Anales del Instituto de Actuarios Españoles, 4ª época, 26, 2020/103-126

CAP & TRAP AND ALTERNATIVES IN PRICE DISCRIMINATION

CAP & TRAP Y ALTERNATIVAS EN LA DISCRIMINACIÓN DE PRECIOS

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Reception date: August 8th 2020
Acceptance date: October 10th 2020

Abstract

Cap and Trap is a mechanism for smoothing rate change at policy renewal. However, this method still can expose an insurance company to a potential conduct risk. Individual clients could be overcharged or undercharged on an individual basis for their risk. This paper discusses alternative methods to better smooth prices with the aim of reducing the effect of price discrimination.

Keywords: Cap and Trap, Price Discrimination, Smooth pricing

Resumen

El Cap & Trap es un mecanismo para atenuar las variaciones en el precio de renovación de la póliza. Sin embargo, este método puede exponer a una compañía de seguros a un riesgo potencial sobre su conducta. A los clientes se les podría cobrar de más o de menos dependiendo de su riesgo de forma individual por su riesgo. En este artículo se describen métodos alternativos para suavizar las variaciones en precio con el objetivo de reducir el efecto de reducir la discriminación por precios.

Palabras clave: Cap & Trap, discriminación de precios, suavizado de precios

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DOI: 10.26360/2020_5

1 Introduction

Cap and Trap is a method used in the insurance industry to smooth rate changes. While, this method can partially mitigate the price discrimination, it cannot be considered as a proper solution as it can still yield dual prices.

Let us discuss an example. Supposing the amount Z, which is dependent on a rating structure P between limits A and B, such that

$$Z = \max(A, \min(P, B)). \tag{1.1}$$

Here the Cap is A and the Trap is B. Let's discuss a simple example to illustrate the above formula - A customer has a Pound-Sterling 100 premium for a policy from 1st January 2018 to 31st December 2018. The actuaries may decide during 2019 to change the policy holders premiums. To prevent a large change being seen by the customers insurers, a Cap and Trap method is used. There could be a maximum of 50% increase in any one renewal (the maximum price of Pound-Sterling 150) or a 20% decrease in one premium (minimum price of Pound-Sterling 80) from one year to the next. So, regardless of the amount of the calculated renewal premium - for example, Pound-Sterling 200 it would be capped at Pound-Sterling 150; if it were Pound-Sterling 50, it would be increased to the minimum value of Pound-Sterling 80.

As one can see even though the Cap and Trap method has partially removed the price discrimination, however, it can very likely result in dual pricing. This puts the insurance companies at the conduct risk.

In principle, an insurer aims to limit the price charges of a policy holders premium to avoid causing complaints; if the price increases and customers have not made a claim, they will be unhappy; if the price decreases significantly, customers may believe they have been overcharged the previous year. Price elasticity means the customer is less likely to renew the higher the increases, and regulators may see an increase as being unfair to the customer.

Additionally, reducing prices at renewal is not optimal for the insurer; the increase in retention from lower premiums tends not to offset the premium lost by reducing prices, the retention effect tends to be similar for no price change as for a large price cut at renewal.

A policyholder's premium may change for several reasons, for example, a change given risk from new information, updated risk models, new data, etc.

The premium may also change due to reallocation in a company's expenses or removal of discounts for a new customer.

The Cap and Trap values are calculated by taking in many considerations such as renewal price elasticity. Retention rates tend not to improve much when prices reduce at renewal, so if optimizing profit, the insurer would want to stop premiums reducing. The insurer also considers the customers expectations and the acceptable, expected profit or loss from a given policy. The insurer would investigate how the rest of the market behaves, how far the premium is from the market price and get an opinion from the underwriter. The Cap and Trap methodology, unfortunately, can still cause price discrimination against loyal customers.

The problem with the Cap and Trap methodology is that if the prices were to rise significantly for the policyholders, too much business would be lost for insurance companies. If the premium increases significantly, the brokers will refuse future business, which would shrink and thus reduce the quality of the insurers portfolio.

The current Cap and Trap methodology used by insurance companies may expose them to a potential conduct risk because individual clients could be overcharged or undercharged on an individual basis for their risk. Some insurance companies do not have confidence in the current rating structure due to the size of the current portfolio and pricing risks to market rates can create inequality in rating. This paper aims to research a method to assist an insurance company where they have had to increase premiums due to high expenses, thus penalizing loyal customers with no claims. By introducing a smoothing process for pricing, we try to solve the dual pricing issue by proposing a single price.

The paper is organized as follows: in section 2 we review the literature on discrimination and also price discrimination in insurance. In section 3 we discuss the methodology and the smoothing method that is introduced for the first time in this paper. In section 4 we simulate the losses and the prices to see the impact of the smoothing methodology on removing price discrimination. In section 5 we conclude.

2 Price discrimination, literature review

The Financial Conduct Authority (FCA) defines price discrimination as *where firms charge different prices to different customers, who cost the same to serve, based on differences in the customers price sensitivity. Under price discrimination customers who are less price-sensitive to a product pay more for it than those who are more price sensitive,* FCA (2020).

The risk of price discrimination can potentially increase with the amount of data collected for an individual. The larger opportunity to obtain data on wider factors of the insured can put vulnerable customers at risk of facing discrimination. Price discrimination is a practice that is not a unique concept to the insurance market but occurs in several other industries such as the airline and the rail industry.

Procedural fairness looks at the insurance company's conduct and the treatment of its customers, whereas *distributive* fairness focuses on the reasons for a specific customer charged differently.

Some pricing related issues the GRIP (General Insurance Premium Rating Issues Working Party) considered include how accurately premium rates attempt to reflect the expected cost of risk, and how this contradicts the practice of charging premiums which are not directly linked to the cost of the risk or the characteristics of the customer, for example, No Claims Discount, Anderson et al. (2007).

Buzzacchi and Valletti (2005) analyzed the welfare problem of price discrimination in a compulsory insurance market (i.e., car insurance where it is essential to have at least third-party coverage). It was discussed that there is a strong incentive for firms to introduce a classification based on characteristics such as sex. However, this article was written in 2004, and from 21 December 2012, EU regulations stated that insurance companies were no longer allowed to price insurance products based on gender, Edmonds (2015). At the time, insurers argued that banning gender as a risk factor would cause the pricing to be less accurate and expose them to greater risk. The ban would mean the reserves that the insurers held would require more capital and those extra costs would be passed onto the policyholders.

Schmeiser, Störmer, and Wagner (2014) examined the customer's point of view through a survey to investigate the consumers view on the ethics of price discrimination within the insurance industry. Their results found that, while most consumers accept risk differentiation in pricing, premiums using gender as a factor is the least acceptable criteria, followed by age.

Störmer (2015) analyses the acceptance of commonly used risk-rating factors and how willing consumers within three European countries are to provide personal information to insurers for pricing purposes. It was discovered that consumers prefer the risk-factors that are easier to understand for the impact on their premium price. Some different risk-rating factors were found to be preferred, compared to those commonly used. Many rejected person-specific risk-rating factors such as hobbies, home-ownership, and marital status.

Etgar (1975) discusses the existing laws subjecting inner-city residents to paying higher insurance premiums and suffering from price discrimination.
Inertia pricing is a controversial form of price discrimination in financial services in which loyal existing customers are charged more than new customers. This discriminatory practice penalizes customers who fail to frequently switch their providers, as switching gains the best possible deal, FCA (2020).

The FCA's analysis of home insurance concluded that the premiums for existing customers have increased over the years, suggesting that loyal customers are penalized for not shopping around. Regulators have noticed this practice impacts, vulnerable customers, the most, as these are less likely to be able to shop around for the best deal.

Investigations by Thomas (2012) have shown that for inertia pricing, where renewal prices exceed those of new customers with the same level of risk, price discrimination may benefit customers in some circumstances where competition is increased. Switching insurance companies is not always simple for consumers, taking into consideration the additional monetary and financial costs that come with the personalized nature of pricing. One either has to perfectly time the transition between providers or suffer the consequences of paying both the old and new provider at once. This difficulty puts consumers off from switching and thus subjects them to the risk of price discrimination.

The idea of dual pricing, where the same insurance policy is sold at two different prices depending on the loyalty of the consumer, being under scrutiny by the FCA[1].

[1] https://ethicsandinsurance.info/2018/04/10/dual-pricing-in-insurance/

New customers to an insurance policy are often incentivized with a lower premium to encourage them to take on a policy. After the initial year has passed the original price increases, thus recovering these losses.

3 Methodology

The statement of profit and loss of an insurance company is divided up into technical accounts for each line of business. These include the incoming items include investment income, and net of premiums income, and the outgoing items include incurred claims, expenses, dividends to policyholder or shareholders, Schnieper (2000). However, an insurance company optimizes the company's portfolio to find the minimum amount of capital to satisfy restraints from the Solvency II regulations and portfolio performance constraints, Asanga, Asimit, Badescu, and Haberman (2014). The Solvency II pillars are based on ruin probability, expected shortfall or conditional Value-at-Risk, and expected policyholder deficit ratio. The ruin probability constraint is used due to the Solvency II capital requirements to identify this target level over some time. So, the first set of constraints are dictated by standard solvency insurance requirements, i.e., Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). The second set is based on a lower bound for the expected return on capital.

Given some confidence level, α, the VaR of a portfolio with loss L is the minimum value x such that the probability the loss L exceeds x is no larger than $1-\alpha$ McNeil, Frey and Embrechts (2015).

$$\text{VaR}_\alpha(L) = \inf\{x : P(L > x) \leq (1-\alpha)\} \tag{3.1}$$

For the Conditional Value-at-Risk (or the Expected Shortfall), one takes the average of all losses that can happen in the $1-\alpha$% McNeil et al. (2015).

$$\begin{aligned} \text{CVaR}_\alpha(L) &= \frac{1}{1-\alpha}\int_\alpha^1 \text{VaR}_u(L)\text{du} \\ &= \frac{1}{\alpha}\int_0^\alpha \text{VaR}_u(L)\text{du} \\ &= E[L|L \geq \text{VaR}_\alpha(L)]. \end{aligned} \tag{3.2}$$

3.1 A simple model for Cap and Trap

Assuming a loyal customer has a premium of P_0 in the previous year. The insurer calculates the new premium with the new rates provided; this is given by P_c.

Underwriting profit is the difference between premiums and associated costs from the claims and the expenses. For the new calculated premium

$$U = P_c - L - E, \tag{3.3}$$

where P_c is the calculated premium income, L is the total losses, and E is the associated expenses (such as brokerage costs).

Investment income is determined from the bonds that an insurer chooses to invest in, assuming due to the regular future cash-flow

$$I = r_B P_c, \tag{3.4}$$

where r_B is the interest rate, McCabe and Witt (1980). Therefore, with denoting π for profit

$$\pi = U + I. \tag{3.5}$$

So, the expected profit that we wish to maximize is given by

$$\mathrm{E}[\pi] = \mathrm{E}[U + I] = \mathrm{E}[P_c - L - E + r_B P_c]. \tag{3.6}$$

Therefore, the insurance company considers maximizing the expected profit i.e., $\mathrm{E}[\pi]$. However, there are different regulatory and market constraints need to be considered. First of all, we consider a cap and trap restriction

$$0 \le \frac{P_c - P_0}{P_0} \le C,$$

where $C \in (0,\ 1)$ is a constant to capture the cap. This shows what is the percentage of the prices changes allowed according to a cap and trap scheme. In this paper we mainly are concerned with the cap price and charging more the loyal costumers. Similar approach by including $-C \le \frac{P_c - P_0}{P_0}$ can capture the trap. In our simulations we usually consider $C = 10\%$. Changing C will not change the analysis we present in this paper and also it is totally dependent on a particular insurance line of business. However, we have adopted 10 percent increase based on the motor insurance increase rate in the UK based on Consumer Intelligence website as being less than 10 percent[2].

The second constraint is the Solvency II condition, as used in Asanga et al. (2014). For that we assume for a risk tolerance parameter α, the shortfall risk is less than zero i.e.,

$$\mathrm{CVaR}_{\alpha}(-\pi) \leq 0,$$

where CVaR_{α} is the conditional value at risk. We usually set $\alpha = 99.5\%$ to be consistent with the Solvency II accord requirement. Here π refers to the shortfall. If it happens that π is negative, then this means that the loss is negative. The risk of shortfall should not be positive. If π is in deficit, then one must make this positive for a sustainable business. To improve the accuracy, one could impose $\max(-\pi,0)$ within this constraint. The third and the fourth constraints are the market indicators to balance the demand and supply. Regarding the supply side (insurer) we consider the loss ratio, inputting the random variables to produce

$$\mathrm{LR} = \frac{\mathrm{Claims}}{\mathrm{Premiums}}. \tag{3.7}$$

Usually companies want to keep the expected loss ratio be less than a target LR_T, which is usually equal to 60% Assa and Wang (2020) as for a sustainable insurance business. The final constraint is regarding the demand side (insured) which is the premium rate defined by equation as

$$\mathrm{PR} = \frac{\mathrm{Premiums}}{\mathrm{Losses}} \tag{3.8}$$

Companies want to keep the premium rate as low as possible in order to keep the demand side happy. We set a limit PR_T, and usually we consider this to be equal to 8%, see Assa and Wang (2020).

So we can set our objective as follows:

$$\max \mathrm{E}[\pi] \tag{3.9}$$

2 https://www.consumerintelligence.com/articles/uk-motor-premiums-continue-to-rise-amidst-covid-19-pandemic

subject to the following constraints:

$$\begin{cases} P_0 \le P_c \le (1+C)P_0 \\ \text{CVaR}_\alpha(-\pi) \le 0 \\ \text{E[LR]} \le \text{LR}_T \\ \text{E[PR]} \le \text{PR}_T \end{cases} . \tag{3.10}$$

The first condition is actually the cap condition. For the trap condition we could have added a new constraint like $P_0(1-C) \le P_c$. However, we now only discuss the cap condition as the trap condition can be discussed in a similar manner.

To solve Equation (3.9) subject to the constraints first we look at the expectation of the profit:

$$\begin{aligned} \text{E}[\pi] &= \text{E}[U+I] \\ &= \text{E}[P_c - L - E + r_B P_c] \\ &= (1+r_B)P_c - E - E[L]. \end{aligned} \tag{3.11}$$

Then, applying the cash invariance property

$$\begin{aligned} \text{CVaR}_\alpha(-\pi) &= \text{CVaR}_\alpha(-P_c + L + E - r_B P_c) \\ &= -(1+r_B)P_c + E + \text{CVaR}_\alpha(L). \end{aligned} \tag{3.12}$$

Taking the explanation of the Loss Ratio, we get

$$\begin{aligned} \text{E[LR]} &= \text{E}\left[\frac{\text{Claims}}{\text{Premiums}}\right] \\ &= \text{E}\left[\frac{\text{Claims}}{P_c}\right] \\ &= \frac{\text{EC}}{P_c} \end{aligned} , \tag{3.13}$$

when the notation EC is the expectation of the claims.

Then taking the expectation of the Premium Rate, we have

$$\begin{aligned} \text{E[PR]} &= \text{E}\left[\frac{\text{Premiums}}{\text{Losses}}\right] \\ &= \text{E}\left[\frac{1}{\text{Losses}}\right] P_c \\ &= \text{EL}P_c \end{aligned} , \tag{3.14}$$

where the notation EL is the expectation of 1 over the losses. Using simulation we can find 1/sample and take the mean.

Therefore, the problem becomes,

$$\max\{(1+r_B)P_c - E - E[L]\} \tag{3.15}$$

subject to

$$\begin{cases} P_0 \leq P_c \leq (1+C)P_0 \\ -(1+r_B)P_c + E + \text{CVaR}_\alpha(L) \leq 0 \\ \frac{\text{EC}}{P_c} \leq \text{LR}_T \\ \text{EL} \times P_c \leq \text{PR}_T \end{cases} \tag{3.16}$$

This problema simpliflies to

$$\max\,(P_c) \tag{3.17}$$

subject to

$$\begin{cases} P_0 \leq P_c \leq (1+C)P_0 \\ \frac{E+\text{CVaR}_\alpha(L)}{(1+r_B)} \leq P_c \\ \frac{\text{EC}}{\text{LR}_T} \leq P_c \leq \frac{\text{PR}_T}{\text{EL}} \end{cases} \tag{3.18}$$

As a result, the solution is

$$P_c = \min\left\{(1+C)P_0, \frac{\text{PR}_T}{\text{EL}}\right\}, \tag{3.19}$$

if

$$\begin{cases} P_0 \leq P_c \\ \frac{E+\text{CVaR}_\alpha(L)}{(1+r_B)} \leq P_c \\ \frac{\text{EC}}{\text{LR}_T} \leq P_c \end{cases} . \tag{3.20}$$

Note that we always have to consider the feasibility condition, which means the intersection of the intervals for the solution is non-empty. This means that we need always to check the following feasibility condition

$$\max\left\{P_0, \frac{E+\mathrm{CVaR}_\alpha(L)}{(1+r_B)}, \frac{\mathrm{EC}}{\mathrm{LR}_T}\right\} \leq \min\left\{(1+C)P_0, \frac{\mathrm{PR}_T}{\mathrm{EL}}\right\}.$$

3.2 Smoothing Rate Change, an improved approach

Multiple changes to the rate structure over time can create a never-ending cycle of Cap and Trap; the method of working to price risks to market rates can create inequality in the rating. These rates are usually calculated based on previous losses and using statistical analysis. Often the actuaries will realize they need to increase the rates to meet the needs of the business. This can mean that the lower risk customers that the insurer wants to keep are subject to discrimination.

To decrease this impact on the lower risk customers, smoothing mechanisms are used. This reduces the swings for the customers when changing the rate from the previous rate to the newly calculated rate. In the past, the Cap and Trap methodology has been used as a smoothing mechanism, but this still negatively impacts the lower risk customers, again putting them at risk of leaving the portfolio. This issue impacts niche insurers more so as they wish to retain as many good customers as possible, whereas less specialized insurers, such as the motor insurance business, can afford to take on a large number of higher-risk customers. The good customers leaving would reduce the quality of the portfolio for a niche insurer and hurt the overall business.

One method that is often used in the industry is creating a hybrid model, which is a model that incorporates the ideas for the new model but at a mid-point with the old model to allow for a smoothing mechanism. This could reduce the impact of the rate change on the policyholders. When the actuaries have decided on a new rating structure, instead of implementing this with a Cap and Trap, they could produce a model which is a combination of the new and old rating structure. This would smooth the transition for the policyholders, and eventually, one could implement the new rating structure later on.

For the model described in Section 3.1 the objective function in Equation 3.9 maximizing E[π] is nothing but maximizing P_c, given the constraints. Therefore this section aims to produce a new method that smooths the objective function.

The idea of a hybrid model can be used. When a rate change is implemented, this hybrid can be used as a smoothing mechanism in order to create a new premium, P_s, the smoothed premium. This will allow the rates to be adjusted to the smoothed premium before it reaches the calculated premium, P_c, which is determined by the rate change calculations. The smoothed premium will be determined by the following equation,

$$P_s = \lambda P_0 + (1-\lambda)P_c. \tag{3.21}$$

Here a smoothing factor λ is used. The parameter λ can be considered as a function of P_0 and P_c. However, to have a robust ground for our discussions in terms of the decision making fundamentals, a new objective function can be considered:

$$\min \quad \{(P_s - P_c)^2 + \beta(P_s - P_0)^2\}. \tag{3.22}$$

Here, the parameter β depends on the company's preference. In general, the objective wants to keep the difference between P_s and P_0 on one hand, and between P_s and P_c on the other hand, as small as possible. Larger β means we want the prices to be closer to P_0 and as noted depends on the insurance company attitude towards pricing. However, the choice of the β can also reflect the companies preference towards price discrimination as well. Let us consider that we have N_{New} potentially new costumers and N_{Loyal} existing loyal costumers. If the company applies a less discriminatory approach by not penalizing the loyal and rewarding the new ones then, the objective would be:

$$\min \quad \{N_{New}(P_s - P_c)^2 + N_{Loyal}(P_s - P_0)^2\} . \tag{3.23}$$

So, one can consider $\beta = \frac{N_{Loyal}}{N_{New}}$. On the contrary, if the pricing is more prone to discrimination i.e., the new clients are rewarded and the loyal are penalized, and we have the opposite:

$$\min \; \left\{N_{Loyal}(P_s - P_c)^2 + N_{New}(P_s - P_0)^2\right\}. \tag{3.24}$$

As such β is given by $\beta = \frac{N_{New}}{N_{Loyal}}$

Incorporating (3.22) into the constraints described and adapting them to the following for the smoothed hybrid model;

$$\begin{cases} P_0 \leq P_s \leq P_c \\ \mathrm{CVaR}_\alpha(-\pi) \leq 0 \\ \mathrm{E[LR]} \leq \mathrm{LR}_T \\ \mathrm{E[PR]} \leq \mathrm{PR}_T \end{cases} . \tag{3.25}$$

The cap (and trap) constraint has been removed as the smoothing is used as an alternative method here. To solve the smoothing problem we have

$$\min\,(P_s - P_c)^2 + \beta(P_s - P_0)^2, \tag{3.26}$$

subject to,

$$\begin{cases} P_0 \leq P_s \leq P_c \\ \frac{E+\mathrm{CVaR}_\alpha(L)}{(1+r_B)} \leq P_s \\ \frac{\mathrm{EC}}{\mathrm{LR}_T} \leq P_s \leq \frac{\mathrm{PR}_T}{\mathrm{EL}} \end{cases} . \tag{3.27}$$

To solve this problem note that the solutions are either at the boundary or not. For that we have to verify P_s at the objective for the following four values and take the mínimum.

1.The first solution is found by taking the minimum using the second inequality in (3.27):

$$P_1 = \frac{E + \mathrm{CVaR}_\alpha(L)}{(1 + r_B)}. \tag{3.28}$$

2.The second solution is found by taking the minimum using the lower limit of the third equation of (3.22)

$$P_2 = \frac{\mathrm{EC}}{\mathrm{LR}_T}. \tag{3.29}$$

3.The third solution is found by taking the upper limit of the third equation of 3.29

$$P_3 = \frac{\mathrm{PR}_T}{\mathrm{EL}} \tag{3.30}$$

4.And finally using the following first order condition:

$$(P_s - P_c) + \beta(P_s - P_0) = 0, \tag{3.31}$$

we find that

$$P_4 = \frac{P_c + \beta P_0}{(1+\beta)}. \tag{3.32}$$

We always have to consider the feasibility condition, which is nothing but to make sure if the intersection of all constraints are non-empty. If the feasibility condition holds then the minimum can be found as follows:

- if $\max\{P_1, P_2, P_3\} < P_4$, then $P_s = \max\{P_1, P_2, P_3\}$;
- if $P_4 < \min\{P_1, P_2, P_3\}$, then $P_s = \min\{P_1, P_2, P_3\}$;
- Otherwise, $P_s = P_4$.

In the following figures we show the cases that is explained here:

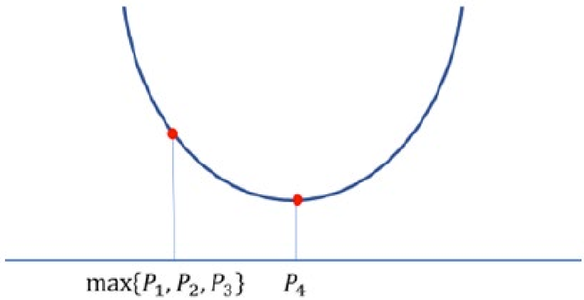


Figure 3.1: The three cases to find the optimal solution with feasibility

condition when max $\{P_1, P_2, P_3\} < P_4$.

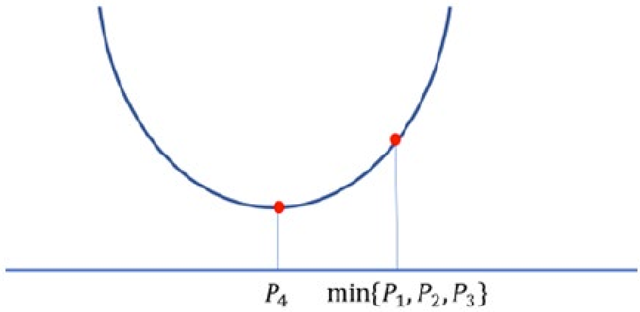


Figure 3.2: The three cases to find the optimal solution with feasibility condition when $P_4 < \min\{P_1, P_2, P_3\}$.

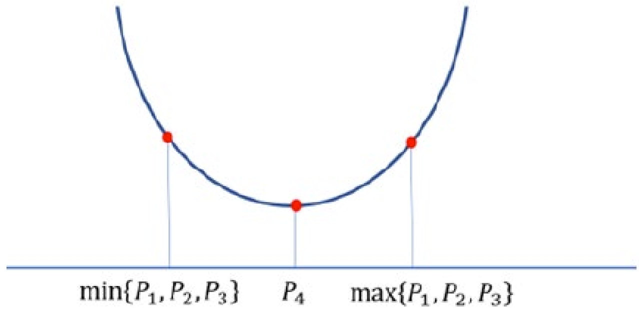


Figure 3.3: The three cases to find the optimal solution with feasibility condition when the first and the second conditions do not hold.

4 Simulation of Mathematical Approach

4.1 Deriving a formula for P_c

In Asanga et al. (2014) it was assumed that claim amounts follows a lognormal distribution. In order to simulate the results produced in Section 3.1 we must evaluate 'EC' and 'EL'. For that we have chosen to use a lognormal distribution for the loss variable.

The losses L are defined as

$$L = e^{\mu+\sigma X}, \tag{4.1}$$

$$\frac{1}{L} = e^{-\mu+\sigma(-X)}, \tag{4.2}$$

where X has a standard normal distribution with mean 0 and variance 1. Taking the expectation, we get,

$$EL = E\left[\frac{1}{L}\right] = E\left[e^{(-\mu+\sigma(-X))}\right] = e^{-\mu+\frac{1}{2}\sigma^2}. \tag{4.3}$$

Note that - X is also a normal standard distribution with mean 0 and variance 1. So the Premium Rate is

$$E[PR] = P_c e^{-\mu+\frac{1}{2}\sigma^2}. \tag{4.4}$$

For the insurance contracts we use a quota-share policy, for a ratio $l \in [0,1]$.

So, we can find the EC as:

$$EC = l e^{\mu+\frac{1}{2}\sigma^2}. \tag{4.5}$$

Therefore, for E[LR] and E[PR] we have

$$E[LR] = \frac{e^{\mu+\frac{1}{2}\sigma^2}}{P_c}, \tag{4.6}$$

$$E[PR] = P_c e^{-\mu+\frac{1}{2}\sigma^2}. \tag{4.7}$$

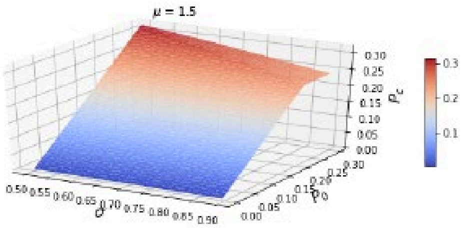
The solution of P_c in Equation (3.19) becomes

$$P_c = \min\left\{(1+C)P_0, PR_T e^{\mu-\frac{1}{2}\sigma^2}\right\}. \tag{4.8}$$

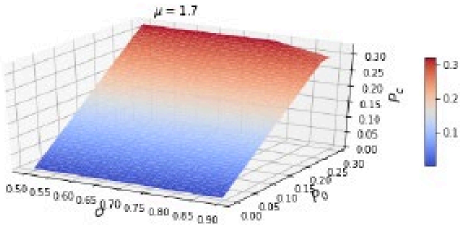
Note that in all cases we assume the parameters are chosen so that the feasibility conditions hold.

4.2 Simulation

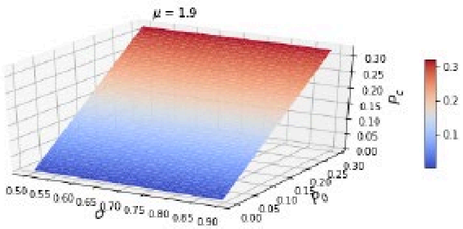
This solution was simulated for values of μ equal to 1.5, 1.7 and 1.9 with σ varying from 0.5 to 0.9 and P_0 varying from 0 to 0.3. We also take $E = 0$ and α= 99.5%. Note that we have to choose parameters to maintain the feasibility conditions. Here we present the results in the following figures:



(a)



(b)



(c)

Figure 4.1: The surfaces for the varying values of μ. Here the x axis is σ, y axis is P_0 and the z axis is the outcome of the minimizing function.

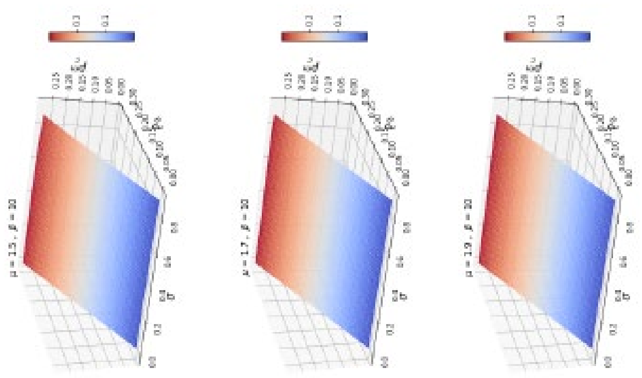
4.3 Simulation of Improved Approach

Now we define a function from Equation (3.30).

$$f(x) = (x - P_c)^2 + \beta(x - P_0)^2 \tag{4.9}$$

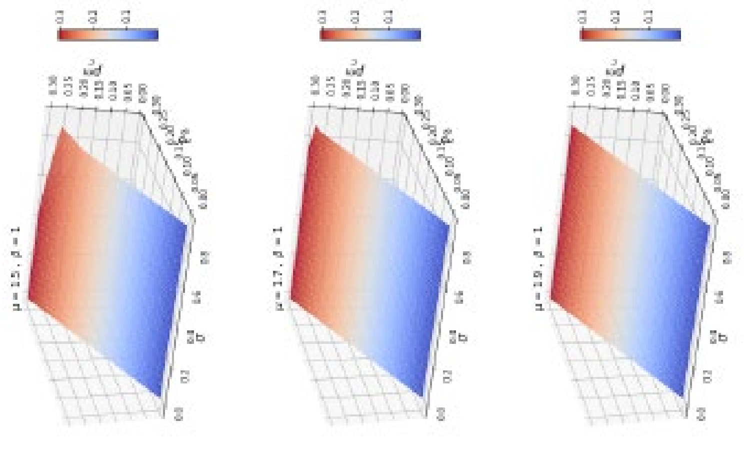
This is then minimized, and we set β as 0.1,1 and 10. Here μ, σ, E, α and P_0 take values as in the previous simulation.

If β is set as 10 then Equation (4.9) is closer to P_0 in order to make it minimum, so a company that chooses this would wish to be closer to the original premium. This could be used in the case of a gradual rate change. Whereas if β is set as 0.1 then Equation (4.9) is closer to P_c the newly calculated premium. However, for a selection of $\beta = 1$, the prices would be something in between. We have done this for all three parameters $\beta = 0.1, 1, 10$ for the parameters of the example in the Figure 4.1. For simplicity we also have considered $l = 1$. You can observe the differences in the Figures 4.2-4.4.



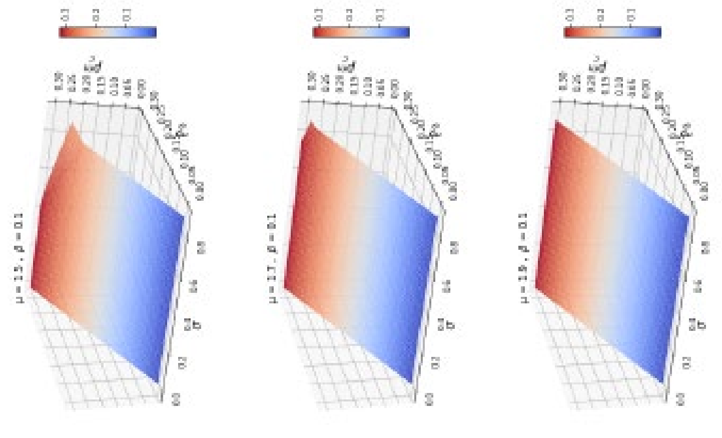
(a) (b) (c)

Figure 4.2: The surfaces for the varying values of μ and β



(a) (b) (c)

Figure 4.3: The surfaces for the varying values of μ and β.



(a) (b) (c)

Figure 4.4: The surfaces for the varying values of μ and β.

4.4 Discussing results

The idea of the improved method was built upon in Section 3.2, adapting the previous Cap and Trap method to a newly proposed smoothing method, taking inspirations from the idea of a hybrid premium P_s.

Then using the lognormal distribution, expressions for E[LR] and E[PR] were found by Equations (4.6) and (4.7), which were then used to find the full expression for Equation (3.19) in terms of these values given in Equation (4.8).

By introducing a smoothing mechanism into the simulation, it reduces the sharp jump in the rate change from one rate to another. Therefore, the customers' price would not increase as dramatically as it would with the Cap and Trap methodology and possibly reduce the price discrimination.

The size of μ has little effect on the overall surface in Figures 4.1-4.4 , as μ increases this stretches in the z-axis direction; $\frac{8}{100}e^{\mu-\frac{1}{2}\sigma^2}$ therefore, this increases the value of P_c. When $1.1P_0$ is greater than then σ does not have any effect on $1.1P_0$ hence the constant line on the y-axis.

As described, we consider a quota-share policy. Increasing σ, therefore increasing the risk in the simulation, causes the value for P_c to decrease. This is quite a surprising result as one would expect the opposite result. This may be due to the constraints of E[PR] and E[LR]. Additionally, it must be acknowledged that there are limitations to this solution as it only applies to the $P_0 \leq P_c$. points where

For the improved simulation results detailed in the Figures 4.2-4.4 the larger β the outcome of minimizing the function is also larger, shown by the z-axis being larger for $\beta = 0.1$ than that of $\beta = 10$.

Figures 4.2-4.4 concluded that the solution to the minimum of P_c produces a surface of Equation (3.29). Therefore, in an insurance context, the solution to the smoothing approach is Equation (3.29) is the ideal smoothed premium to be used to improve the issues presented by the Cap and Trap.

As explained in Section 2, the main problem with the Cap and Trap methodology is that applying multiple changes to the rate structure can risk the cycle of the Cap and Trap, thus risking the company losing good customers. Therefore, alternative methods to smoothing the rate change is discussed, building on from the mathematical approaches described in Sections 3.1 and 3.2. In most of the cases that we study the optimal solution is attained at

$$P_s = \frac{P_c + \beta P_0}{(1+\beta)}.$$

Using the smoothing terminology this means the smoothing parameter is equal to $\lambda = \frac{\beta}{1+\beta}$. Now if we use the cases for non-discriminatory and discriminatory smoothing as discussed above we get:

- **Non-discriminatory smoothing**:

$$\beta_{ND} = \frac{N_{Loyal}}{N_{New}} \Rightarrow \lambda_{ND} = \frac{N_{Loyal}}{N_{New} + N_{Loyal}}.$$

- **Discriminatory smoothing:**

$$\beta_D = \frac{N_{New}}{N_{Loyal}} \Rightarrow \lambda_D = \frac{N_{New}}{N_{New} + N_{Loyal}}.$$

As one can see, a smoothing pricing approach will put more weight on P_0 in the non-discriminatory approach if the number of Loyal customers are relatively higher, while the opposite holds for a discriminatory approach.

5 Concluding remarks

The Cap and Trap methodology was investigated in this paper. There were some limitations, however, with no published literature detailing the methodology involved for this pricing process, this can be regarded as the first attempt in this topic. We have discussed the problems that may arise by using the Cap and Trap methodology; many of the explanations and inspirations for the various ideas detailed come from discussions with qualified actuaries and other senior insurance professionals.

As an alternative to Cap and Trap methodology, we proposed a smoothing process for pricing that is based on the insurance company's preference. This can better clarify if the smoothing process is towards less or more price discrimination. We have seen this can readily be linked to proportion of the new customers to the total number of new and loyal customers. Our framework also can explain how the evolution of the risk variable parameters including variance and mean changes the smoothing process. We also have observed that the regulatory and market restrictions have a great role in such changes.

Discussing the effects of current pricing methods on price discrimination is becoming increasingly important. In July 2019, the Financial Conduct Authority announced that they would act to protect all consumers who are adversely affected by price discrimination. This includes those loyal and timepoor consumers as well as those deemed vulnerable. Therefore, researching into whether the Cap and Trap methodology subjects consumers to price discrimination is important for all insurers who use this methodology to consider. Actuaries work with rate changes for many reasons, one is to indicate how well a company is performing and the other is for formulating a loss ratio projection for a book of business, Bodoff (2009). This process allows them to calculate the minimum premium subject to the insurance company's constraints. Multiple factors are considered, which could impact the risk of a customer and their probability of future losses. Often lower risk groups can obtain lower premiums, as these groups will benefit the insurer's portfolio of risks and lower their expenses and losses.

Additional research is recommended on the Cap and Trap methodology as this is an area currently lacking in academia. This methodology should be analyzed and discussed whether or not it is an appropriate pricing method for the current insurance industry. For this project, data was simulated using the lognormal distribution for the simplicity of the model. It is recommended that this could be repeated with real-life pricing data, or this could be compared to an exponential distribution to analyze which produces the most accurate results. To improve the simulation and to model it as close as possible to a real-life simulation, one could take a subset of the distribution so that the premiums are more clustered as they will be in reality.

For the future one can also consider other price smoothing processes. For instance, another alternative to the smoothing process we proposed in this paper, is the credibility theory. This is particularly useful to make up for risks such as the limited size of the historical data and additional risks. So, this way insurers may use market information to complement their current portfolio data Parodi (2014) which is known as Credibility Rating (see Bühlmann and Gisler (2006)). However, one issue with this methodology is that the data is not always relevant to the policies that insurance companies handle, particularly in the case of specialty insurers, who are most likely to face such an issue, this is an additional problem.

To summarize, the Cap and Trap method was analyzed and simulated. This was compared with an improved method introducing an alternative smoothing approach which produced a possible calculated premium that insurers could use. This approach produced less harsh of a jump from the original premium to the newly calculated premium from the rate change. This significant jump is what can cause price discrimination against customers.

6 Acknowledgments

The authors would like to thank the anonymous referee(s) who have contributed to the development of the contents with helpful suggestions. Many thanks to the team at Azur Underwriting Ltd, including Lara Korz, Kate Wells and Dimitris Schizas who provided us with some insight on various topics throughout the research. Additionally the authors would like to thank Tom Baines, Actuary at Co-operative Insurance, for his help in understanding of the fundamentals in actuarial methods. Thanks also goes to Malcolm Slee, Retired Actuary and Institute and Faculty of Actuaries Council Member, Tahvia O'Hart, Claims Analyst Manager at RAC WA and Michael Tripp, Partner at Mazars, Chair of the GI Board at the Institute and Faculty of Actuaries for their insight into alternative methods to the Cap and Trap and smoothing rate change. Finally, we thank Margot Agnew, who has proofread the paper.

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