarificación. 2014

200. El componente transfronterizo de las relaciones aseguradoras. 2014

199. El seguro basado en el uso (Usage Based Insurance). 2014

198. El seguro de decesos en la normativa aseguradora. Su encaje en Solvencia II. 2014

197. El seguro de responsabilidad civil en el arbitraje. 2014

196. La reputación corporativa en empresas aseguradoras: análisis y evaluación de factores explicativos. 2014

195. La acción directa del perjudicado en el ordenamiento jurídico comunitario. 2013

194. Investigaciones en Seguros y Gestión del Riesgo: RIESGO 2013

193. Viability of Patent Insurance in Spain. 2013

192. Viabilidad del seguro de patentes en España. 2013

191. Determinación de zonas homogéneas de riesgo para los rendimientos de distintos cultivos de la región pampeana en Argentina. 2013

190. Género y promoción en los sectores financiero y asegurador. 2013

189. An Introduction to Reinsurance. 2013

188. El control interno y la responsabilidad penal en la mediación de seguros privados. 2013

187. Una introducción al gobierno corporativo en la industria aseguradora en América Latina. 2013

186. Mortalidad de jóvenes en accidentes de tráfico. 2012

185. Las reclamaciones derivadas de accidentes de circulación por carretera transfronterizos. 2012

184. Efecto disuasorio del tipo de contrato sobre el fraude. 2012

183. Claves del Seguro Español: una aproximación a la Historia del Seguro en España. 2012

182. La responsabilidad civil del asegurador de asistencia sanitaria. 2012

181. Colaboración en el contrato de Reaseguro. 2012

180. Origen, situación actual y futuro del seguro de Protección Jurídica. 2012

179. Experiencias de microseguros en Colombia, Perú y Brasil. Modelo socio agente. 2012

178. El agente de seguros y su Responsabilidad Civil. 2012

177. Riesgo operacional en el marco de Solvencia II. 2012

176. Un siglo de seguros marítimos barceloneses en el comercio con América. (1770-1870). 2012

175. El seguro de Caución. 2012

174. La contabilidad de los corredores de seguros y los planes y fondos de pensiones. 2012

173. El seguro de Vida en América Latina. 2011

172. Gerencia de riesgos sostenibles y Responsabilidad Social Empresarial en la entidad aseguradora. 2011

171. Investigaciones en Seguros y Gestión del Riesgo. RIESGO 2011

170. Introdução ao Resseguro. 2011

169. La salud y su aseguramiento en Argentina, Chile, Colombia y España. 2011

168. Diferencias de sexo en conductas de riesgo y tasa de mortalidad diferencial entre hombres y mujeres. 2011

167. Movilización y rescate de los compromisos por pensiones garantizados mediante contrato de seguros. 2011

166. Embedded Value aplicado al ramo No Vida. 2011

165. Las sociedades cautivas de Reaseguro. 2011

164. Daños del amianto: litigación, aseguramiento de riesgos,y fondos de compensación. 2011

163. El riesgo de tipo de interés: experiencia española y Solvencia II. 2011

162. I Congreso sobre las Nuevas Tecnologías y sus repercusiones en el Seguro: Internet, Biotecnología y Nanotecnología. 2011

161. La incertidumbre bioactuarial en el riesgo de la longevidad. Reflexiones bioéticas. 2011

160. Actividad aseguradora y defensa de la competencia. La exención antitrust del sector asegurador. 2011

159. Estudio empírico sobre la tributación de los seguros de vida. 2010

158. Métodos estocásticos de estimación de las provisiones técnicas en el marco de Solvencia II. 2010

157. Introducción al Reaseguro. 2010

156. Encuentro Internacional sobre la Historia del Seguro. 2010

155. Los sistemas de salud en Latinoamérica y el papel del seguro privado. 2010

154. El Seguro de Crédito en Chile. 2010

153. El análisis financiero dinámico como herramienta para el desarrollo de modelos internos en el marco de Solvencia II. 2010

152. Características sociodemográficas de las personas con doble cobertura sanitaria. Un estudio empírico. 2010

151. Solidaridad impropia y seguro de Responsabilidad Civil. 2010

150. La prevención del blanqueo de capitales en las entidades aseguradoras, las gestoras y los corredores de seguros 2010

149. Fondos de aseguramiento agropecuario y rural: la experiencia mexicana en el mutualismo agropecuario y sus organizaciones superiores. 2010

148. Avaliaçao das Provisoes de Sinistro sob o Enfoque das Novas Regras de Solvência do Brasil. 2010

147. El principio de igualdad sexual en el Seguro de Salud: análisis actuarial de su impacto y alcance. 2010

146. Investigaciones históricas sobre el Seguro español. 2010

145. Perspectivas y análisis económico de la futura reforma del sistema español de valoración del daño corporal. 2009

144. Contabilidad y Análisis de Cuentas Anuales de Entidades Aseguradoras (Plan contable 24 de julio de 2008). 2009

143. Mudanças Climáticas e Análise de Risco da Indústria de Petróleo no Litoral Brasileiro. 2009

142. Bases técnicas dinámicas del Seguro de Dependencia en España. Una aproximación en campo discreto. 2009

141. Transferencia Alternativa de Riesgos en el Seguro de Vida: Titulización de Riesgos Aseguradores. 2009

140. Riesgo de negocio ante asegurados con múltiples contratos. 2009

139. Optimización económica del Reaseguro cedido: modelos de decisión. 2009

138. Inversiones en el Seguro de Vida en la actualidad y perspectivas de futuro. 2009

137. El Seguro de Vida en España. Factores que influyen en su progreso. 2009

136. Investigaciones en Seguros y Gestión de Riesgos. RIESGO 2009.

135. Análisis e interpretación de la gestión del fondo de maniobra en entidades aseguradoras de incendio y lucro cesante en grandes riesgos industriales. 2009

134. Gestión integral de Riesgos Corporativos como fuente de ventaja competitiva: cultura positiva del riesgo y reorganización estructural. 2009

133. La designación de la pareja de hecho como beneficiaria en los seguros de vida. 2009

132. Aproximación a la Responsabilidad Social de la empresa: reflexiones y propuesta de un modelo. 2009

131. La cobertura pública en el seguro de crédito a la exportación en España: cuestiones prácticas-jurídicas. 2009

130. La mediación en seguros privados: análisis de un complejo proceso de cambio legislativo. 2009

129. Temas relevantes del Derecho de Seguros contemporáneo. 2009

128. Cuestiones sobre la cláusula cut through. Transferencia y reconstrucción. 2008

127. La responsabilidad derivada de la utilización de organismos genéticamente modificados y la redistribución del riesgo a través del seguro. 2008

126. Ponencias de las Jornadas Internacionales sobre Catástrofes Naturales. 2008

125. La seguridad jurídica de las tecnologías de la información en el sector asegurador. 2008

124. Predicción de tablas de mortalidad dinámicas mediante un procedimiento bootstrap. 2008

123. Las compañías aseguradoras en los procesos penal y contencioso-administrativo. 2008

122. Factores de riesgo y cálculo de primas mediante técnicas de aprendizaje. 2008

121. La solicitud de seguro en la Ley 50/1980, de 8 de octubre, de Contrato de Seguro. 2008

120. Propuestas para un sistema de cobertura de enfermedades catastróficas en Argentina. 2008

119. Análisis del riesgo en seguros en el marco de Solvencia II: Técnicas estadísticas avanzadas Monte Carlo y Bootstrapping. 2008

118. Los planes de pensiones y los planes de previsión asegurados: su inclusión en el caudal hereditario. 2007

117. Evolução de resultados tecnicos e financieros no mercado segurador iberoamericano. 2007

116. Análisis de la Ley 26/2006 de Mediación de Seguros y Reaseguros Privados. 2007

115. Sistemas de cofinanciación de la dependencia: seguro privado frente a hipoteca inversa. 2007

114. El sector asegurador ante el cambio climático: riesgos y oportunidades. 2007

113. Responsabilidade social empresarial no mercado de seguros brasileiro influências culturais e implicações relacionais. 2007

112. Contabilidad y análisis de cuentas anuales de entidades aseguradoras. 2007

111. Fundamentos actuariales de primas y reservas de fianzas. 2007

110. El Fair Value de las provisiones técnicas de los seguros de Vida. 2007

109. El Seguro como instrumento de gestión de los M.E.R. (Materiales Específicados de Riesgo). 2006

108. Mercados de absorción de riesgos. 2006

107. La exteriorización de los compromisos por pensiones en la negociación colectiva. 2006

106. La utilización de datos médicos y genéticos en el ámbito de las compañías aseguradoras. 2006

105. Los seguros contra incendios forestales y su aplicación en Galicia. 2006

104. Fiscalidad del seguro en América Latina. 2006

103. Las NIC y su relación con el Plan Contable de Entidades Asegura-doras. 2006

102. Naturaleza jurídica del Seguro de Asistencia en Viaje. 2006

101. El Seguro de Automóviles en Iberoamérica. 2006

100. El nuevo perfil productivo y los seguros agropecuarios en Argentina. 2006

99. Modelos alternativos de transferencia y financiación de riesgos “ART”: situación actual y perspectivas futuras. 2005

98. Disciplina de mercado en la industria de seguros en América Latina. 2005

97. Aplicación de métodos de inteligencia artificial para el análisis de la solvencia en entidades aseguradoras. 2005

96. El Sistema ABC-ABM: su aplicación en las entidades aseguradoras. 2005

95. Papel del docente universitario: ¿enseñar o ayudar a aprender? 2005

94. La renovación del Pacto de Toledo y la reforma del sistema de pensiones: ¿es suficiente el pacto político? 2005

92. Medición de la esperanza de vida residual según niveles de dependencia en España y costes de cuidados de larga duración. 2005

91. Problemática de la reforma de la Ley de Contrato de Seguro. 2005

90. Centros de atención telefónica del sector asegurador. 2005

89. Mercados aseguradores en el área mediterránea y cooperación para su desarrollo. 2005

88. Análisis multivariante aplicado a la selección de factores de riesgo en la tarificación. 2004

87. Dependencia en el modelo individual, aplicación al riesgo de crédito. 2004

86. El margen de solvencia de las entidades aseguradoras en Iberoamérica. 2004

85. La matriz valor-fidelidad en el análisis de los asegurados en el ramo del automóvil. 2004

84. Estudio de la estructura de una cartera de pólizas y de la eficacia de un Bonus-Malus. 2004

83. La teoría del valor extremo: fundamentos y aplicación al seguro, ramo de responsabilidad civil autos. 2004

81. El Seguro de Dependencia: una visión general. 2004

80. Los planes y fondos de pensiones en el contexto europeo: la necesidad de una armonización. 2004

79. La actividad de las compañías aseguradoras de vida en el marco de la gestión integral de activos y pasivos. 2003

78. Nuevas perspectivas de la educación universitaria a distancia. 2003

77. El coste de los riesgos en la empresa española: 2001.

76. La incorporación de los sistemas privados de pensiones en las pequeñas y medianas empresas. 2003

75. Incidencia de la nueva Ley de Enjuiciamiento Civil en los procesos de responsabilidad civil derivada del uso de vehículos a motor. 2002

74. Estructuras de propiedad, organización y canales de distribución de las empresas aseguradoras en el mercado español. 2002

73. Financiación del capital-riesgo mediante el seguro. 2002

72. Análisis del proceso de exteriorización de los compromisos por pensiones. 2002

71. Gestión de activos y pasivos en la cartera de un fondo de pensiones. 2002

70. El cuadro de mando integral para las entidades aseguradoras. 2002

69. Provisiones para prestaciones a la luz del Reglamento de Ordenación y Supervisión de los Seguros Privados; métodos estadísticos de cálculo. 2002

68. Los seguros de crédito y de caución en Iberoamérica. 2001

67. Gestión directiva en la internacionalización de la empresa. 2001

65. Ética empresarial y globalización. 2001

64. Fundamentos técnicos de la regulación del margen de solvencia. 2001

63. Análisis de la repercusión fiscal del seguro de vida y los planes de pensiones. Instrumentos de previsión social individual y empresarial. 2001

62. Seguridad Social: temas generales y régimen de clases pasivas del Estado. 2001

61. Sistemas Bonus-Malus generalizados con inclusión de los costes de los siniestros. 2001

60. Análisis técnico y económico del conjunto de las empresas aseguradoras de la Unión Europea. 2001

59. Estudio sobre el euro y el seguro. 2000

58. Problemática contable de las operaciones de reaseguro. 2000

56. Análisis económico y estadístico de los factores determinantes de la demanda de los seguros privados en España. 2000

54. El corredor de reaseguros y su legislación específica en América y Europa. 2000

53. Habilidades directivas: estudio de sesgo de género en instrumentos de evaluación. 2000

52. La estructura financiera de las entidades de seguros, S.A. 2000

51. Seguridades y riesgos del joven en los grupos de edad. 2000

50. Mixturas de distribuciones: aplicación a las variables más relevantes que modelan la siniestralidad en la empresa aseguradora. 1999

49. Solvencia y estabilidad financiera en la empresa de seguros: metodología y evaluación empírica mediante análisis multivariante. 1999

48. Matemática Actuarial no vida con MapleV. 1999

47. El fraude en el Seguro de Automóvil: cómo detectarlo. 1999

46. Evolución y predicción de las tablas de mortalidad dinámicas para la población española. 1999

45. Los Impuestos en una economía global. 1999

42. La Responsabilidad Civil por contaminación del entorno y su aseguramiento. 1998

41. De Maastricht a Amsterdam: un paso más en la integración europea. 1998

39. Perspectiva histórica de los documentos estadístico-contables del órgano de control: aspectos jurídicos, formalización y explotación. 1997

38. Legislación y estadísticas del mercado de seguros en la comunidad iberoamericana. 1997

37. La responsabilidad civil por accidente de circulación. Puntual comparación de los derechos francés y español. 1997

36. Cláusulas limitativas de los derechos de los asegurados y cláusulas delimitadoras del riesgo cubierto: las cláusulas de limitación temporal de la cobertura en el Seguro de Responsabilidad Civil. 1997

35. El control de riesgos en fraudes informáticos. 1997

34. El coste de los riesgos en la empresa española: 1995

33. La función del derecho en la economía. 1997

32. Decisiones racionales en reaseguro. 1996

31. Tipos estratégicos, orientación al mercado y resultados económicos: análisis empírico del sector asegurador español. 1996

30. El tiempo del directivo. 1996

29. Ruina y Seguro de Responsabilidad Civil Decenal. 1996

28. La naturaleza jurídica del Seguro de Responsabilidad Civil. 1995

27. La calidad total como factor para elevar la cuota de mercado en empresas de seguros. 1995

26. El coste de los riesgos en la empresa española: 1993

25. El reaseguro financiero. 1995

24. El seguro: expresión de solidaridad desde la perspectiva del derecho. 1995

23. Análisis de la demanda del seguro sanitario privado. 1993

22. Rentabilidad y productividad de entidades aseguradoras. 1994

21. La nueva regulación de las provisiones técnicas en la Directiva de Cuentas de la C.E.E. 1994

20. El Reaseguro en los procesos de integración económica. 1994

19. Una teoría de la educación. 1994

18. El Seguro de Crédito a la exportación en los países de la OCDE (evaluación de los resultados de los aseguradores públicos). 1994

16. La legislación española de seguros y su adaptación a la normativa comunitaria. 1993

15. El coste de los riesgos en la empresa española: 1991

14. El Reaseguro de exceso de pérdidas 1993

12. Los seguros de salud y la sanidad privada. 1993

10. Desarrollo directivo: una inversión estratégica. 1992

9. Técnicas de trabajo intelectual. 1992

8. La implantación de un sistema de controlling estratégico en la empresa. 1992

7. Los seguros de responsabilidad civil y su obligatoriedad de aseguramiento. 1992

6. Elementos de dirección estratégica de la empresa. 1992

5. La distribución comercial del seguro: sus estrategias y riesgos. 1991

4. Los seguros en una Europa cambiante: 1990-95. 1991

2. Resultados de la encuesta sobre la formación superior para los profesionales de entidades aseguradoras (A.P.S.). 1991

1. Filosofía empresarial: selección de artículos y ejemplos prácticos. 1991

LIBROS

La Responsabilidad Civil en el ámbito de los ciberriesgos. 2017

Longevidad y envejecimiento en el tercer milenio. 2017

El Ahorro en perspectiva histórica. 2016

Lo bueno, si breve… Microrrelatos de Seguros. 2016

The risk of longevity and its practical application to Solvency II. 2015

Historia de FIDES –Federación Interamericana de Empresas de Seguros. 2015

El riesgo de longevidad y su aplicación práctica a Solvencia II. 2014

Historia del Seguro en España. 2014

Actas del III Congreso Internacional de Nuevas Tecnologías: sus repercusiones en el seguro: internet, biotecnología y nanotecnología: 12 y 13 de noviembre de 2012, Santiago de Chile. 2013

Emergencia y reconstrucción: el antes y después del terremoto y tsunami del 27F en Chile. 2012

Riesgo sistémico y actividad aseguradora. 2012

La historia del seguro en Chile (1810-2010). 2012

Modelo de proyección de carteras de seguros para el ramo de decesos. 2011

Desarrollo comercial del seguro colectivo de dependencia en España. 2010

La mediación de seguros en España: análisis de la Ley 26/2006, de Mediación de Seguros y Reaseguros Privados. 2010

Museo del Seguro. Catálogo. 2010

Diccionario MAPFRE de Seguros. 2008

Teoría de la credibilidad: desarrollo y aplicaciones en primas de seguros y riesgos operacionales. 2008

El seguro de caución: una aproximación práctica. 2007

El seguro de pensiones. 2007

Las cargas del acreedor en el seguro de responsabilidad civil. 2006

Diccionario bilingüe de expresiones y términos de seguros: inglés-español, español-inglés. 2006

El seguro de riesgos catastróficos: reaseguro tradicional y transferencia alternativa de riesgos. 2005

La liquidación administrativa de entidades aseguradoras.2005

INFORMES Y RANKINGS

Desde 1994 se publican anualmente estudios que presentan una panorámica concreta de los mercados aseguradores europeos, de España e Iberoamérica y que pueden consultarse en formato electrónico en castellano ey en inglés desde la página Web:
www.fundacionmapfre.org

Mercado español de seguros

Mercado asegurador latinoamericano

Ranking de grupos aseguradores europeos

Ranking de grupos aseguradores iberoamericanos

Panorama económico y sectorial 2018

Regímenes de regulación de solvencia 2018

Elementos para el expansión del seguro en América Latina 2017

Sistemas de Pensiones 2017

Los millennials y el seguro en España. 2016

Tendencias de crecimiento de los mercados aseguradores de América Latina para 2016

Estudio social sobre la jubilación: expectativas y experiencias. 2015

La percepción social del seguro en España 2014

Informe de predicción de la actividad aseguradora en España. 2014

La internacionalización de la empresa española: riesgos y oportunidades. 2014

El seguro en la sociedad y la economía españolas. 2013

Papel del seguro en el desarrollo sostenible. ICEA, 2013

Emprender en momentos de crisis: riesgos y factores de éxito. 2012

La percepción social del seguro en España. 2012

You can download the digital version in the
Documentation Center
www.fundacionmapfre.org/documentacion

Fundación MAPFRE

Fundación **MAPFRE**

C/223

Paseo de Recoletos, 23
28004 Madrid (España)
www.fundacionmapfre.org

P.V.P.: 30 €
ISBN 978-84-9844-683-8
9 788498 446838