

UNIVERSIDAD CARLOS III DE MADRID, PUERTA DE TOLEDO

Quantification of Model Risk with Bootstrapping Method

Trabajo Fin de Master

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Quantification of Model Risk with Bootstrapping Method

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A handwritten signature in black ink, appearing to be the name '徐东' (Xu Dong).

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Part I Introduction

Before the 2007 crisis, traditional financial institution relied heavily on standard risk models. Failures of these models in the crisis era resulted in significant financial loss, as standard risk models are insensitive and slow to respond to the sudden change in volatility, often resulting in overestimated required capital in post-crisis era. These failures led to the interest of both academic world and financial institutions to understand the model risk, which was regarded as one of the most important reasons that contributed to the failures of standard risk models.

The concept of the model risk has been debated and there is no single unified definition of model risk. In general, model risk is often used in a wide sense and refers to any potential errors, misspecification of model or improper operation that may lead to inaccuracy in the results of the model. In this paper, model risk is defined as excessive frequency of hits, when actual return is less than VaR forecasts, and clustering of the hits / violations. In mathematical terms, the model risk of excessive hits and violation clustering can be tested by likelihood ratio test of unconditional coverage and independence proposed by Christoffersen (1998) .

The concept of the optimal capital adjustment q is introduced in order to quantify the model selection criteria mentioned before. Backtesting procedure is implemented to search for a minimal capital adjustment q that allows the series of VaR forecasts to pass the likelihood ratio test of unconditional coverage and independence using a one-year moving window. With the backtesting procedure, this paper intends to quantify the model risk for various risk model and thus facilitate the model selection process.

This paper uses Dow Jones Industrial Average (DJI) and S&P 500 (GSPC) indexes as original data. Log return of each index is used as input for modeling and forecast process. Common GARCH family models, historical simulation and normal distribution are employed as standard risk modes. The VaR forecasts are then calculated using a four-year moving window (1020 points). The parameters of the GARCH models were calibrated every single year (255 points) since it is common in market practice.

Asymmetric GARCH model is expected to outperform standard GARCH model given the empirically observed asymmetric volatility. However, the findings of this paper suggest that the standard GARCH model performs relatively well within GARCH family models as it derives relatively stable capital adjustment. Since only frequency of hits and independence of hits are taken into consideration when searching for optimal capital adjustment, further study could incorporate the magnitude criteria to study its impact on the capital adjustment.

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This paper also studies the relationship between model risk and significance level of VaR estimates. The findings indicate that the model risk boosts when significance level decreases.

The optimal capital adjustment method offers an applicable method to quantify model risk. By tailoring the tests used in backtesting, it allows a subjective method to evaluate model risk and study the evolution of the model risk across time.

Part II Literature Review

Insufficient understanding of model risk has emerged as a fundamental reason of the failures observed in prevalent risk models in the 2007 crisis. As a result, the concept of model risk has been brought into the scope of regulatory and academic interest.

According to Green and Figlewski (1999), Cont (2006) and Hull and Suo (2002), there are three distinct types of model risk. First of all, the model could be poorly specified, and therefore it could not serve its purpose of risk management or asset valuation. Moreover, the application of a given model is under question if some strong assumptions have to be made without solid quantitative support. This usually happens when crucial inputs are not observable from the market. Furthermore, the choice of model can also be problematic and usually subject to expert judgment. Calibration and estimation error of parameters and inappropriate assumptions of the theoretical framework also contribute to the uncertainty of a given quantitative model (Alexander and Sarabia, 2012). In general, model risk is often used in a wide sense and refers to any potential errors, misspecification of model or improper operation that may lead to inaccuracy in the results of the model.

It is widely recognized that various candidate models may lead to significant difference in risk forecasts, especially in crisis era when volatility peaks. Therefore, the selection of the most suitable model, as mentioned above, contributes to one of the most prominent model risks. Boucher et al.(2014) explored an applicable methodology to quantify the degree of suitability of candidate models through backtesting techniques. In their study, a series of dynamical adjustment of risk capital was calculated by comparing historical realized return and imperfect risk forecasts generated by candidate models. Considering some subjective desirable features of risk model, such as frequency, independence and magnitude of violations, they proposed that the adjustment of risk capital could be used as an indicator of the suitability of the candidate models. Inspired by Boucher et al.(2014), this paper employs the risk capital adjustment method via backtesting techniques to compare popular value at risk (VaR) models, a standard measure to quantify financial risks.

Introduced by J.P Morgan in 1994 as a method of risk management, VaR has quickly become the most applied practical tool to quantify risk. VaR is defined as the maximum potential deviation in return of a financial instrument with a given probability over a fixed horizon. The measure has gained popularity as a sophisticated risk management tools thanks to the increased volatility observed in the financial market during last decades. VaR measures serve as an internal risk management tool, at the same time, some regulatory bodies obligate financial institutions to calculate required capital based on VaR estimates. The accuracy of the

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VaR estimates is therefore of paramount importance to achieve optimal capital allocation.

Given the wide application of VaR measures in financial industry, especially for risk management and regulatory purposes, various methodologies were proposed to provide accurate estimates of VaR. According to Manganelli (2001), existing models for calculating VaR can be divided into three categories: parametric, Nonparametric and semiparametric models.

Some typical parametric VaR models include RiskMetrics and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) family models. As discovered by Mandelbrot (1963) and Fama (2007), distributions of financial return are typically leptokurtotic and negatively skewed, hence fat-tailed distributions, for instance, student t distribution or normal inverse Gaussian distribution, could depict better financial returns than widely applied normal distribution. Some research was focusing on comparing VaR models within GARCH family. Awartani and Corradi (2005) suggested that asymmetric GARCH model outperformed standard symmetric GARCH model. Their findings are aligned with empirical market observation that stock returns are usually negatively correlated with volatility.

Historical simulation is one of the most popular nonparametric methods for VaR estimation. It significantly simplifies the calculation as no parametric assumption has to be made. However, as summarized by Manganelli (2001), some implicit hidden assumptions of this approach should be taken into considerations. First of all, although no explicit assumption of return distributions is made, all the returns within the rolling window are assumed to follow the same distribution. Moreover, the length of the window is a delicate issue. On the one hand, the quantile estimator only is consistent when the window size is infinite. On the other hand, the window size must not be too large since the forecasts are meaningful only the historical data follow the same distribution.

Part III Data and Descriptive Statistics

Daily adjusted closing prices of the Dow Jones Industrial Average (\hat{DJI}) and the Standard & Poor's 500 (\hat{GSPC}) are derived from Yahoo API. Log return of the indices is calculated and used as input of the VaR measures. The market return using the log function is defined as:

$$R_t = \log \frac{p_t}{p_{t-1}}$$

Where p_t denotes price on day t .

Descriptive statistics of the input indices are separately illustrated below.

1. Dow Jones Industrial Average (\hat{DJI})

The Dow Jones Industrial Average is a stock market index created by Wall Street Journal editor and Dow Jones & Company co-founder Charles Dow. Its components are 30 largest public companies listed in the United States. The index is a price-weighted average of its components taking into account the effects of stock splits and other adjustments. Although the index is supposed to generally depict the performance of the industrial sector, it is influenced by other macroeconomic factors of the economy.

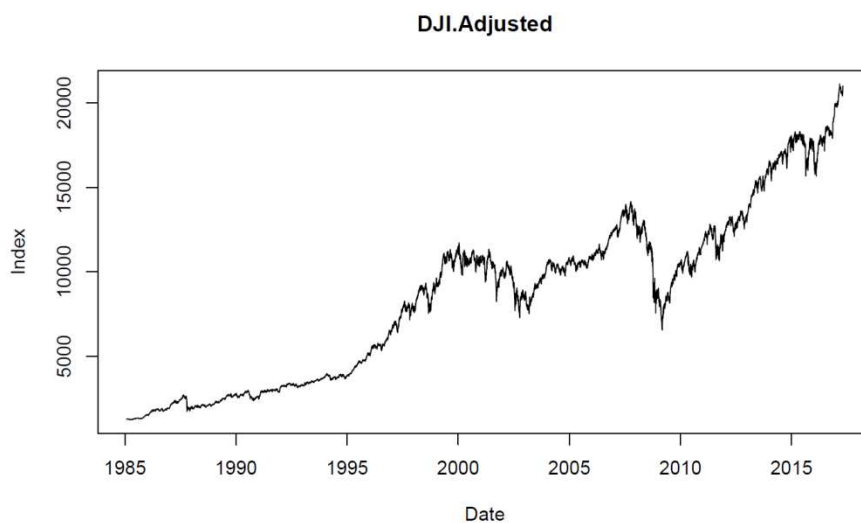


Figure 1

The time series of observed log returns of the Dow Jones Industrial Average index on a daily basis consists of 8129 points (from 29 January 1985 to 26 April 2017). As large changes tend to be followed by large changes while small fluctuations tend to be followed by small fluctuations, volatility clustering is observable, which is typical in financial time series.

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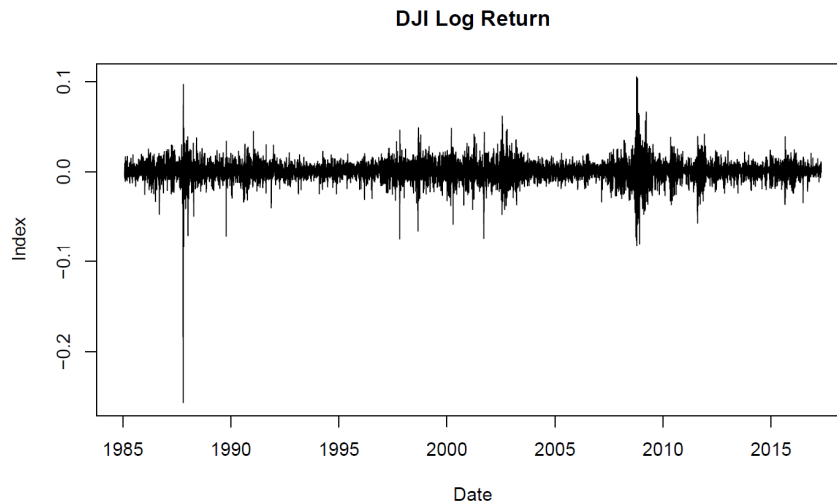


Figure 2

The autocorrelation function (ACF) plot of log return suggests potential first order auto-regression while the plot of partial autocorrelation function (PACF) of log return does not demonstrate the existence of moving average.

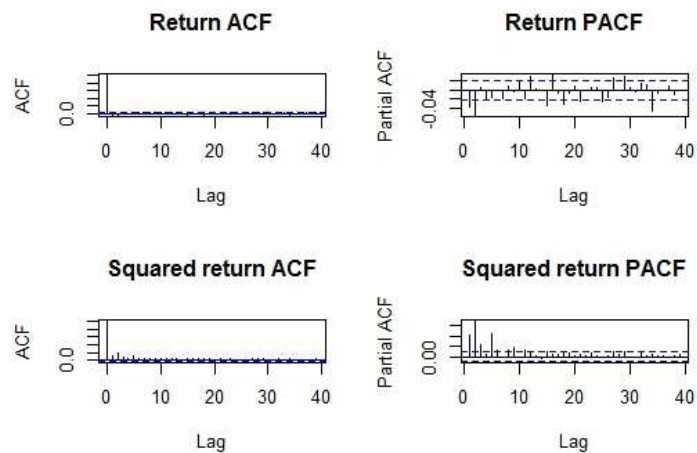
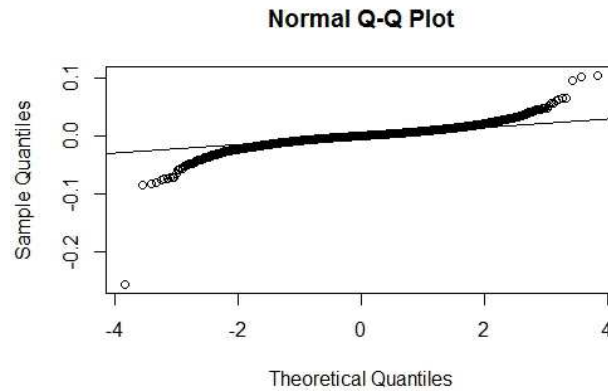


Figure 3

The non-linear Q-Q plot shows that the empirical distribution differs from normal distribution. The result of the KS test ($p < 0.05$) confirms the deviation by rejecting the null hypothesis of normality.

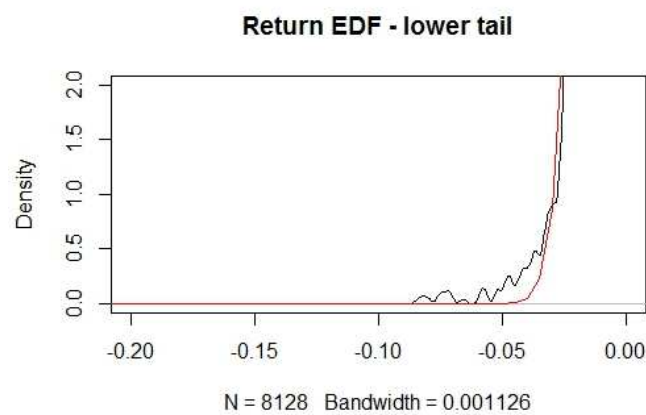
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*Figure 4*

One-sample Kolmogorov-Smirnov test
data: log return of Dow Jones Industrial Average (DJI)
D = 0.085066, p-value < 2.2e-16
alternative hypothesis: two-sided
FAIL, REJECT THE NULL HYPOTHESIS AND LD.DJI IS NOT NORMAL.

Table 1

As shown in the figure below, the density curve of the empirical distribution has more extreme values in its lower tail than the theoretical normal distribution curve, suggesting fat-tailed nature of the observed logarithmic return series.

*Figure 5*

2. Standard & Poor's 500 (^GSPC)

The Dow Jones Industrial Average is a stock market index based on the market capitalizations of 500 large companies having common stock listed on

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the NYSE or NASDAQ. The weightings of its components are determined by S&P Dow Jones Indices. The index is considered as one of the most representative indices of the U.S stock market and an general indicator of the U.S economy.

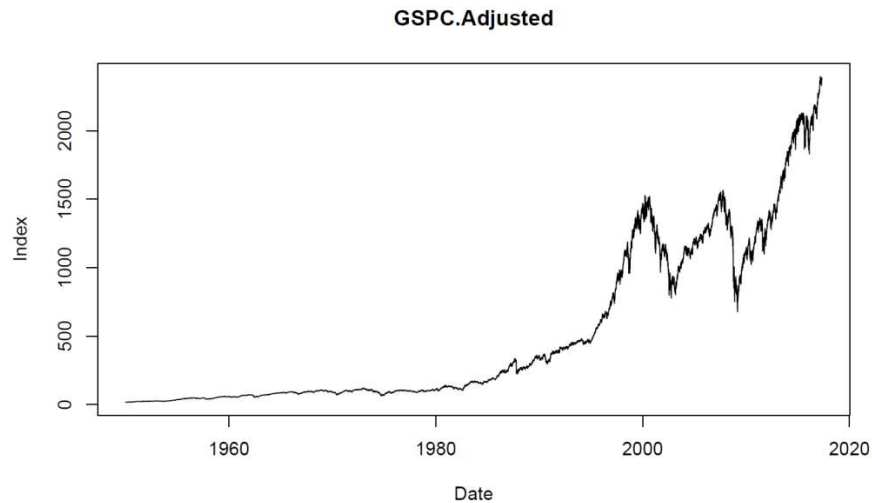


Figure 6

The time series of observed log returns of the Standard & Poor's 500 index on a daily basis consists of 16939 points (from 3 January 1950 to 26 April 2017). As large changes tend to be followed by large changes while small fluctuations tend to be followed by small fluctuations, volatility clustering is observable, which is typical in financial time series.

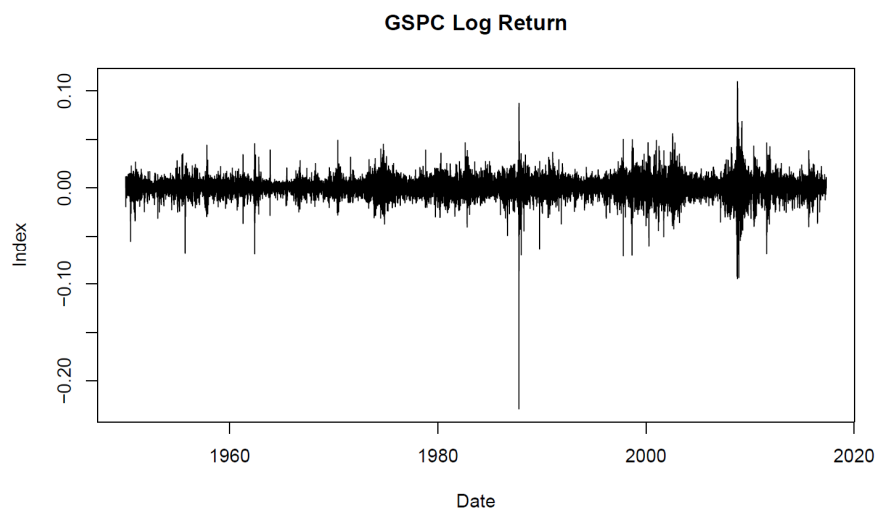


Figure 7

The autocorrelation function (ACF) plot of log return suggests potential first order auto-regression while the plot of partial autocorrelation function (PACF) of log return does not demonstrate the existence of moving average.

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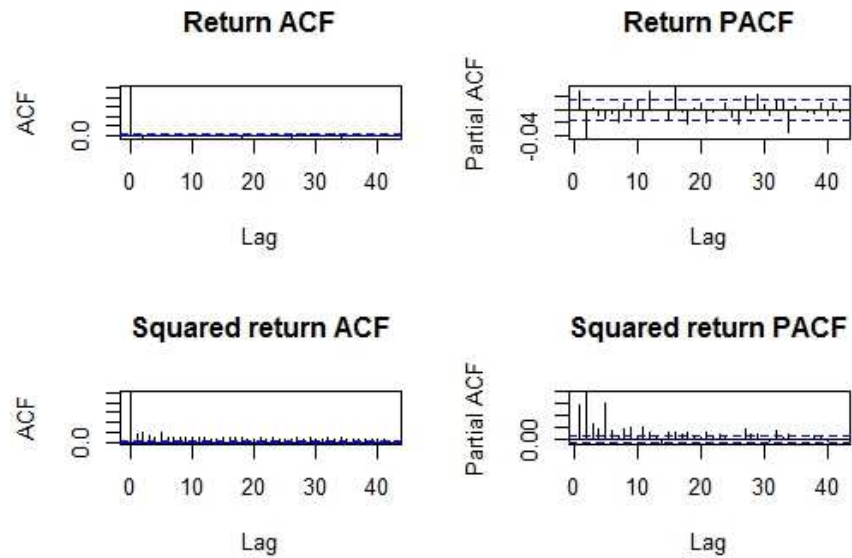


Figure 8

The non-linear Q-Q plot shows that the empirical distribution differs from normal distribution. The result of the KS test ($p < 0.05$) confirms the deviation by rejecting the null hypothesis of normality.

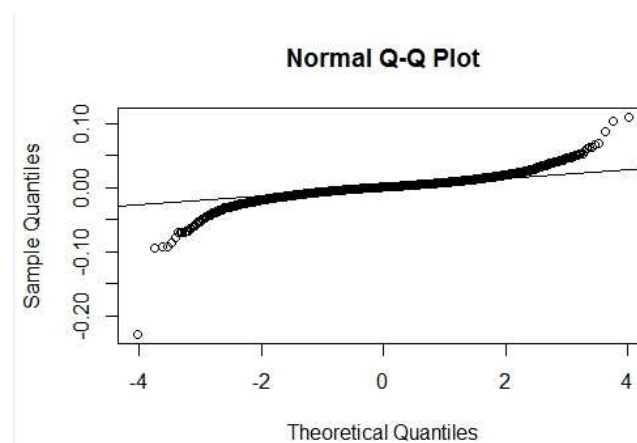


Figure 9

One-sample Kolmogorov-Smirnov test
data: log return of S&P 500 (GSPC)
D = 0.07595, p-value < 2.2e-16
alternative hypothesis: two-sided
FAIL, REJECT THE NULL HYPOTHESIS AND LD.GSPC IS NOT NORMAL.

Table 2

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As shown in the figure below, the density curve of the empirical distribution has more extreme values in its lower tail than the theoretical normal distribution curve, suggesting fat-tailed nature of the observed logarithmic return series.

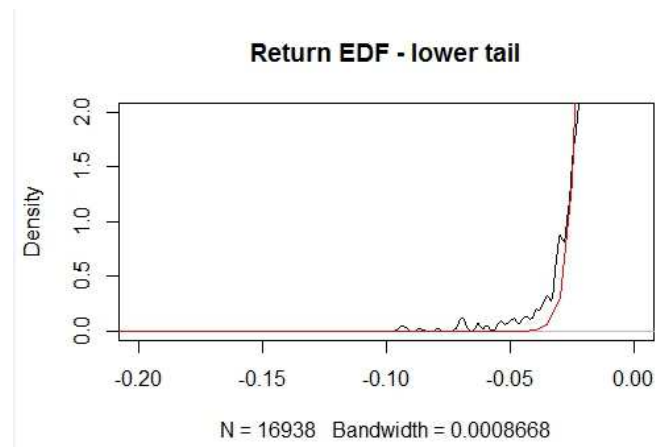


Figure 10

Part IV Methodology

4.1 Definition of VaR

VaR is the maximum potential deviation in return of a financial instrument with a given probability over a fixed horizon. As defined in previous section, R_t is the log return series of the index in time t . The random loss over period $[t, t + t']$ is $\Delta R(t') = -(R_{t+t'} - R_t)$. Let $F_{\Delta R}$ be the cumulative function of the random loss and $F_{\Delta R}(x) = P(\Delta R \leq x)$. Then VaR at significance level α ($\alpha \in (0,1)$) can be defined as the smallest real number satisfying the equation:

$$F_{\Delta R}(x) \geq \alpha$$

Therefore,

$$VaR_\alpha = \inf(x | F_{\Delta R}(x) \geq \alpha)$$

Common market practice of VaR estimation includes econometric evaluation, quantile estimation based on historical simulation and extreme value method.

4.2 Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model

Engle (1982) developed the autoregressive conditional heteroscedasticity model (ARCH). Let r_t be the log return at time t and a_t represent the random factor. The autoregressive moving-average (ARMA) of order p and q is defined as:

$$r_t = \varphi_0 + \sum_{i=1}^p \varphi_i r_{t-i} + a_t - \sum_{j=1}^q \theta_j a_{t-j}$$

where parameters of the equation are given as $\varphi_0, \varphi_1, \dots, \varphi_p, \theta_1, \dots, \theta_q$.

The random factor a_t can be further specified as :

$$a_t = \sigma_t \varepsilon_t$$

where ε_t is an independently and identically distributed random variable with certain distribution. σ_t is the conditional variance of return r_t , defined as $\sigma_t = E((r_t - E(r_t))^2 | \Omega_{t-1})$, where Ω_{t-1} represents all information available until moment $t-1$. GARCH model of order u and v , in general, defines the conditional variance σ_t as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^u \alpha_i a_{t-i}^2 + \sum_{j=1}^v \beta_j \sigma_{t-j}^2$$

where parameters $\alpha_0, \alpha_1, \dots, \alpha_u, \beta_1, \dots, \beta_v$ meet the requirement of:

$$\alpha_0 > 0,$$

$$\alpha_1, \dots, \alpha_u \geq 0,$$

$$\beta_1, \dots, \beta_v \geq 0,$$

$$\sum_{i=1}^{\max(u,v)} (\alpha_i + \beta_i) < 1.$$

In the following sections, different GARCH family models will be presented and their formula will be given based on the definition of GARCH model defined above.

4.2.1 The standard GARCH model

The standard GARCH model (Bollerslev,1986), as mentioned above, is written as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^u \alpha_i a_{t-i}^2 + \sum_{j=1}^v \beta_j \sigma_{t-j}^2.$$

In this paper, only GARCH(1,1) is used for VaR estimation and ε_t is assumed to follow standardized normal distribution. The standard GARCH (1,1) model has the form of:

$$r_t = \varphi_0 + \sum_{i=1}^p \varphi_i r_{t-i} + a_t - \sum_{j=1}^q \theta_j a_{t-j}$$

$$a_t = \sigma_t \varepsilon_t$$

$$\varepsilon_t \rightarrow N(0,1)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

4.2.2 The exponential GARCH model

Nelson (1991) defines the exponential GARCH model as:

$$\log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^u (\alpha_i a_{t-i} + \gamma_i (|a_{t-i}| - E|a_{t-i}|)) + \sum_{j=1}^v \beta_j \log(\sigma_{t-j}^2)$$

where γ_i captures the size effect. The expected value of the absolute standardized random factor a_t is calculated as:

$$E|a_t| = \int_{-\infty}^{\infty} |a| f(a, 0, 1, \dots) da$$

In this paper, only exponential GARCH(1,1) is used for VaR estimation and ε_t is assumed to follow standardized normal distribution. The exponential GARCH (1,1) model has the form of:

$$r_t = \varphi_0 + \sum_{i=1}^p \varphi_i r_{t-i} + a_t - \sum_{j=1}^q \theta_j a_{t-j}$$

$$a_t = \sigma_t \varepsilon_t$$

$$\varepsilon_t \rightarrow N(0,1)$$

$$\log(\sigma_t^2) = \alpha_0 + \alpha_1 a_{t-1} + \gamma_1 (|a_{t-1}| - E|a_{t-1}|) + \beta_1 \log(\sigma_{t-1}^2)$$

4.2.3 The GJR GARCH model

Glosten et al. (1993) proposed the GJR GARCH model which allows the model to asymmetrically incorporate positive and negative shocks by applying the indicator function I .

The conditional variance is defined as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^u (\alpha_i \varepsilon_{t-i}^2 + \gamma_i I_{t-i} \varepsilon_{t-i}^2) + \sum_{j=1}^v \beta_j \sigma_{t-j}^2$$

where γ_i captures the leverage effect. The indicator function is given as:

$$I = \begin{cases} 1, & \text{if } \varepsilon \leq 0 \\ 0, & \text{otherwise} \end{cases}$$

In this paper, only GJR GARCH(1,1) is used for VaR estimation and ε_t is assumed to follow standardized normal distribution. The GJR GARCH (1,1) model has the form of:

$$r_t = \varphi_0 + \sum_{i=1}^p \varphi_i r_{t-i} + a_t - \sum_{j=1}^q \theta_j a_{t-j}$$

$$a_t = \sigma_t \varepsilon_t$$

$$\varepsilon_t \rightarrow N(0,1)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 I_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

4.2.4 The threshold GARCH model

According to Zakoian (1994), the threshold GARCH model is defined as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^u \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}) + \sum_{j=1}^v \beta_j \sigma_{t-j}^2$$

In this paper, only threshold GARCH(1,1) is used for VaR estimation and ε_t is assumed to follow standardized normal distribution. The threshold GARCH (1,1) model has the form of:

$$r_t = \varphi_0 + \sum_{i=1}^p \varphi_i r_{t-i} + a_t - \sum_{j=1}^q \theta_j a_{t-j}$$

$$a_t = \sigma_t \varepsilon_t$$

$$\varepsilon_t \rightarrow N(0,1)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1(|\varepsilon_{t-1}| - \gamma_1\varepsilon_{t-1}) + \beta_1\sigma_{t-1}$$

In this paper, daily VaR estimates are calculated for all GARCH family models mentioned above. The parameters of the GARCH model are derived from a moving window of 4 years (1020 points) and recalibrated every year (255 points).

Daily VaR is estimated with conditional variance. For instance, if the significance level is α , VaR forecast at time t is:

$$VaR_t^\alpha = \text{Value of financial position} \times \Phi^{-1}(\alpha) \times \sigma_t$$

where $\Phi^{-1}()$ is the inverse standard normal distribution.

4.3 Parametric model – Normal distribution

Under the parametric model, daily returns are assumed to be normally distributed. A moving window of 4 year (1020 points) is used to recalibrate parameters every single day. VaR forecasts with the significance level α is obtained as:

$$VaR_t^\alpha = \text{Value of financial position} \times \Phi^{-1}(\alpha) \times \sigma_t$$

where $\Phi^{-1}()$ is the inverse standard normal distribution.

4.4 Nonparametric model – historical simulation of VaR

The historical simulation technique assumes no specific return distribution. Given a moving window of t periods, the VaR estimates of significance level α are calculated as:

$$VaR_t^\alpha = \text{percentil}\{\{r_i\}_{i=1}^t, (100 \times \alpha)\%$$

where $\{r_i\}_{i=1}^t$ is the series of log return defined above.

Compared with other methodologies, the historical simulation technique largely simplifies the estimation process and gives relatively reliable results. However, the implicit underlying assumption is that future returns continue behaving in the same way as historical returns thus historical returns can be used to predict future. The result of the VaR estimates varies a lot depending on the chosen window.

In this paper, moving windows of 4 years (1020 points) and 10 years (2550 points) are used for VaR estimates.

4.5 General Backtesting Procedure

Backtesting is applied to check the reliability of distinct VaR models. Various criteria and tests have been proposed to measure and compare the results of distinct VaR models (Pérignon and Smith, 2010). In this paper, likelihood ratio test of unconditional coverage and independence proposed by Christoffersen (1998) are

used to gauge the accuracy the VaR forecasts. More details regarding the mathematical explanation of the LR tests could be found in the following sector.

The backtesting procedure is implemented to search for a minimal capital adjustment q that allows the series of VaR forecasts to pass the likelihood ratio test using a one-year moving window (255 points). In other words, a sequence of VaR forecasts in time t can be derived from the methodologies mentioned above, given $\{VaR_t(\alpha): t = [1, \dots, T]\}$. The backtesting technique looks for the set of values $\in \mathbb{R}$, such that the sequence: $\{VaR_t(\alpha) - q: t = [1, \dots, T]\}$ passes all the backtests of the LR test. The search for q starts from a relatively big negative value (less prudent) and successively increases by 0.01% until all tests are passed.

4.6 Adjustment Criteria – Frequency and Independence

From the perspective of risk management, a desirable VaR model should satisfy at least two properties. First of all, the actual frequency of violations should be consistent with the chosen expected level of violation (α). Moreover, the violation should be sporadic, which means no violation clustering should be observed.

The indicator variable of hit at time t , I_t is defined as,

$$I_t(\alpha) = \begin{cases} 1, & \text{if } R_t < VaR_{t|t-1}(\alpha) \\ 0, & \text{if } R_t \geq VaR_{t|t-1}(\alpha) \end{cases}$$

where R_t is the return at time t , with $t = \{1, 2, \dots, T\}$, $VaR_{t|t-1}(\alpha)$ is the VaR estimation of time t made at time $t - 1$ for threshold α .

Christoffersen (1998) proposed an operational testing criterion in order to develop easily implementable testing procedures. The sequence of the VaR forecasts $VaR_{t|t-1}(\alpha)$ is efficient if $E[I_{t|t-1, t-2, \dots}] = \alpha$ for all t . This is equivalent to testing that the variable I_t follows Bernoulli distribution with parameter α ,

$$\{I_t(\alpha)\} \sim \text{Bern}(\alpha),$$

Therefore, the total number of VaR exceptions, named Hit_t^α , follows a binomial distribution,

$$\left\{ Hit_t^\alpha = \prod_{t=1}^T I_t(\alpha) \right\} \sim B(T, \alpha)$$

4.6.1 The likelihood ratio test of unconditional coverage

Given a sequence of indicator variable, $\{I_t(\alpha)\}_{t=1}^T$, the null hypothesis is

$$E[I_t] = \alpha.$$

The likelihood under the null hypothesis is

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$$L(\alpha; I_1, I_2, \dots, I_T) = (1 - \alpha)^{n_0} \alpha^{n_1},$$

where n_1 is the number of hits, $n_1 + n_0 = T$.

The alternative hypothesis is

$$E[I_t] \neq \alpha.$$

The likelihood under the alternative hypothesis is

$$L(p; I_1, I_2, \dots, I_T) = (1 - p)^{n_0} p^{n_1}$$

The standard likelihood ratio test can be written as

$$LR_{uc} = -2 \log \left[\frac{L(\alpha; I_1, I_2, \dots, I_T)}{L(\hat{p}; I_1, I_2, \dots, I_T)} \right] \rightarrow \chi^2(1)$$

where $\hat{p} = n_1 / (n_1 + n_0)$. Under the null hypothesis, LR_{uc} follows the asymptotic distribution.

The procedure above only compares the number of violations observed with expected frequency of violations, while it is indifferent to the presence of violations clustering. The test of independence presented below makes up the deficiency by capturing the dynamics in the higher-order moments.

4.6.2 The likelihood ratio test of Independence

Christoffersen (1998) proposed the independence test using an explicit first-order Markov alternative. The transition probability matrix of a binary first-order Markov chain is defined as

$$\Pi = \begin{bmatrix} 1 - \pi_{01} & \pi_{01} \\ 1 - \pi_{11} & \pi_{11} \end{bmatrix}$$

where $\pi_{ij} = \text{Prob}(I_t = j \mid I_{t-1} = i)$.

The transition probability matrix under the null hypothesis of independence is

$$\Pi_0 = \begin{bmatrix} 1 - \pi_n & \pi_n \\ 1 - \pi_n & \pi_n \end{bmatrix}$$

where $\pi_n = (n_{01} + n_{11}) / (n_{01} + n_{11} + n_{00} + n_{10})$, n_{ij} is the number of observations with value i followed by j .

The likelihood under the null hypothesis is

$$L(\Pi_0; I_1, I_2, \dots, I_T) = (1 - \pi_n)^{n_{00} + n_{10}} \pi_n^{n_{01} + n_{11}}.$$

The transition probability matrix under the alternative hypothesis of independence is

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$$\Pi_1 = \begin{bmatrix} \frac{n_{00}}{n_{00} + n_{01}} & \frac{n_{01}}{n_{00} + n_{01}} \\ \frac{n_{10}}{n_{10} + n_{11}} & \frac{n_{11}}{n_{10} + n_{11}} \end{bmatrix}$$

The likelihood under the alternative hypothesis is

$$L(\Pi_1; I_1, I_2, \dots, I_T) = (1 - \pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1 - \pi_{11})^{n_{10}} \pi_{11}^{n_{11}}.$$

The likelihood ratio test of independence is asymptotically distributed and can be specified as

$$LR_{ind} = -2 \log \left[\frac{L(\Pi_0; I_1, I_2, \dots, I_T)}{L(\Pi_1; I_1, I_2, \dots, I_T)} \right] \rightarrow \chi^2(1)$$

4.6.3 The joint test of Coverage and Independence

Christoffersen (1998) proved that the above tests of unconditional coverage and independence can be combined and jointly expressed a single test. The combine likelihood ratio test is also asymptotical, which can be specified as

$$LR_{cc} = LR_{uc} + LR_{ind} \rightarrow \chi^2(2)$$

Thanks to the simplicity of the joint form, the search of the sequence of capital adjustment q is based on this joint test.

Part V Results

5.1 Result of GARCH family models

In order to calculate daily VaR forecasts, it is necessary to determine the best model which can fully represent the observed daily return of the historical time series. All historical data were used for the model estimation to avoid any bias towards a given time period. Although there is no fixed rule for model selection, usually a set of commonly used statistical criteria is taken into account to reflect different aspects of the model estimation, including no remaining autocorrelations, no remaining arch patterns, significant coefficients and AIC.

After various ARMA and GARCH models have been compared, ARMA(0,0)-GARCH(1,1) model gives best result of Dow Jones industrial Average (DJI) and ARMA(0,1)-GARCH(1,1) model fits better S&P 500.

The results of the fitted GARCH family models are demonstrated below. The model with least AIC value is ranked as the best fitted model within GARCH family. The asymmetric models outperform the standard symmetric GARCH model. This makes sense since shock returns are negatively correlated with volatility and asymmetric volatility is observable in the market.

5.1.1 Dow Jones Industrial Average (DJI)

1. Standard GARCH (1,1) – ARMA(0,0)

As demonstrated below, the normal standard GARCH(1,1) gives relatively well estimation of the historical DJI series. All coefficients are significant. Neither arch pattern nor serial correlation can be observed in the residuals. The news impact curve of standard GARCH(1,1) suggests that the change of the conditional variance σ_t is indifferent to the signal of the random component ε_{t-1} .

Standard GARCH(1,1), ARMA(0,0)					RESIDUALS	
Conditional Variance Dynamics					weighted Ljung-Box Test on Standardized Residuals	
-----					-----	
GARCH Model	: sGARCH(1,1)				statistic p-value	
Mean Model	: ARFIMA(0,0,0)				Lag[1]	0.1455 0.7028
Distribution	: norm				Lag[2*(p+q)+(p+q)-1][2]	0.2103 0.8479
					Lag[4*(p+q)+(p+q)-1][5]	1.9728 0.6249
					d.o.f=0	
					H0 : No serial correlation	
Optimal Parameters					weighted Ljung-Box Test on Standardized Squared Residuals	
-----					-----	
	Estimate	Std.Error	tvalue	Pr(> t)	statistic p-value	
mu	0.000624	0.000089	7.0185	0	Lag[1]	0.0007824 0.9777
omega	0.000002	0.000001	3.2003	0.001373	Lag[2*(p+q)+(p+q)-1][5]	0.9057724 0.8806
alpha1	0.0987	0.00786	12.5571	0	Lag[4*(p+q)+(p+q)-1][9]	2.1466910 0.8868
beta1	0.886063	0.008391	105.5923	0	d.o.f=2	

Figure 11

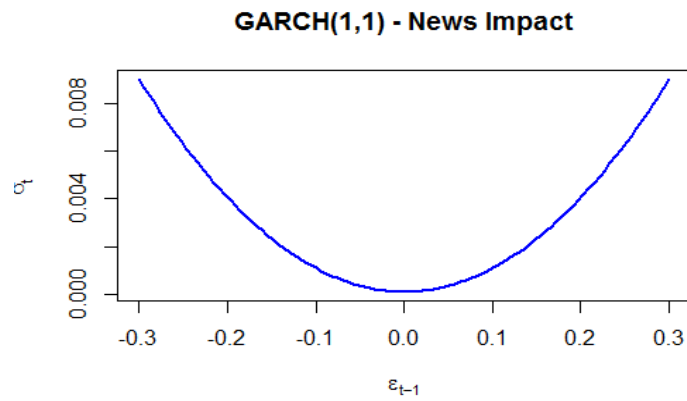


Figure 12

2. Exponential GARCH(1,1) – ARMA (0,0)

As demonstrated below, the normal exponential GARCH(1,1) gives relatively well estimation of the historical DJI series. All coefficients are significant. Neither arch pattern nor serial correlation can be observed in the residuals. The news impact curve of exponential GARCH(1,1) suggests that the change of the conditional variance σ_t is only sensitive to the negative ϵ_{t-1} .

Exponential GARCH(1,1), ARMA(0,0)					RESIDUALS		
Conditional Variance Dynamics					weighted Ljung-Box Test on Standardized Residuals		
-----					-----		
GARCH	Model:	eGARCH(1,1)			Lag[1]	statistic	p-value
Mean	Model:	ARFIMA(0,0,0)			Lag[2*(p+q)+(p+q)-1][2]	0.0001112	0.9916
Distributor	norm				Lag[4*(p+q)+(p+q)-1][5]	0.1049849	0.9162
					Lag[4*(p+q)+(p+q)-1][5]	1.4621303	0.7492
					d.o.f=0	H0 : No serial correlation	
Optimal	Parameters				weighted Ljung-Box Test on Standardized Squared Residuals		
-----					-----		
	Estimate	Std.Error	t value	Pr(> t)	Lag[1]	statistic	p-value
mu	0.00034	0.000085	4.0215	5.80E-05	Lag[2*(p+q)+(p+q)-1][5]	0.1770	0.6740
omega	-0.23213	0.010151	-22.8684	0.00E+00	Lag[4*(p+q)+(p+q)-1][9]	0.2599	0.9876
alpha1	-0.10251	0.005935	-17.2719	0.00E+00	d.o.f=2	1.4199	0.9619
beta1	0.97453	0.001044	933.2272	0.00E+00			
gamma1	0.15463	0.010851	14.2502	0.00E+00			

Figure 13

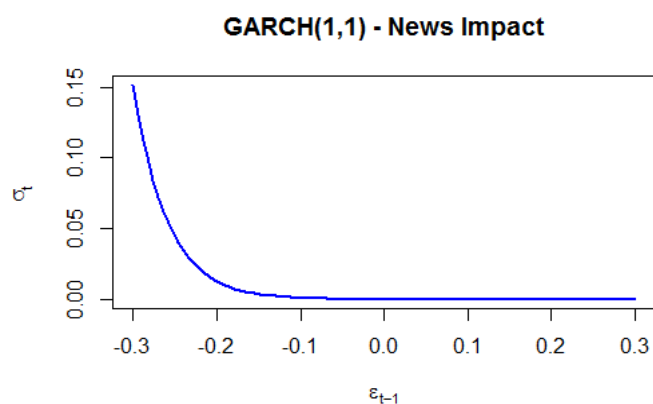


Figure 14

Quantification of Model Risk with Bootstrapping Method

3. GJR GARCH(1,1) – ARMA (0,0)

The normal GJR GARCH(1,1) incorporates an indicator function and gives relatively well estimation of the historical DJI series. All coefficients are significant. Neither arch pattern nor serial correlation can be observed in the residuals. The news impact curve of GJR GARCH(1,1) suggests that the change of the conditional variance σ_t is more sensitive to the negative ε_{t-1} , which represents well the empirical observation of asymmetric volatility.

GJR GARCH(1,1), ARMA(0,0)					RESIDUALS		
Conditional Variance Dynamics					weighted Ljung-Box Test on Standardized Residuals		
-----					-----		
GARCH	Model:	gjrGARCH(1,1)				statistic	p-value
Mean	Model:	ARFIMA(0,0,0)			Lag[1]	0.0555	0.8137
Distribution	norm				Lag[2*(p+q)+(p+q)-1][2]	0.0835	0.9314
					Lag[4*(p+q)+(p+q)-1][5]	1.5393	0.7302
					d.o.f=0		
					H0 : No serial correlation		
Optimal	Parameters				weighted Ljung-Box Test on Standardized Squared Residuals		

		Estimate	Std.Error	t value	Pr(> t)		
mu		0.000383	0.000069	5.5385	0	Lag[1]	0.8903 0.3454
omega		0.000002	0.000001	2.4797	0.013151	Lag[2*(p+q)+(p+q)-1][5]	1.5736 0.7218
alpha1		0.02158	0.005899	3.6581	0.000254	Lag[4*(p+q)+(p+q)-1][9]	2.9498 0.7672
beta1		0.890865	0.003092	288.1625	0	d.o.f=2	
gamma1		0.129326	0.009183	14.0825	0		

Figure 15

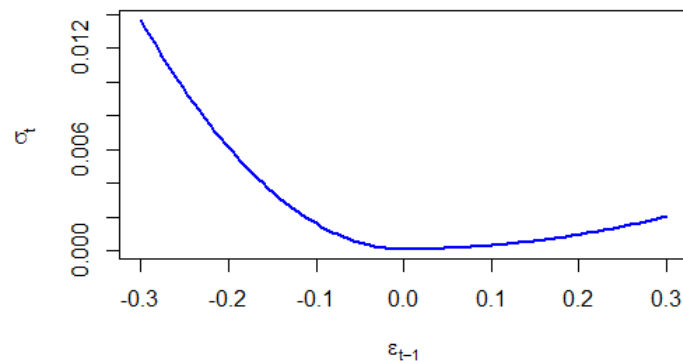
GARCH(1,1) - News Impact

Figure 16

4. Threshold GARCH(1,1) – ARMA (0,0)

The normal threshold GARCH(1,1) incorporates an indicator function and gives relatively well estimation of the historical DJI series. All coefficients are significant. Neither arch pattern nor serial correlation can be observed in the residuals. The news impact curve of threshold GARCH(1,1) suggests that the change of the conditional variance σ_t is more sensitive to the negative ε_{t-1} , which represents well the empirical observation of asymmetric volatility.

Quantification of Model Risk with Bootstrapping Method

Threshold GARCH(1,1), ARMA(0,0)					RESIDUALS				
Conditional Variance Dynamics					weighted Ljung-Box Test on Standardized Squared Residuals				
GARCH	Model:	fGARCH(1,1)			Statistic		p-value		
fGARCH	Sub-Model:	TGARCH			Lag[1]	0.0005143	0.9819		
Mean	Model:	ARFIMA(0,0,0)			Lag[2*(p+q)+(p+q)-1][5]	0.0599028	0.9993		
Distribution norm					Lag[4*(p+q)+(p+q)-1][9]	1.3661302	0.9658		
					d. o. f=2				
Optimal	Parameters				weighted ARCH LM Tests				
	Estimate	Std.Error	t value	Pr(> t)	Statistic		Shape	Scale	P-value
mu	0.000316	0.000082	3.8749	0.000107	ARCH Lag[3]	0.07789	0.500	2.000	0.7802
omega	0.000266	0.000027	9.9388	0	ARCH Lag[5]	0.11379	1.440	1.667	0.9842
alpha1	0.088072	0.005924	14.8668	0	ARCH Lag[7]	0.15080	2.315	1.543	0.9984
beta1	0.906456	0.006209	145.9999	0					
eta11	0.717891	0.052573	13.6552	0					

Figure 17

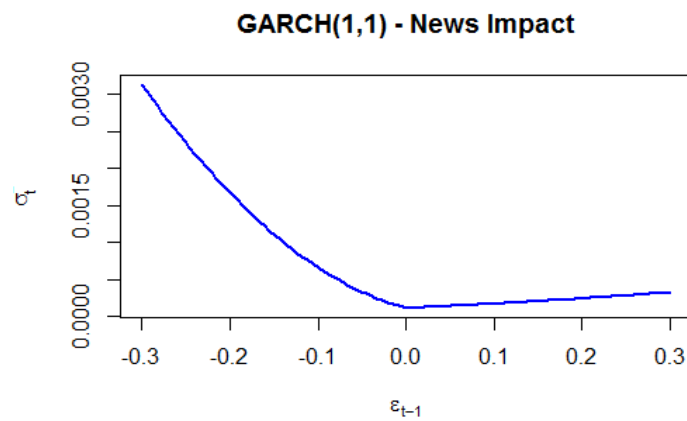


Figure 18

Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) are used to determine the quality of GARCH family models. AIC and BIC give consistent results that threshold GARCH(1,1) – ARMA (0,0) is the preferred model for DJI index. It is worth mentioning that since GJR GARCH model is nested with standard GARCH model, a likelihood ratio test has been conducted. The test result is in line with AIC and BIC that GJR GARCH is better than standard GARCH model.

Summary - model statistics						
		Akaike	Bayes	Shibata	Hannan-Quinn	LogLikelihood
1	Tgarch(1,1), norm	-6.5837	-6.5794	-6.5837	-6.5823	26761.33
2	eGARCH(1,1),norm	-6.5785	-6.5742	-6.5785	-6.5771	26740.21
3	gjrGARCH(1,1),norm	-6.5733	-6.569	-6.5733	-6.5718	26718.84
4	sGARCH(1,1),norm	-6.5515	-6.5481	-6.5515	-6.5504	26629.42

Table 3

Quantification of Model Risk with Bootstrapping Method

5.1.2 S&P 500 (GSPC)

1. Standard GARCH(1,1) – ARMA (0,1)

As demonstrated below, the normal standard GARCH(1,1) gives relatively well estimation of the historical GSPC series. Most coefficients are significant. Neither arch pattern nor serial correlation can be observed in the residuals. The news impact curve of standard GARCH(1,1) suggests that the change of the conditional variance σ_t is indifferent to the signal of the random component ε_{t-1} .

Standard GARCH(1,1), ARMA(0,1)					RESIDUALS	
Condition Variance Dynamics					Weighted Ljung-Box Test on Standardized Residuals	
-----					-----	
GARCH	Model:	sGARCH(1,1)			Lag[1]	statistic p-value
Mean	Model:	ARFIMA(0,0,1)			Lag[2*(p+q)+(p+q)-1][2]	0.5173 0.4720
Distribut	norm				Lag[4*(p+q)+(p+q)-1][5]	1.5212 0.4256
Optimal Parameters					d.o.f=1	2.9067 0.4551
-----					H0 : No serial correlation	
	Estimate	Std.Error	t value	Pr(> t)	Weighted Ljung-Box Test on Standardized Squared Residuals	

mu	0.000483	0.000057	8.4736	0	Lag[1]	statistic p-value
ma1	0.093391	0.008268	11.2958	0	Lag[2*(p+q)+(p+q)-1][5]	10.19 0.001413
omega	0.000001	0.000001	1.2425	0.21406	Lag[4*(p+q)+(p+q)-1][9]	10.54 0.006667
alpha1	0.085701	0.013917	6.1582	0	d.o.f=2	12.56 0.013507
beta1	0.906399	0.013698	66.1706	0		

Figure 19

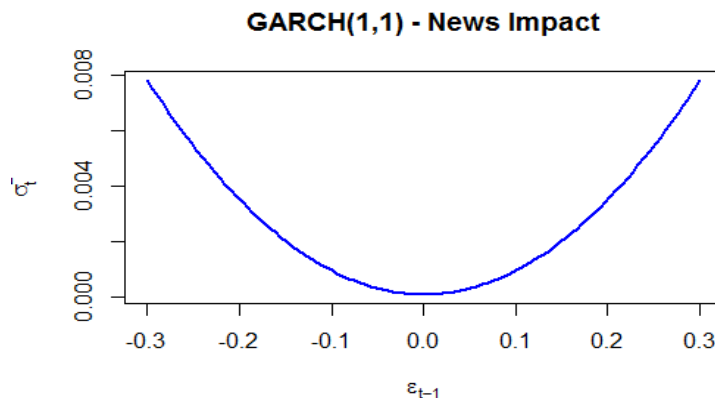


Figure 20

2. Exponential GARCH(1,1) – ARMA (0,1)

As demonstrated below, the normal exponential GARCH(1,1) gives relatively well estimation of the historical GSPC series. All coefficients are significant. Neither arch pattern nor serial correlation can be observed in the residuals. The news impact curve of standard GARCH(1,1) suggests that the change of the conditional variance σ_t is only sensitive to the negative random component ε_{t-1} .

Quantification of Model Risk with Bootstrapping Method

Exponential GARCH(1,1), ARMA(0,0)					RESIDUALS			
Conditional Variance Dynamics					Weighted Ljung-Box Test on Standardized Residuals			
-----					-----			
GARCH	Model:	eGARCH(1,1)				statistic	p-value	
Mean	Model:	ARFIMA(0,0,1)			Lag[1]	0.0006512	0.9796	
Distributor	norm				Lag[2*(p+q)+(p+q)-1][2]	1.2762086	0.5690	
					Lag[4*(p+q)+(p+q)-1][5]	2.8991330	0.4571	
					d.o.f=1			
					H0 : No serial correlation			
Optimal	Parameters				Weighted Ljung-Box Test on Standardized Squared Residuals			
		Estimate	Std.Error	tvalue	Pr(> t)	-----		
						statistic	p-value	
mu		0.000252	0.000033	7.7152	0.00E+00	Lag[1]	19.77	8.756e-06
ma1		0.089484	0.008238	10.8627	0.00E+00	Lag[2*(p+q)+(p+q)-1][5]	19.81	2.191e-05
omega		-0.16646	0.006889	-24.1649	0.00E+00	Lag[4*(p+q)+(p+q)-1][9]	20.71	1.315e-04
alpha1		-0.07393	0.003696	-20.005	0.00E+00	d.o.f=2		
beta1		0.982091	0.000837	1173.174	0.00E+00			
gamma1		0.151232	0.018111	8.3501	0.00E+00			

Figure 21

GARCH(1,1) - News Impact

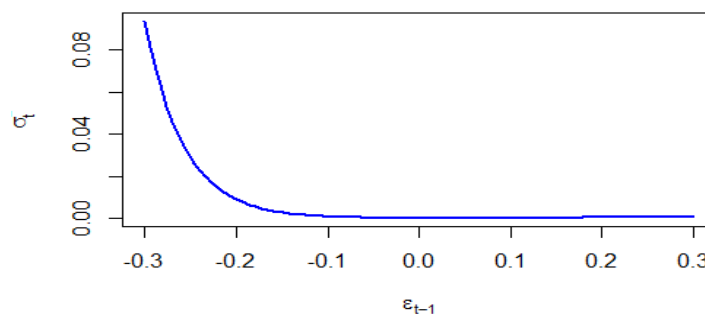


Figure 22

3. GJR GARCH(1,1) – ARMA (0,1)

As demonstrated below, the normal GJR GARCH(1,1) gives relatively well estimation of the historical GSPC series. Most coefficients are significant. Neither arch pattern nor serial correlation can be observed in the residuals. The news impact curve of standard GARCH(1,1) suggests that the change of the conditional variance σ_t is more sensitive to the negative ϵ_{t-1} , which represents well the empirical observation of asymmetric volatility.

GJR GARCH(1,1), ARMA(0,0)					RESIDUALS			
Conditional Variance Dynamics					Weighted Ljung-Box Test on Standardized Residuals			
-----					-----			
GARCH	Model:	gjrGARCH(1,1)				statistic	p-value	
Mean	Model:	ARFIMA(0,0,1)			Lag[1]	0.00937	0.9229	
Distributor	norm				Lag[2*(p+q)+(p+q)-1][2]	0.75915	0.8713	
					Lag[4*(p+q)+(p+q)-1][5]	1.94873	0.7261	
					d.o.f=1			
					H0 : No serial correlation			
Optimal	Parameters				Weighted Ljung-Box Test on Standardized Squared Residuals			
		Estimate	Std.Error	tvalue	Pr(> t)	-----		
						statistic	p-value	
mu		0.00029	0.00006	4.8222	0.000001	Lag[1]	6.136	0.01325
ma1		0.095039	0.0084	11.3142	0	Lag[2*(p+q)+(p+q)-1][5]	6.801	0.05796
omega		0.000001	0.000001	1.5303	0.125935	Lag[4*(p+q)+(p+q)-1][9]	8.576	0.09911
alpha1		0.030134	0.007778	3.8742	0.000107	d.o.f=2		
beta1		0.91012	0.012312	73.9237	0			
gamma1		0.09549	0.010631	8.9818	0			

Figure 23

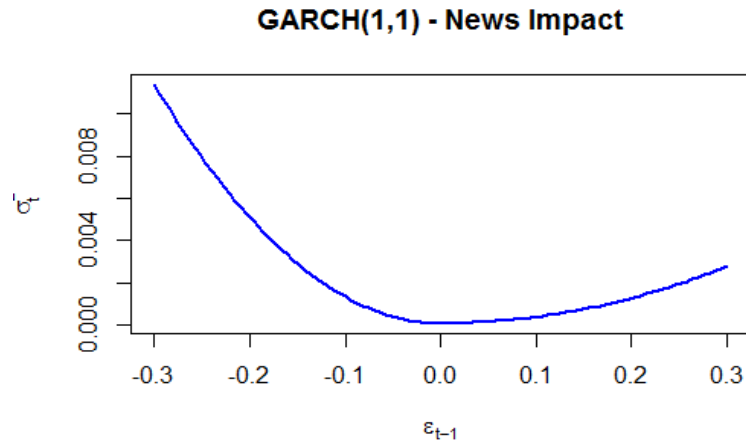


Figure 24

4. Threshold GARCH(1,1) – ARMA (0,1)

The normal threshold GARCH(1,1) gives relatively well estimation of the historical GSPC series. All coefficients are significant. Neither arch pattern nor serial correlation can be observed in the residuals. The news impact curve of standard GARCH(1,1) suggests that the change of the conditional variance σ_t is more sensitive to the negative ε_{t-1} , which represents well the empirical observation of asymmetric volatility.

Threshold GARCH(1,1), ARMA(0,0)					RESIDUALS		
Conditional Variance Dynamics					weighted Ljung-Box Test on Standardized Residuals		
-----					-----		
GARCH	Model:	fGARCH(1,1)			statistic p-value		
fGARCH	Sub-Model:	TGARCH			Lag[1]	0.05067	0.8219
Mean	Model:	ARFIMA(0,0,1)			Lag[2*(p+q)+(p+q)-1][2]	1.44684	0.4672
Distribution	norm				Lag[4*(p+q)+(p+q)-1][5]	3.06462	0.4152
-----					d.o.f=1		
Optimal Parameters					H0 : No serial correlation		
-----					weighted Ljung-Box Test on Standardized Squared Residuals		
-----					-----		
	Estimate	Std. Error	t	value	Pr(> t)	statistic	p-value
mu	0.00025	0.000051	4.8635	0.000001		Lag[1]	27.95 1.247e-07
ma1	0.085578	0.009248	9.2536	0		Lag[2*(p+q)+(p+q)-1][5]	28.37 9.114e-08
omega	0.000159	0.000005	3.2076	0.001338		Lag[4*(p+q)+(p+q)-1][9]	29.49 6.018e-07
alpha1	0.082816	0.016205	5.1106	0		d.o.f=2	
beta1	0.918678	0.018326	50.1284	0			
eta11	0.522215	0.027586	18.9305	0			

Figure 25

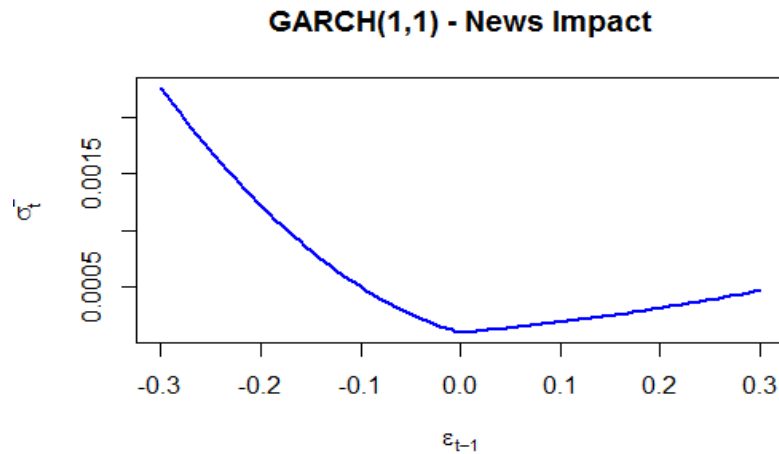


Figure 26

Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) are used to determine the quality of GARCH family models. AIC and BIC give consistent results that exponential GARCH(1,1) – ARMA (0,1) is the preferred model for GSPC index. It is worth mentioning that since GJR GARCH model is nested with standard GARCH model, a likelihood ratio test has been conducted. The test result is in line with AIC and BIC that GJR GARCH is better than standard GARCH model.

Summary - model statistics						
		Akaike	Bayes	Shibata	Hannan-Quinn	LogLikelihood
1	eGARCH(1,1),norm	-6.8365	-6.8337	-6.8365	-6.8356	57904.19
2	Tgarch(1,1), norm	-6.8363	-6.8335	-6.8363	-6.8354	57902.38
3	gjrGARCH(1,1),norm	-6.8341	-6.8314	-6.8341	-6.8332	57884.35
4	sGARCH(1,1),norm	-6.8107	-6.8089	-6.8107	-6.8101	57684.1

Table 4

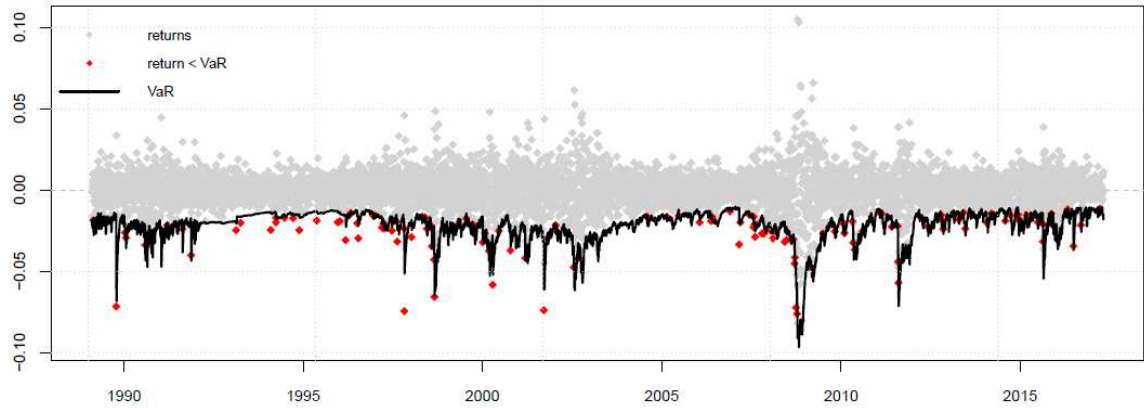
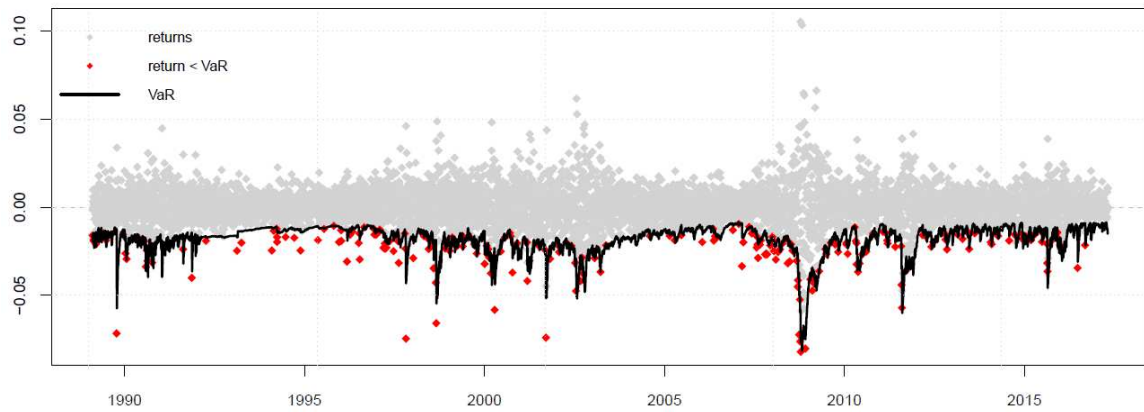
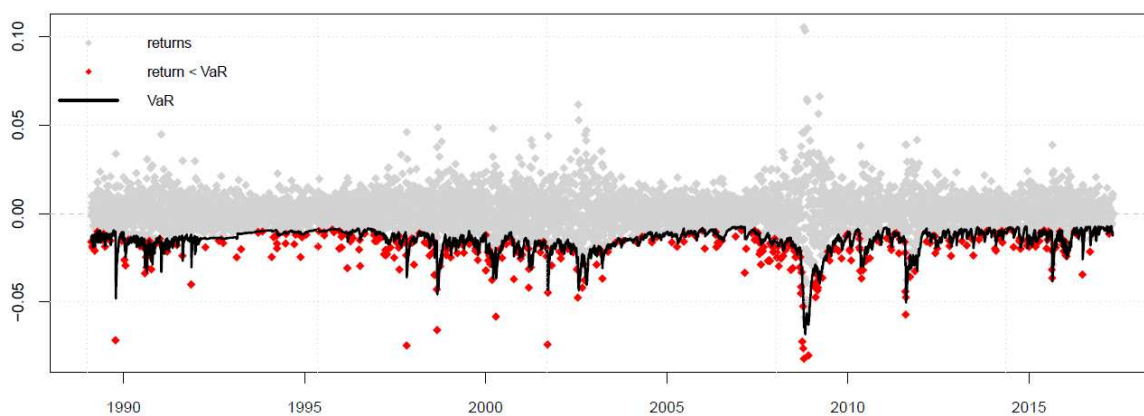
5.2 Result of VaR estimates

For GARCH family models, daily VaR forecasts were calculated using a four-year moving window (1020 points). The parameters of the GARCH models were calibrated every single year (255 points) since it is common in market practice. Daily VaR forecasts at significance level 1%, 2.5% and 5% were separately estimated.

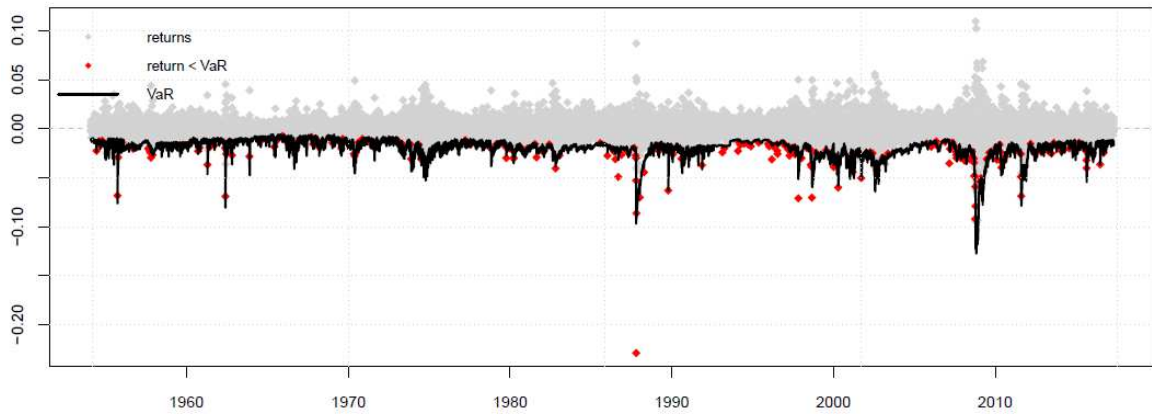
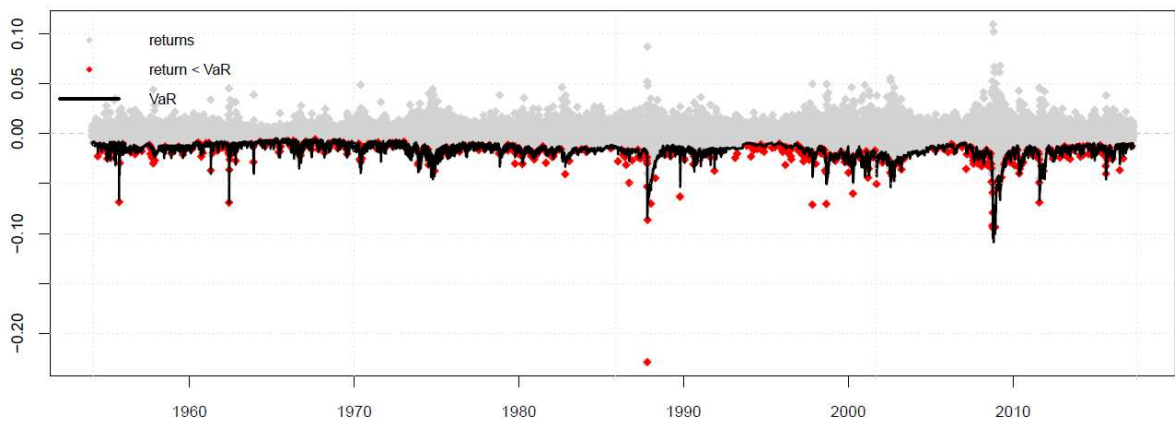
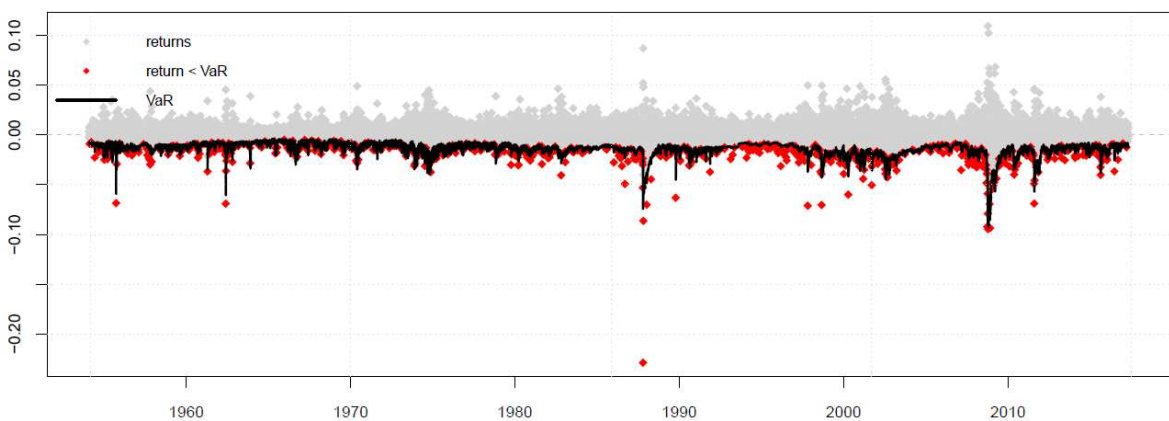
As demonstrated below, for DJI and GSPC index, the frequency of hits (actual return < VaR forecasts) increases with significance level α . Moreover, for α level 2.5% and 5%, hits tend to cluster together especially when volatility is relatively high (e.g. in crisis era). The graphs below only illustrate results of normal standard GARCH model of DJI and GSPC while similar conclusions can be drawn from other GARCH family models.

DJI ($\alpha = 1\%$) – Standard GARCH (1,1) – ARMA (0,0)

Quantification of Model Risk with Bootstrapping Method

*Figure 27*DJI ($\alpha = 2.5\%$) – Standard GARCH (1,1) – ARMA (0,0)*Figure 28*DJI ($\alpha = 5\%$) – Standard GARCH (1,1) – ARMA (0,0)*Figure 29*GSPC ($\alpha = 1\%$) – Standard GARCH (1,1) – ARMA (0,1)

Quantification of Model Risk with Bootstrapping Method

*Figure 30*GSPC ($\alpha = 2.5\%$) – Standard GARCH (1,1) – ARMA (0,1)*Figure 31*GSPC ($\alpha = 5\%$) – Standard GARCH (1,1) – ARMA (0,1)*Figure 32*

The results of the non-parametric historical simulation demonstrated below were calculated at significance level 1%, 2.5% and 5% with a four-year moving window as

Quantification of Model Risk with Bootstrapping Method

well. As expected, for DJI and GSPC index, the frequency of hits (actual return < VaR forecasts) increases with significance level α . Moreover, for α level 2.5% and 5%, hits tend to cluster together especially when volatility is relatively high (e.g. in crisis era). Although more restricted α level significantly lowers the frequency of hits, such improvement comes at the expense of excessive capital requirement when volatility is relatively low. Compared with the dynamic GARCH family models, the historical simulation method is insensitive to the sudden change of volatility. It responds slowly to the increase in volatility and such stagnation results in insufficient VaR forecasts in times of crisis and excessive capital requirement in post-crisis era.

DJI ($\alpha = 1\%$) - Non-parametric historical simulation

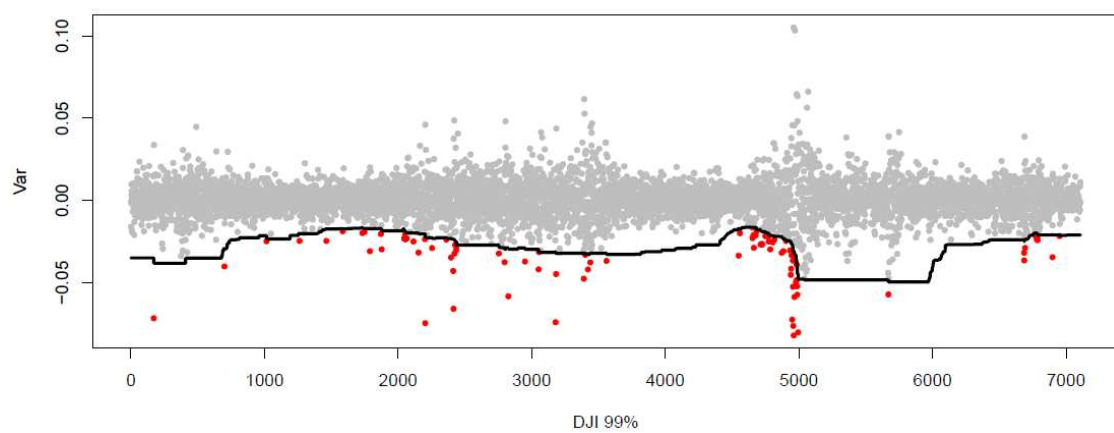


Figure 33

DJI ($\alpha = 2.5\%$) - Non-parametric historical simulation

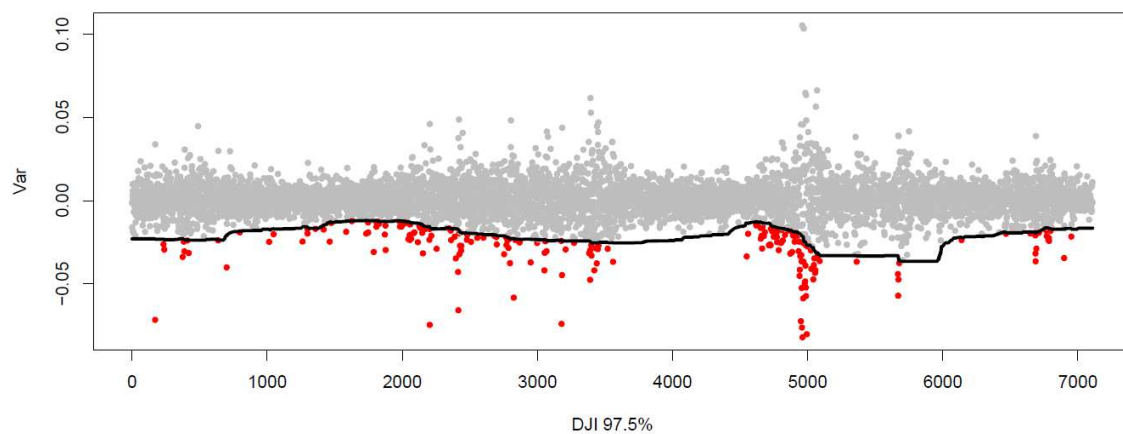
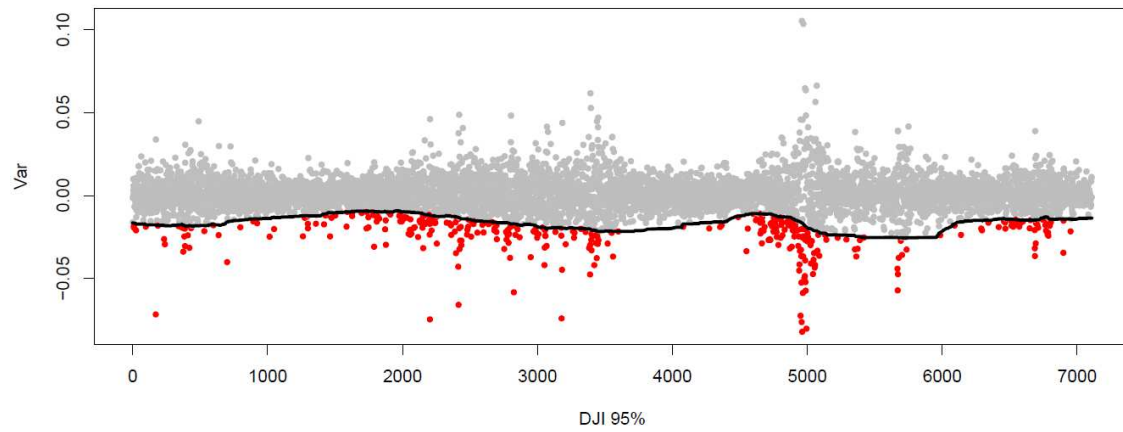
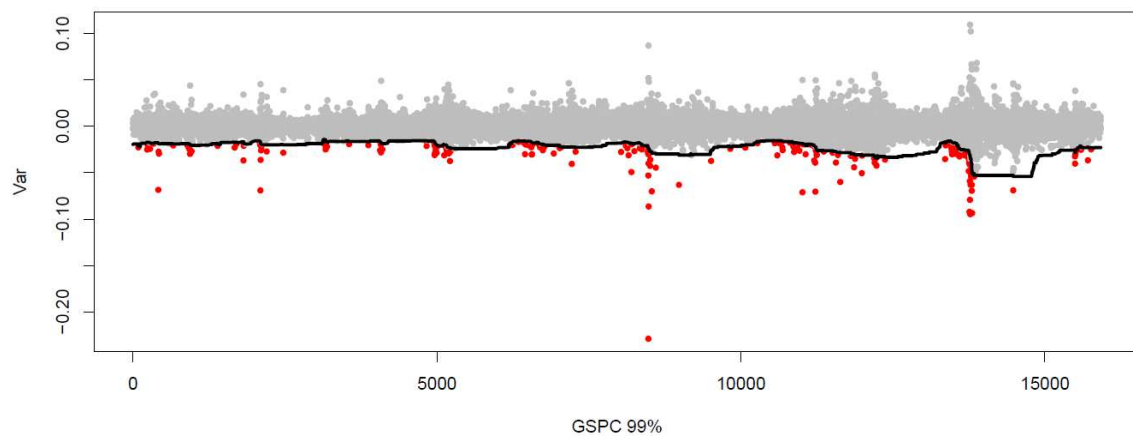
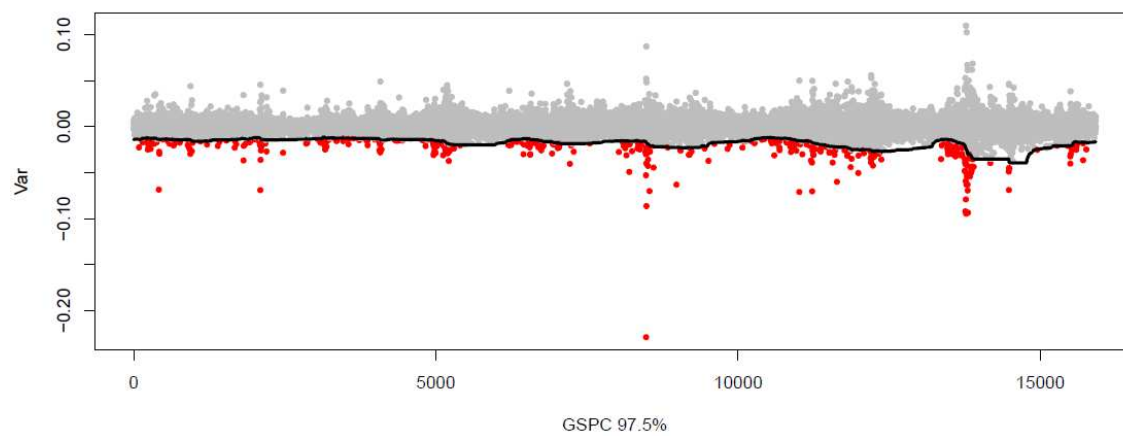


Figure 34

DJI ($\alpha = 5\%$) - Non-parametric historical simulation

Quantification of Model Risk with Bootstrapping Method

*Figure 35*GSPC ($\alpha = 1\%$) - Non-parametric historical simulation*Figure 36*GSPC ($\alpha = 2.5\%$) - Non-parametric historical simulation*Figure 37*

Quantification of Model Risk with Bootstrapping Method

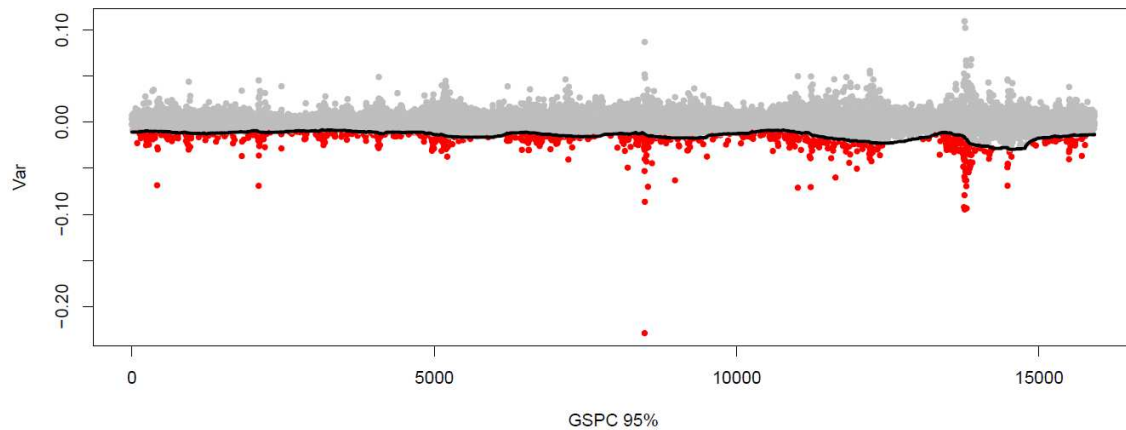
GSPC ($\alpha = 5\%$) - Non-parametric historical simulation

Figure 38

The results of the normal distribution simulation were calculated at significance level 1%, 2.5% and 5% with a four-year moving window as well. The conclusion is similar to the non-parametric simulation that the lower frequency of hits comes at the expense of excessive capital requirement when volatility is relatively low. Compared with the dynamic GARCH family models, this method is insensitive to the sudden change of volatility. It responds slowly to the increase in volatility and such stagnation results in insufficient VaR forecasts in times of crisis and excessive capital requirement in post-crisis era.

The results of VaR forecasts vary with different methodologies. For DJI index, the difference between the maximum and minimum VaR forecasts within GARCH family at significance level 5% can reach 3%. The difference in VaR forecasts among various GARCH family models enlarges to 4% when the significance level is 1%. Also, the difference tends to be larger when more volatility is observed in the market, which highlights the importance of searching for an accuracy VaR estimates. As crisis era is marked with extremely high volatility, the selection of VaR models is critical for risk management.

DJI – GARCH FAMILY

Quantification of Model Risk with Bootstrapping Method

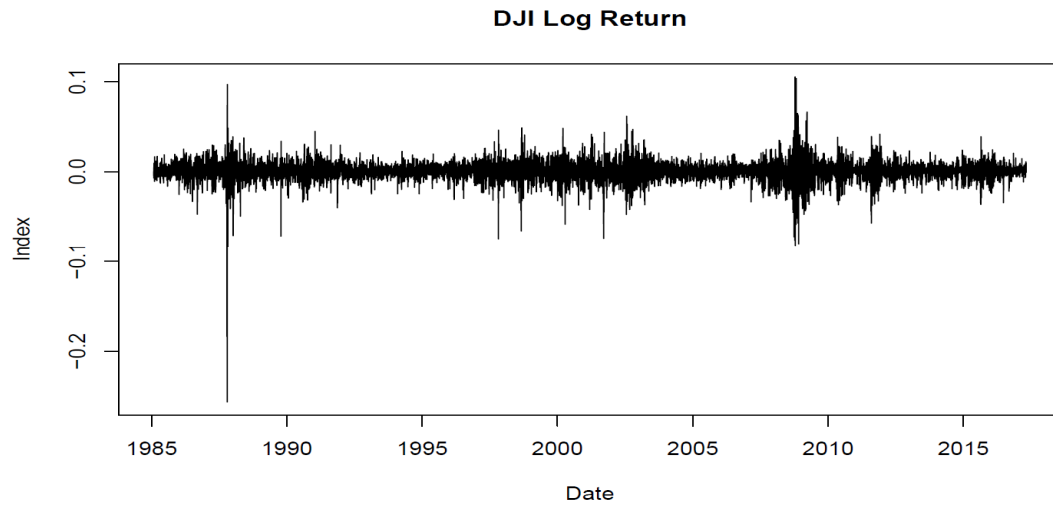


Figure 39

difference between min. and max. VaR (5%)

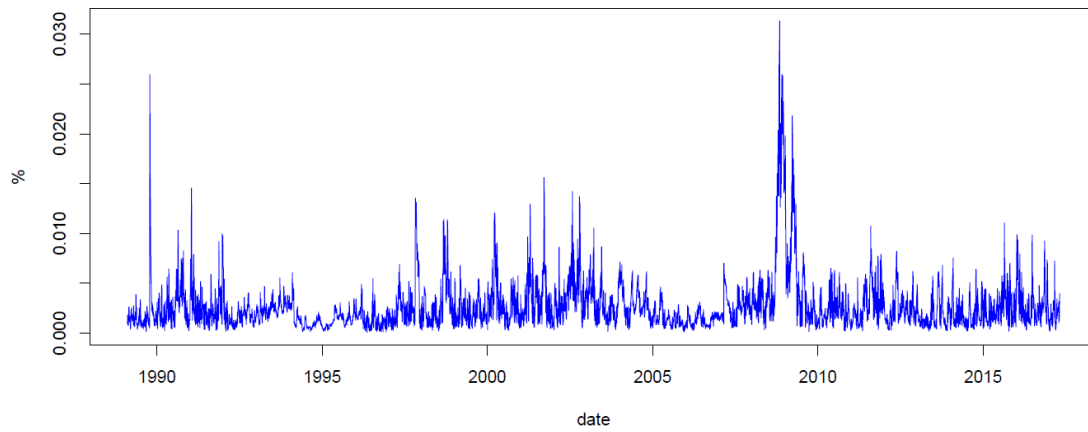


Figure 40

difference between min. and max. VaR (1%)

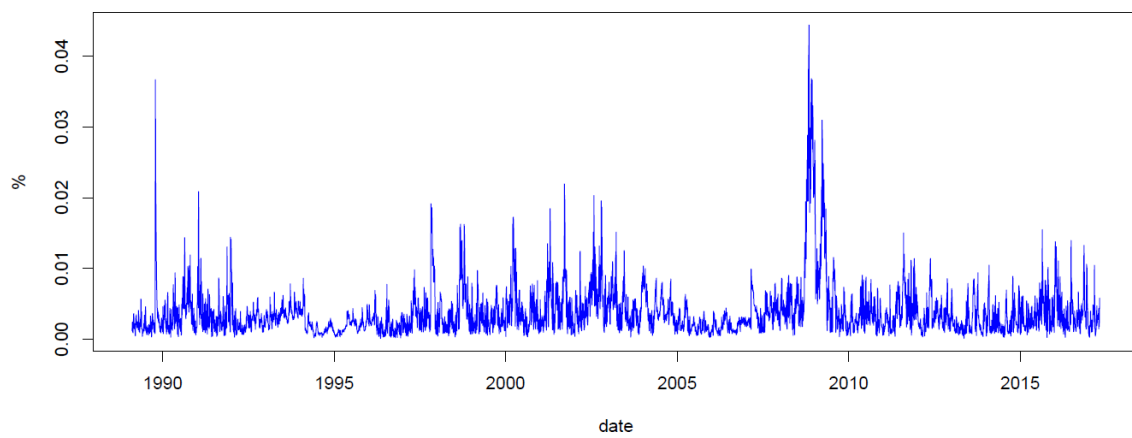


Figure 41

DJI – GARCH FAMILY, Non-parametric and normal distribution models

Quantification of Model Risk with Bootstrapping Method

Comparing estimates results generated by GARCH family models, historical simulations and normal distribution model, the difference between the maximum and minimum VaR forecasts at significance level 5% can reach 5%. The difference in VaR forecasts among various models enlarges to 7% when the significance level is 1%. As mentioned before, the model selection is crucial in times of crisis since the difference of model estimates is larger when volatility is high.

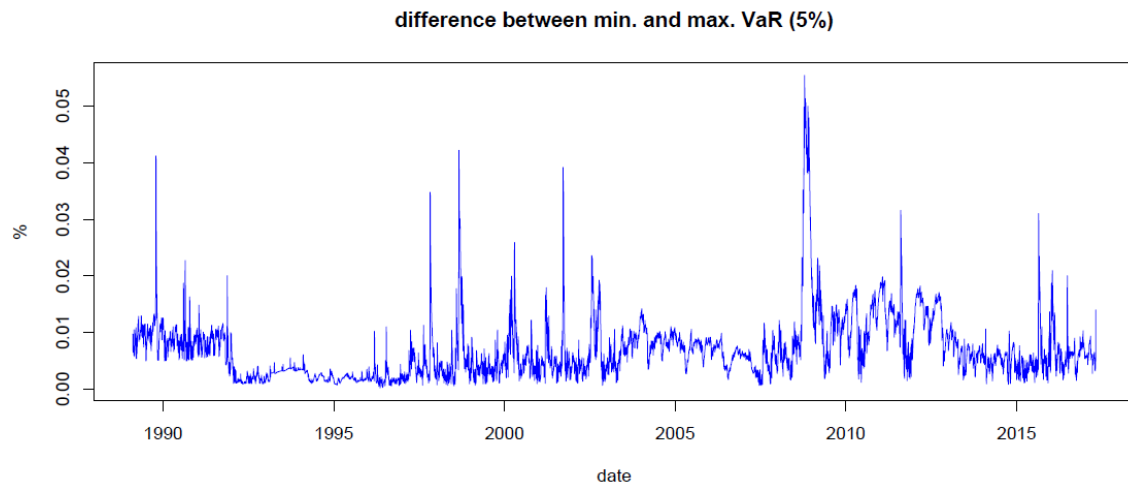


Figure 42

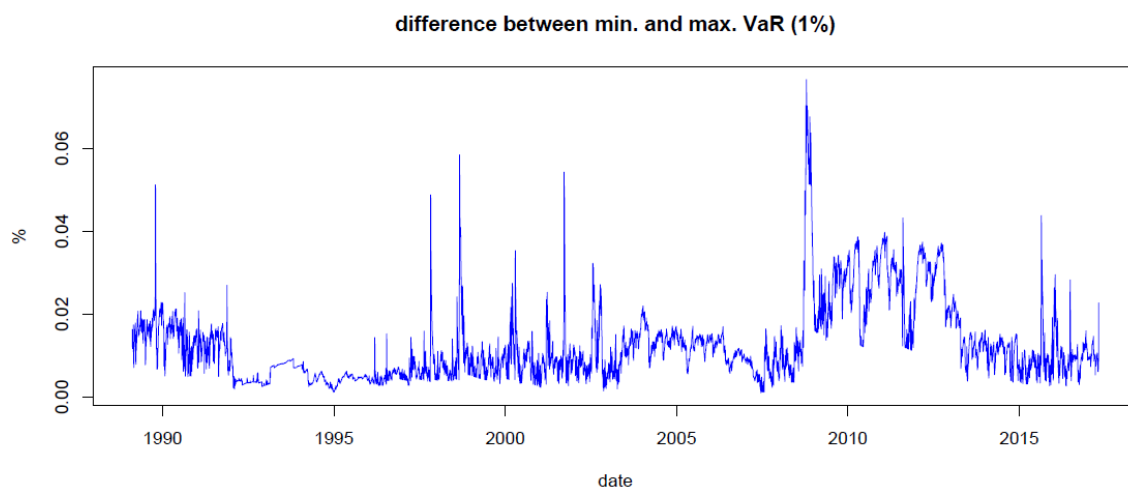


Figure 43

GSPC – GARCH FAMILY

For GSPC index, the difference between the maximum and minimum VaR forecasts within GARCH family at significance level 5% can reach 8%. The difference in VaR forecasts among various GARCH family models can be almost 13% when the significance level is 1%. Also, the difference tends to be larger when more volatility is observed in the market, which highlights the importance of searching for an

Quantification of Model Risk with Bootstrapping Method

accuracy VaR estimates. As crisis era is marked with extremely high volatility, the selection of VaR models is critical for risk management.

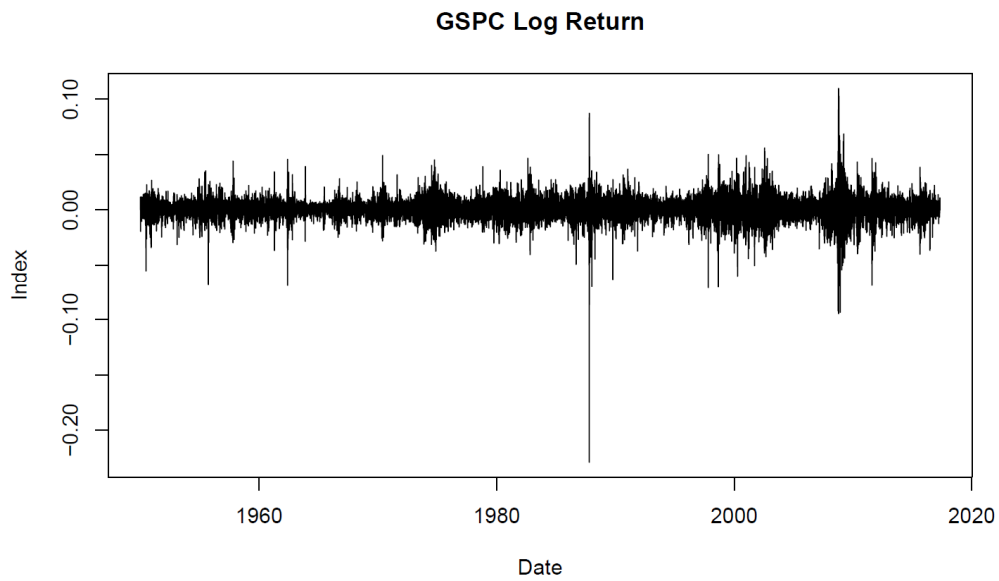


Figure 44

difference between min. and max. VaR (5%)

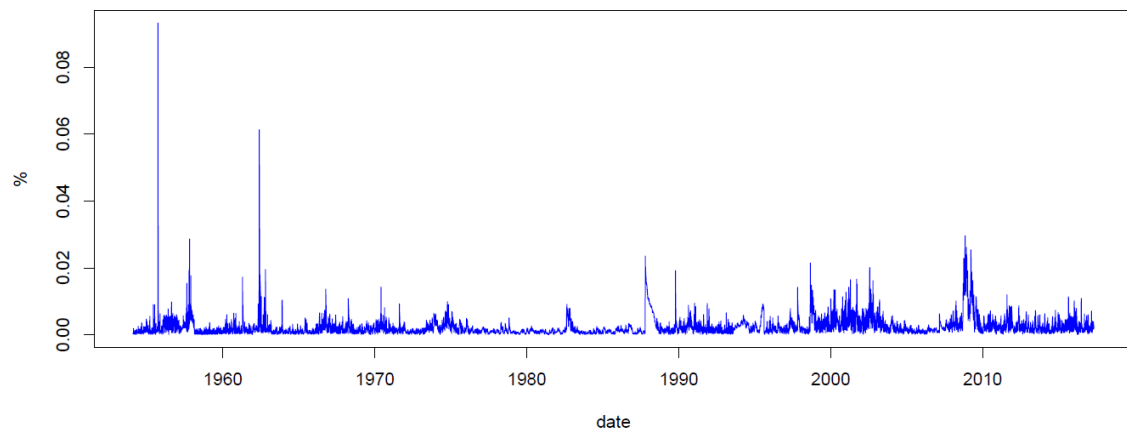


Figure 45

Quantification of Model Risk with Bootstrapping Method

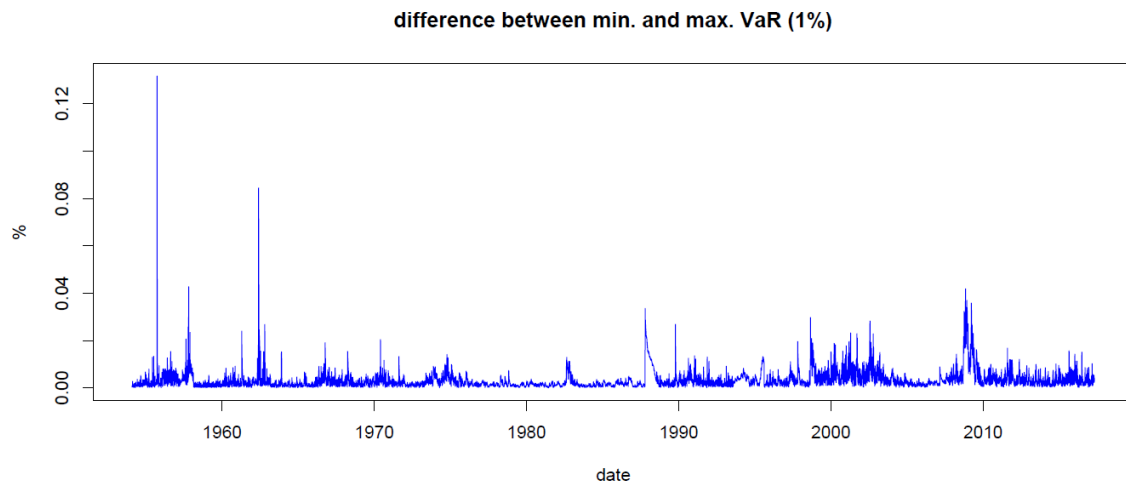


Figure 46

GSPC – GARCH FAMILY, Non-parametric and normal models

Comparing estimates results generated by GARCH family models, historical simulations and normal distribution model, the difference between the maximum and minimum VaR forecasts at significance level 5% can reach 13%. The difference in VaR forecasts among various models enlarges to 16% when the significance level is 1%. As mentioned before, the model selection is crucial in times of crisis since the difference of model estimates is larger when volatility is high.

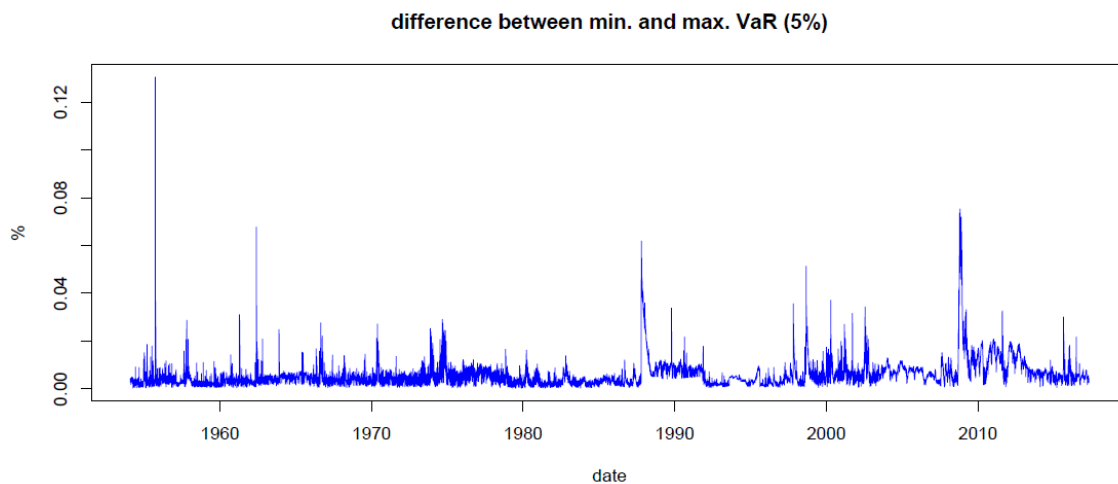
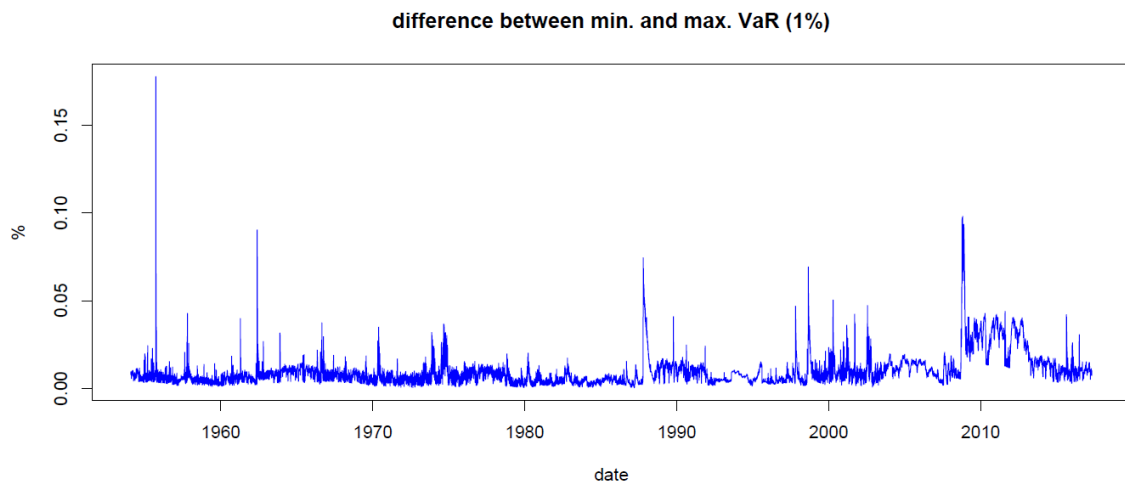


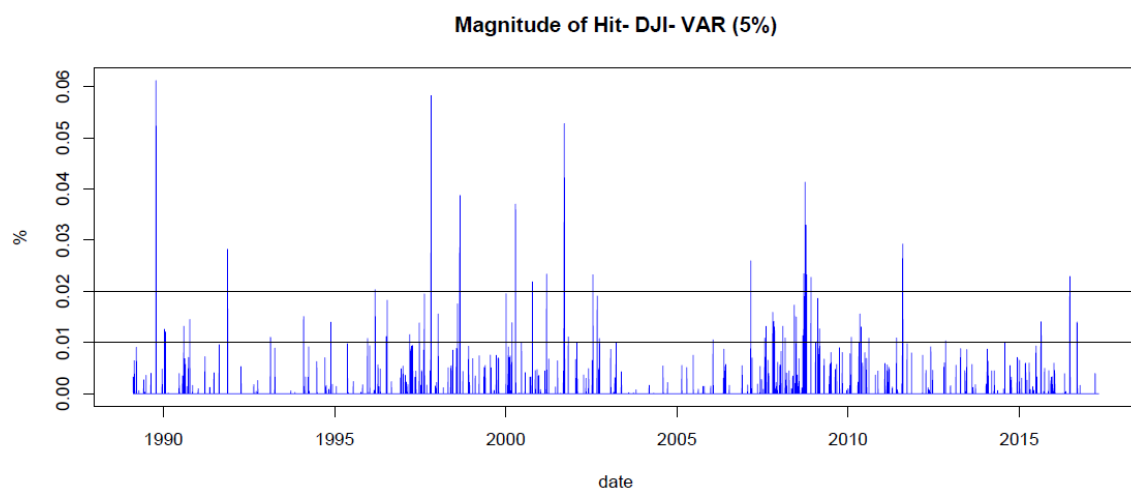
Figure 47

Quantification of Model Risk with Bootstrapping Method

Figure 48**5.3 The concept of capital adjustment q**

Hit of the VaR forecasts occurs when the negative actual return is less than the VaR estimates, which means that the model gives insufficient estimates to cover the risk. Graphs below demonstrated the magnitude of the hits. As predicted, hits are less frequent and less severe for significance level 1% than for significance level 5%. The figure also shows that hits tend to be autocorrelated with high frequency. It would be ideal if an optimal capital adjustment of VaR was applied to increase the capital requirement. Such buffer (represented by the horizontal lines in the following graphs) can significantly reduce the frequency, autocorrelation and magnitude of the hits hence boost the performance of the VaR models. In the following section, the result of the q adjustment is presented for further examination.

DJI – Standard GARCH(1,1) – VaR (5%)

Figure 49

DJI – Standard GARCH(1,1) – VaR (1%)

Quantification of Model Risk with Bootstrapping Method

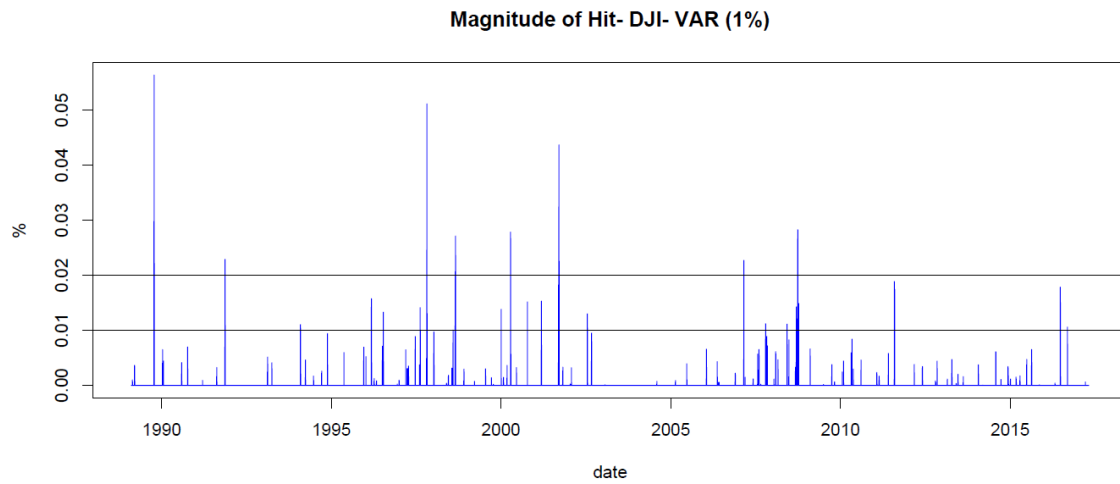


Figure 50

GSPC – Standard GARCH(1,1) – VaR (5%)

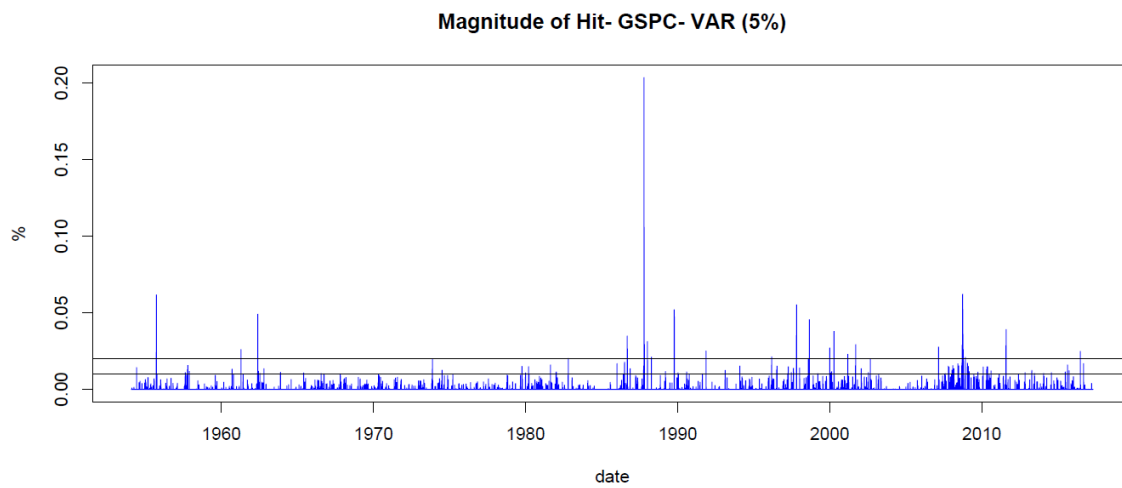


Figure 51

GSPC – Standard GARCH(1,1) – VaR (1%)

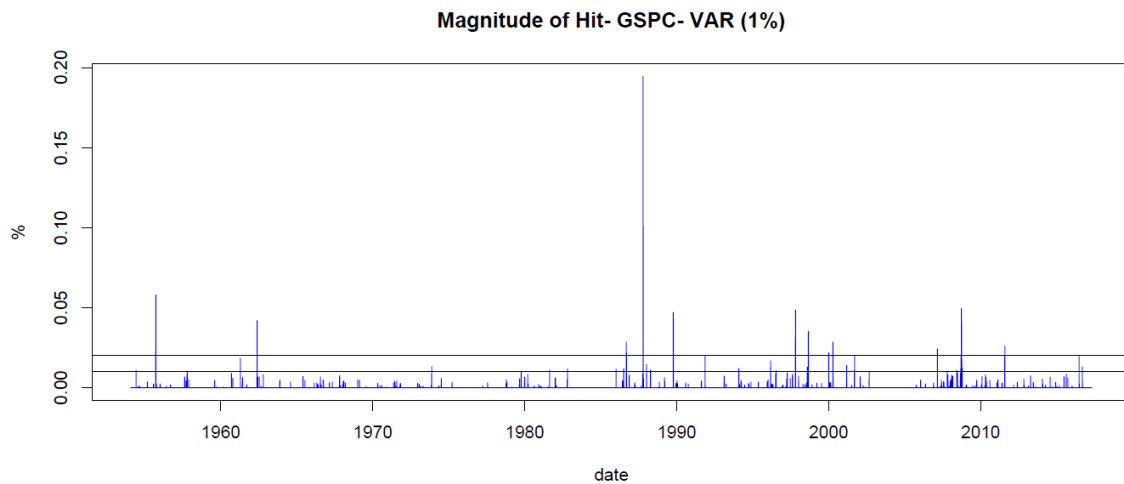


Figure 52

5.4 Result of Backtesting – Adjustment q

The backtesting technique, as mentioned in the methodology section, is applied to calculate the optimal capital adjustment q . The algorithm programmed is using a loop function to exhaustively search for the minimum q that can pass the unconditional coverage and independence test. The search begins with a large negative value and increases by 0.01% each time until the test is passed. If q is negative, less capital is required to pass the test. If q is positive, the VaR forecasts are not sufficient to pass the tests and therefore more capital should be injected.

The VaR forecasts are calculated with a moving window of 4 years (1020 points) while the backtesting is performed with a one-year window (255 points). The significance level of the unconditional and independence test is 5%.

DJI VaR (5%)

Optimal capital adjustment q was calculated separately for different VaR estimates models at significance level 5%. Generally speaking, for DJI index, the GARCH family models require less capital adjustment and the model performance is more stable than historical simulation and normal distribution method. In the following sector, optimal capital adjustment q results of various models are analysed and a general comparison is drawn in the end.

DJI Standard GARCH (1,1)-ARMA(0,0) VaR(95%)

The capital adjustment for standard GARCH model is relatively stable overtime. In most of the cases, q is negative, which suggests that the model overestimates the capital requirement necessary to pass the model. In the times of crisis, the model performance is also stable given the q adjustment is only around 0.2% to pass the conditional coverage and independence test.

Quantification of Model Risk with Bootstrapping Method

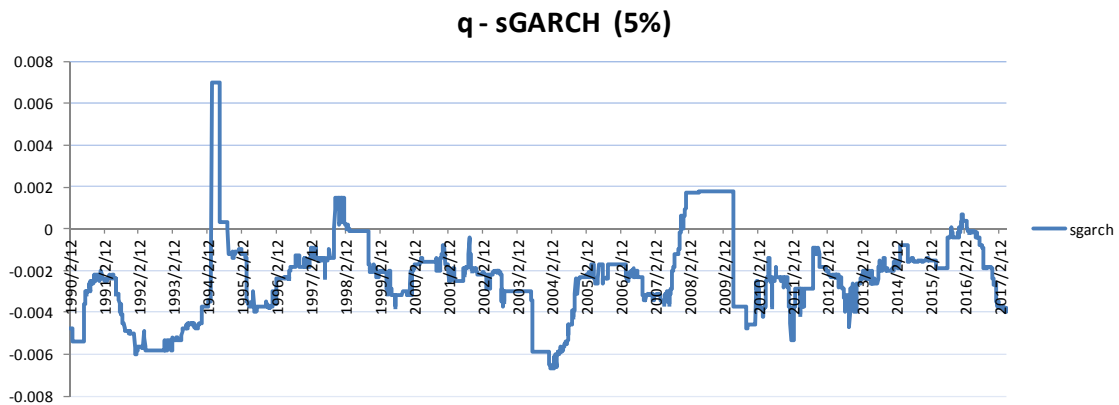


Figure 53

DJI Exponential GARCH (1,1)-ARMA(0,0) VaR(95%)

The exponential GARCH model also demonstrates stable performance over time. In most of the cases, q is negative, which suggests that the model overestimates the capital requirement necessary to pass the model. In recent crisis (2007), the model requires more capital adjustment (almost 1%) than standard GARCH model to pass the conditional coverage and independence test. The sudden boost of q implies that during the 2007 crisis, excessive hits and violation clustering occurred for the exponential GARCH model.

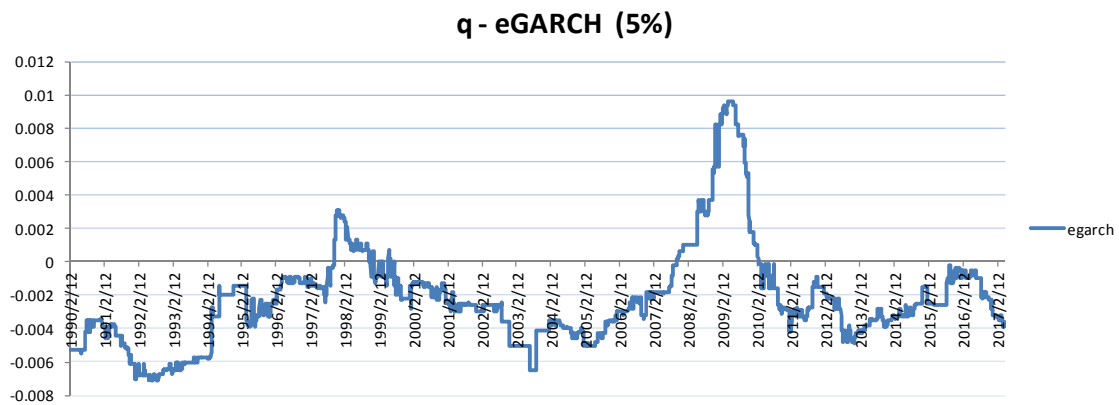


Figure 54

DJI GJR GARCH (1,1)-ARMA(0,0) VaR(95%)

The optimal capital adjustment q of the GJR GARCH model resembles a lot to the q of standard GARCH model. In most of the time, q is negative and immaterial, while in the recent crisis (2007), a sudden jump of q can be observed and its magnitude can reach to 0.04%.

Quantification of Model Risk with Bootstrapping Method

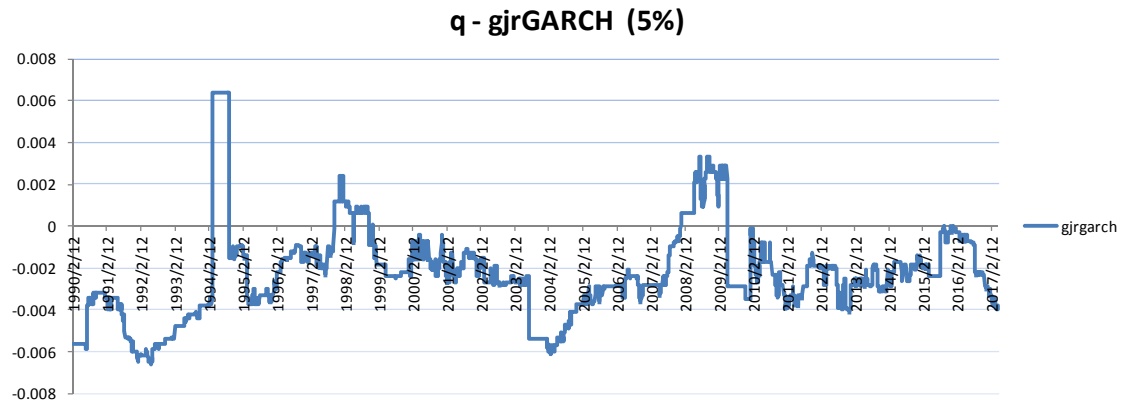


Figure 55

DJI Threshold GARCH (1,1)-ARMA(0,0) VaR(95%)

The optimal capital adjustment q of the Threshold GARCH model resembles a lot to the q of standard and GJR GARCH model. In most of the time, q is negative and immaterial, while in the recent crisis (2007), a sudden jump of q can be observed and its magnitude can reach to 0.04%.

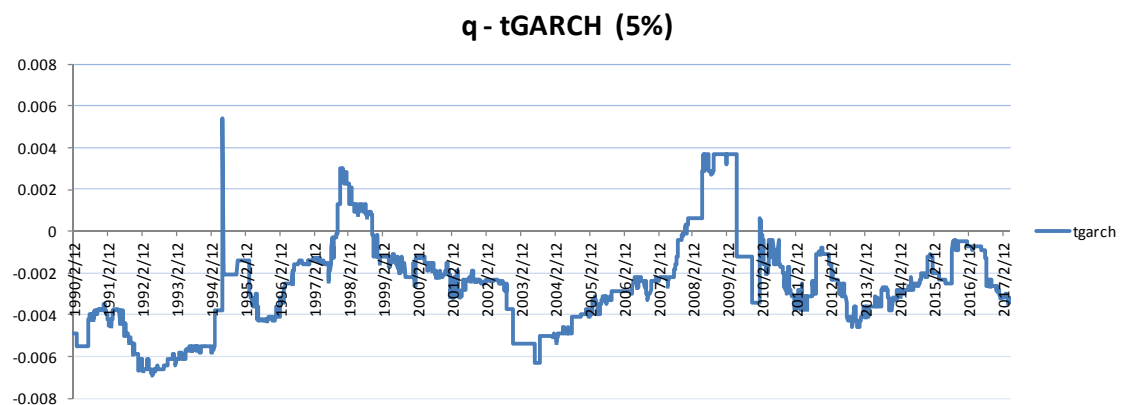


Figure 56

For DJI index, the GARCH family VaR estimates works efficiently to predict the potential risk. Regarding the recent crisis, to our surprise, the standard GARCH model requires less capital adjustment to pass the unconditional and independence test.

Quantification of Model Risk with Bootstrapping Method

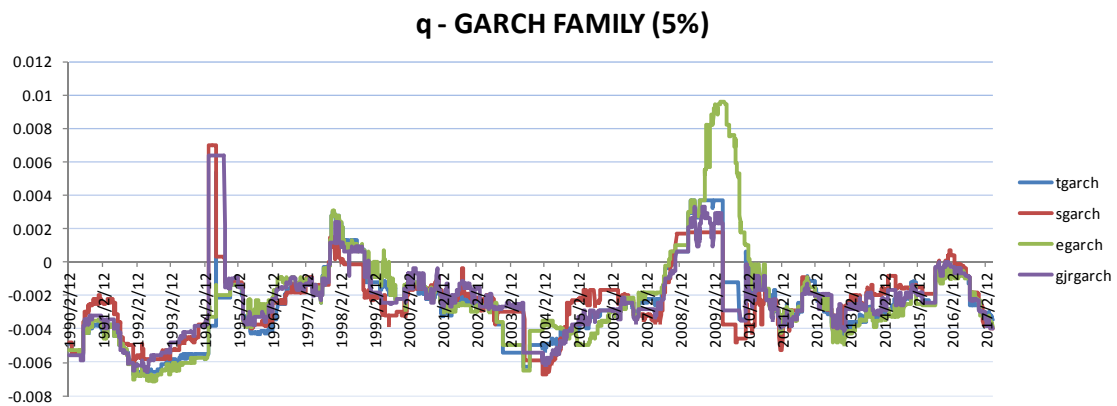


Figure 57

As illustrated in the following graphs, compared with GARCH family models, the optimal capital adjustment q demonstrates more volatility for the historical simulation and normal distribution methods. Excessive capital is required in most cases. Especially in post-crisis era (2010), almost 1.5% unnecessary capital is required by the models while in crisis era the capital adjustment can reach to 1.8%. This phenomenon implicates that the historical simulation and normal distribution methods adapt very slow to the sudden exchange of volatility. As a result, insufficient VaR forecasts are made during crisis while excessive capital is required after crisis.

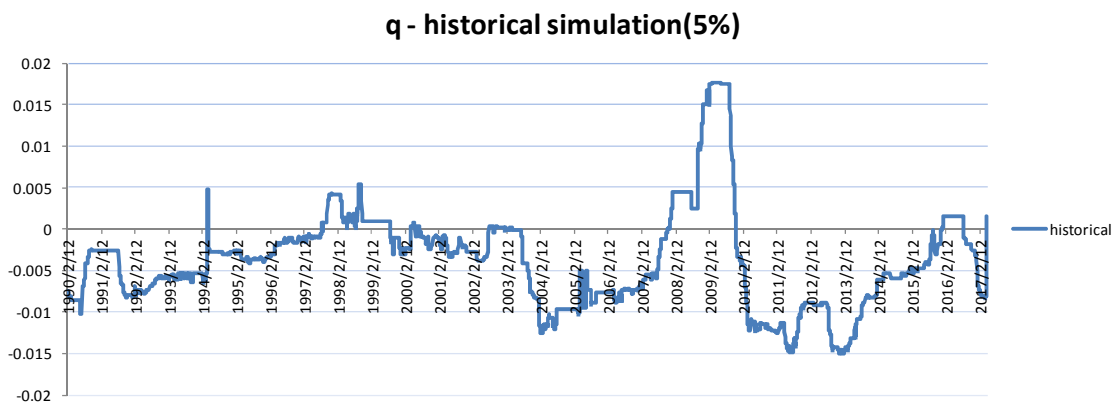


Figure 58

Quantification of Model Risk with Bootstrapping Method

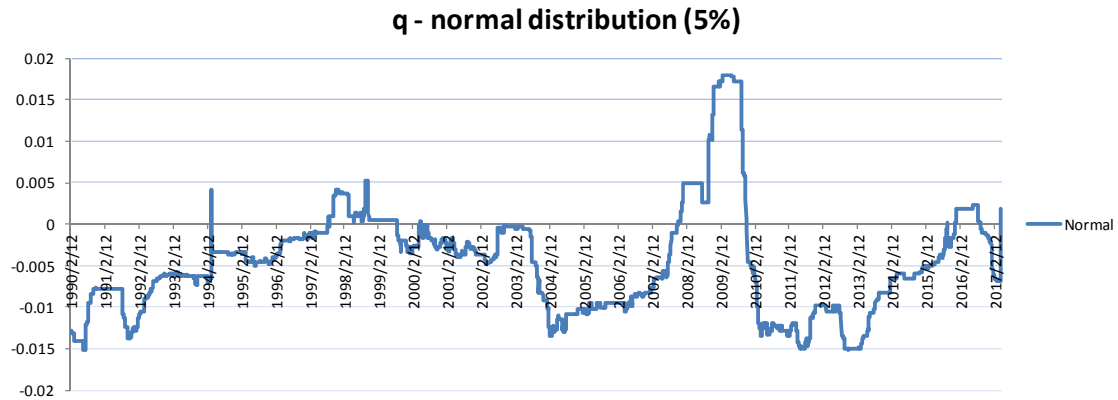


Figure 59

For DJI index, the GARCH family VaR estimates works better to predict the potential risk than historical simulation and normal distribution method. The following graph shows that in average GARCH family models require stable and low capital adjustment while for other two methods volatility is common in the times of crisis and in the post-crisis era.

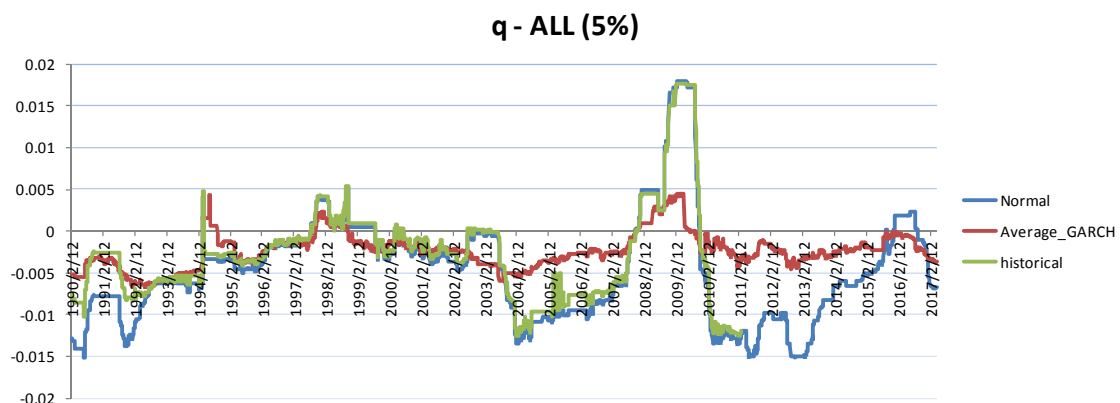


Figure 60

As expected, the GARCH family models result in the least absolute value of the q adjustment, 0.28% in average. When q is positive, the average adjustment q is 0.18% in average for GARCH family models. Compared with historical simulation and normal distribution method, with q adjustment 0.47% and 0.51% respectively, GARCH model forecasts experience less hits and the hits are less autocorrelated.

To our surprise, the standard GARCH model requires least capital adjustment q among GARCH family. The absolute value of the adjustment q is 0.27% in average and its standard deviation is 0.2%, lowest within GARCH family.

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	sGARCH	eGARCH	gjrGARCH	tGARCH	historical	Normal	Average_GARCH
\bar{q}	-0.23%	-0.22%	-0.23%	-0.26%	-0.38%	-0.47%	-0.24%
$ \bar{q} $	0.27%	0.31%	0.28%	0.30%	0.56%	0.65%	0.28%
$\bar{q}, \text{if } q > 0$	0.19%	0.35%	0.23%	0.21%	0.47%	0.51%	0.18%
$\bar{q}, \text{if } q < 0$	-0.28%	-0.31%	-0.28%	-0.31%	-0.59%	-0.68%	-0.29%
σ_q	0.20%	0.29%	0.22%	0.22%	0.60%	0.64%	0.21%

Table 5

According to the AIC and BIC statistics derived in previous chapter, the asymmetric GARCH family model should have made a better estimation than the standard symmetric GARCH model. The reason of the inconsistency is that the optimal capital adjustment q in this paper takes into account frequency of the hit and the independence of the hit, while the magnitude of the hit is neglected. Further study could incorporate the magnitude criteria to study the its impact on the capital adjustment q .

Summary - model statistics						
		Akaike	Bayes	Shibata	Hannan-Quinn	LogLikelihood
1	Tgarch(1,1), norm	-6.5837	-6.5794	-6.5837	-6.5823	26761.33
2	eGARCH(1,1),norm	-6.5785	-6.5742	-6.5785	-6.5771	26740.21
3	gjrGARCH(1,1),norm	-6.5733	-6.569	-6.5733	-6.5718	26718.84
4	sGARCH(1,1),norm	-6.5515	-6.5481	-6.5515	-6.5504	26629.42

Table 6

As shown in the table below, the threshold GARCH model gives the lowest average number of hits with a one year moving window, while the magnitude of the hit is only -0.60%. The average number of hit of standard GARCH model is 13.813, the highest among all models. The reason that the standard GARCH model results in least q adjustment is that it performs relatively well in independence test, while other GARCH family methods get penalized for violation clustering. Here, the optimal adjustment q takes a different perspective than traditional method.

	sgarch	egarch	gjrgarch	tgarch	Historical	Norm
\bar{hit}	13.813	14.054	13.534	13.101	13.322	12.702
σ_{hit}	6.388	8.341	6.272	7.170	12.183	12.579
$mag.(hit)$	-0.62%	-0.59%	-0.60%	-0.60%	-0.68%	-0.67%

Table 7

Quantification of Model Risk with Bootstrapping Method

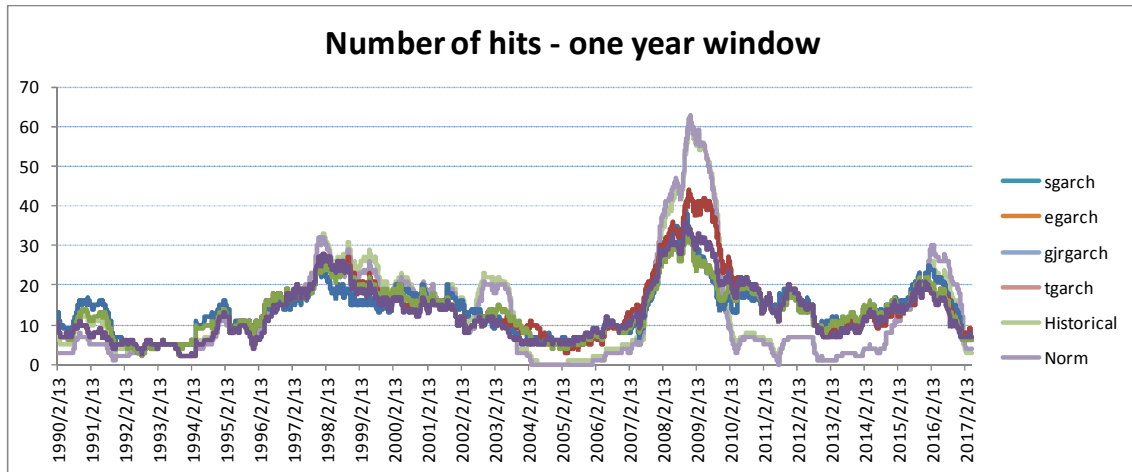


Figure 61

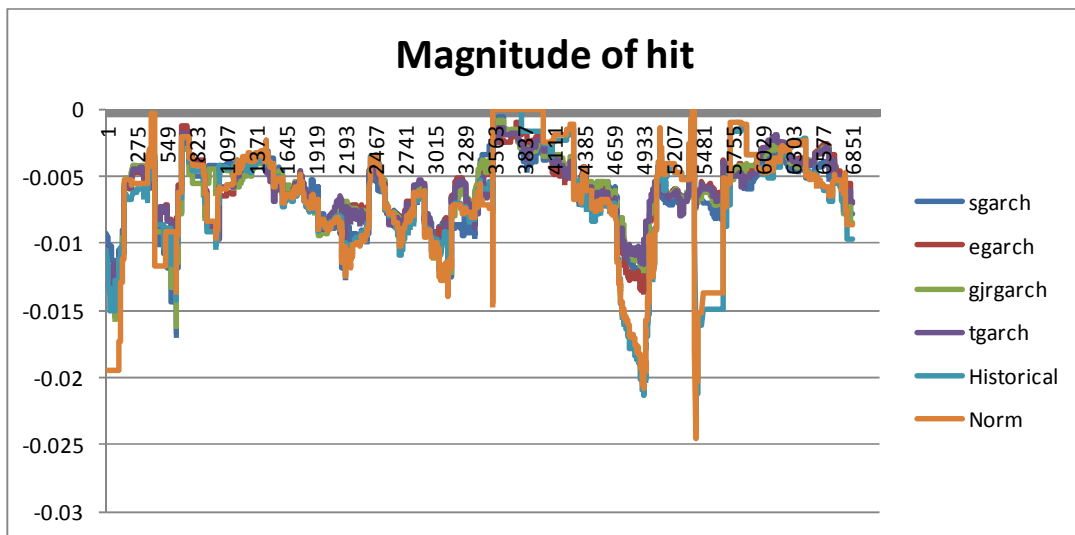


Figure 62

GSPC VaR (5%)

For the GSPC index, the conclusion regarding the performance of capital adjustment q is similar to the DJI index. The optimal adjustment q of GARCH family is usually smaller than the those of historical simulations and normal distribution. Therefore, the study concludes that GARCH family VaR is better than others given least adjustment required to pass the conditional coverage and independence test.

In the next sector, optimal capital adjustment q results of various models are analysed and a general comparison is drawn in the end.

GSPC Standard GARCH (1,1)-ARMA(0,1) VaR(95%)

The capital adjustment for standard GARCH model is relatively stable overtime. In most of the cases, q is negative, which suggests that the model overestimates the

Quantification of Model Risk with Bootstrapping Method

capital requirement necessary to pass the model. A single large jump of q adjustment is observed for the 1987 crisis where q easily reaches to 3.33%. In the 2007 crisis, the model performance is also stable given the q adjustment is only around 0.17% to pass the conditional coverage and independence test.

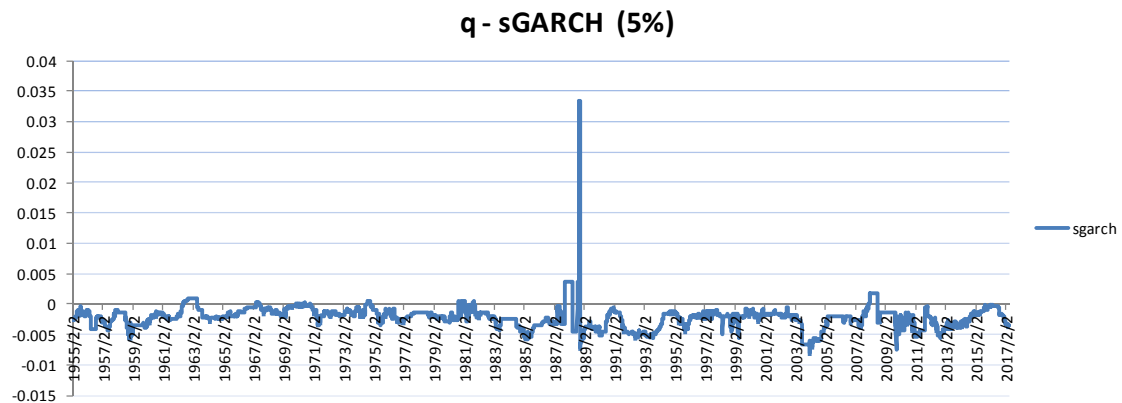


Figure 63

GSPC Exponential GARCH (1,1)-ARMA(0,1) VaR(95%)

The exponential GARCH model also demonstrates stable performance over time. In most of the cases, q is negative, which suggests that the model overestimates the capital requirement necessary to pass the model. In recent crisis (2007), the model requires more capital adjustment (almost 0.7%) than standard GARCH model to pass the conditional coverage and independence test. The sudden boost of q implies that during the 2007 crisis, excessive hits and violation clustering occurred for the exponential GARCH model.

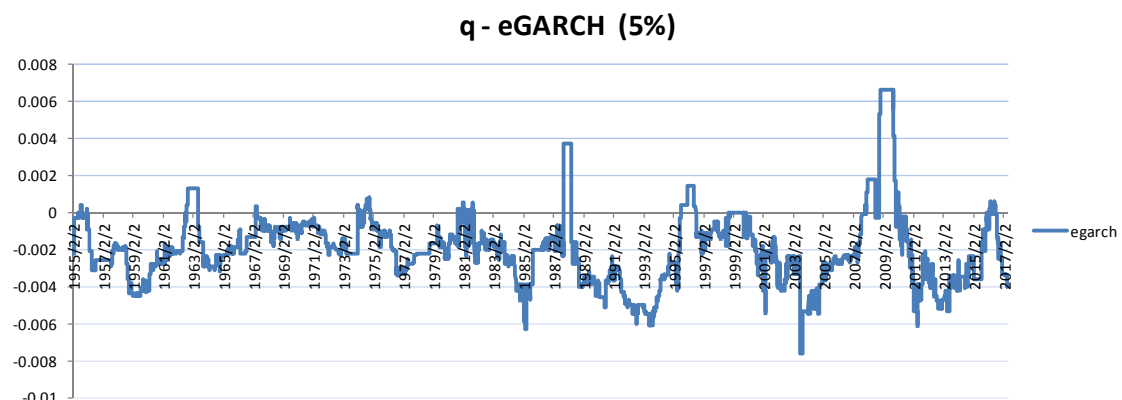


Figure 64

GSPC gjr GARCH (1,1)-ARMA(0,1) VaR(95%)

Quantification of Model Risk with Bootstrapping Method

The optimal capital adjustment q of the GJR GARCH model resembles a lot to the q of standard GARCH model. A single large jump of q adjustment is observed for the 1987 crisis where q easily reaches to 3.32%. In most of the time, q is negative and immaterial, while in the recent crisis (2007), no sudden jump is observed which demonstrates stability of the model.

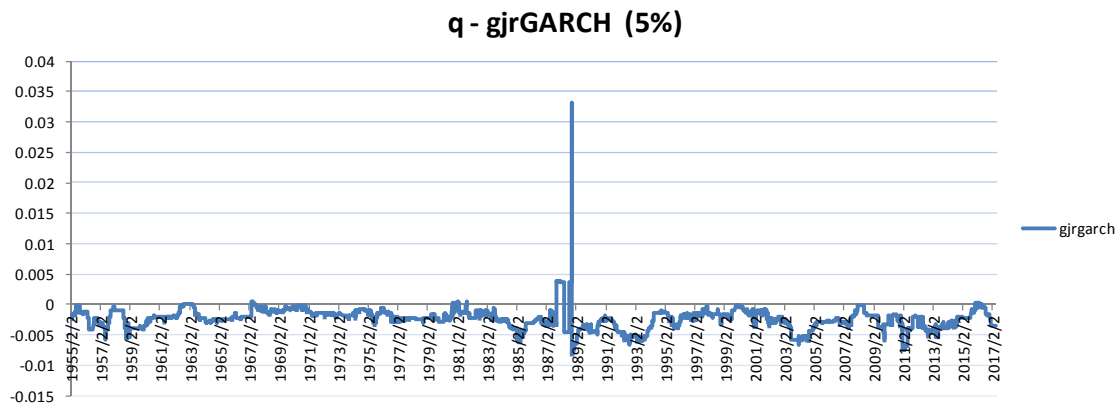


Figure 65

GSPC Threshold GARCH (1,1)-ARMA(0,1) VaR(95%)

The optimal capital adjustment q of the Threshold GARCH model resembles a lot to the q of standard and GJR GARCH model. In most of the time, q is negative and immaterial. There is no sudden jump of q in the 1987 and 2007 crisis. The model is relatively stable.

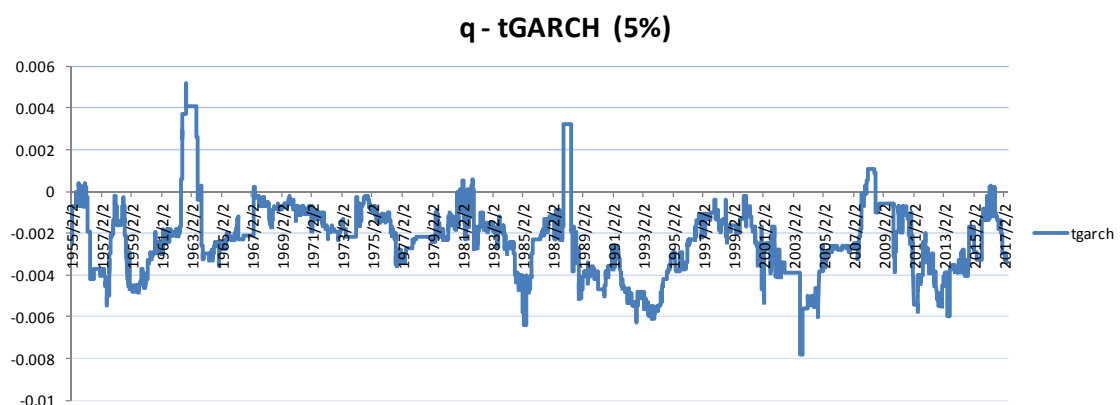


Figure 67

For GSPC index, the GARCH family VaR estimates works efficiently to predict the potential risk. Regarding the recent crisis, to our surprise, the GJR GARCH model and threshold GARCH model do not require extra capital adjustment to pass the unconditional and independence test.

Quantification of Model Risk with Bootstrapping Method

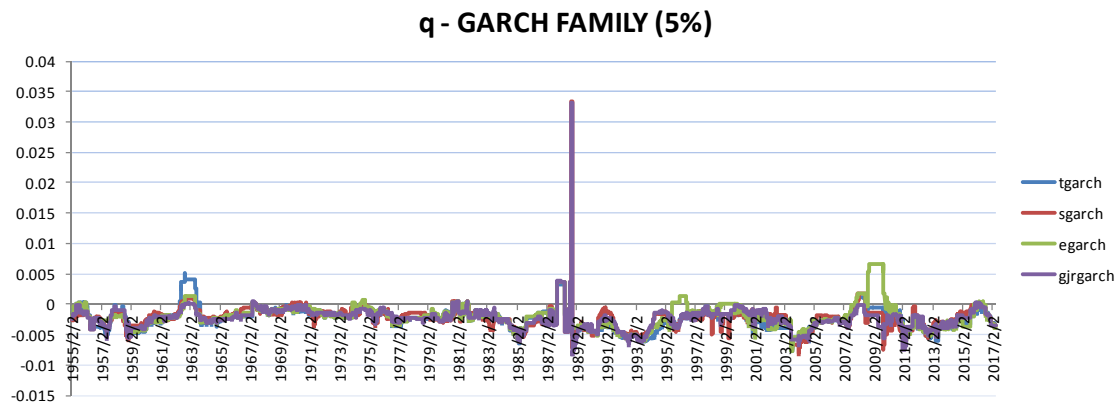


Figure 68

As illustrated in the following graphs, compared with GARCH family models, the optimal capital adjustment q demonstrates more volatility for the historical simulation and normal distribution methods. Excessive capital is required in most cases. In crisis era the capital adjustment can reach to 2%. This phenomenon implicates that the historical simulation and normal distribution methods adapt very slow to the sudden exchange of volatility. As a result, insufficient VaR forecasts are made during crisis while excessive capital is required after crisis.

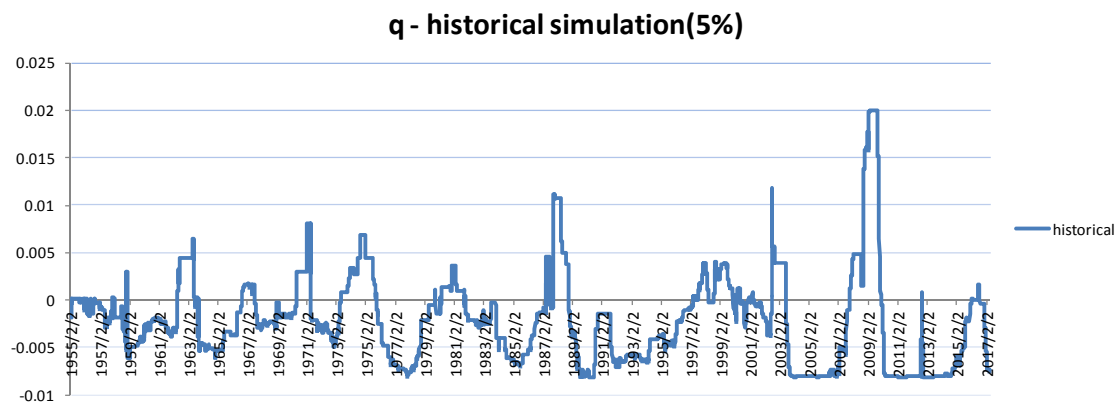


Figure 69

Quantification of Model Risk with Bootstrapping Method

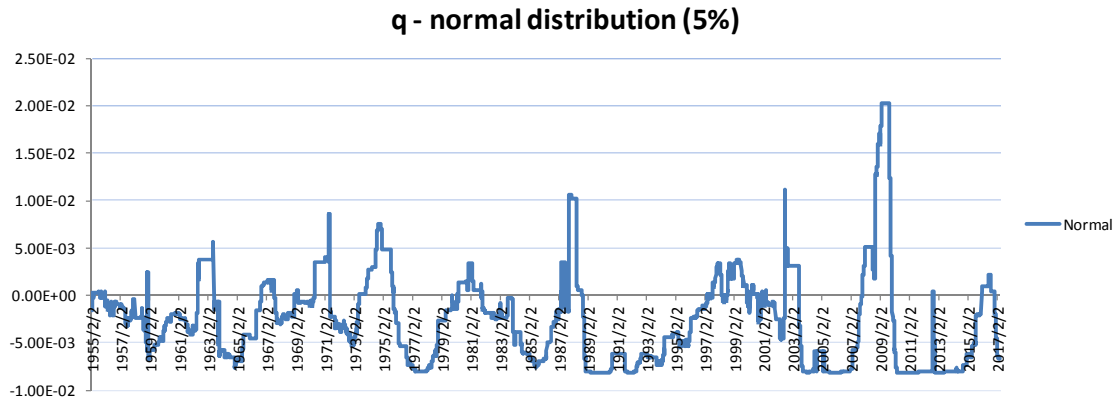


Figure 70

For GSPCindex, the GARCH family VaR estimates works better to predict the potential risk than historical simulation and normal distribution method. The following graph shows that in average GARCH family models require stable and low capital adjustment while for other two methods volatility is common in the times of crisis and in the post-crisis era.

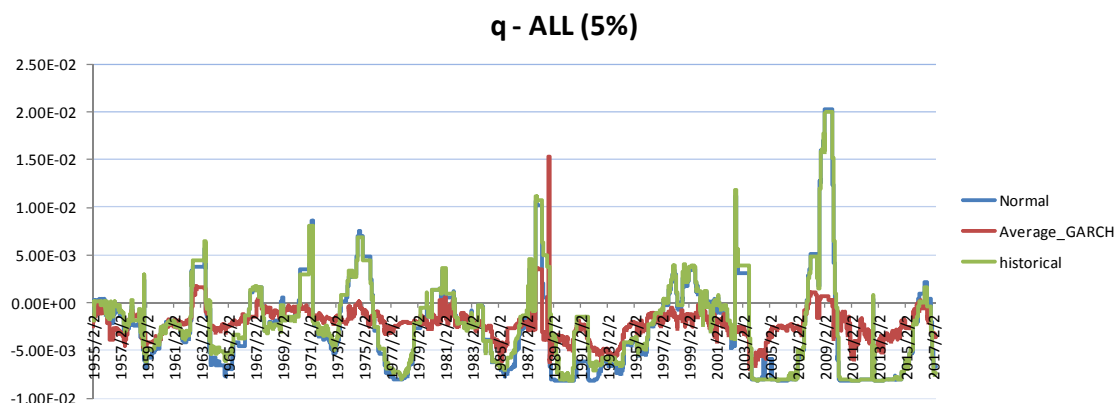


Figure 71

When q is positive, which means more capital is required to pass the tests, the adjustment of GARCH family models is the smallest compared with other two techniques. Generally speaking, GARCH family models demonstrate stability in VaR forecasts in terms of conditional coverage and independence assumption.

As expected, the GARCH family models result in the least absolute value of the q adjustment, 0.24% in average. When q is positive, the average adjustment q is 0.12% in average for GARCH family models. Compared with historical simulation and normal distribution method, with q adjustment 0.41% and 0.36% respectively, GARCH model forecasts experience less hits and the hits are less autocorrelated.

The exponential GARCH model requires least capital adjustment q among GARCH family (-0.2%). The absolute value of the adjustment q is 0.24% in average and its

Quantification of Model Risk with Bootstrapping Method

standard deviation is 0.2%, lowest within GARCH family. The standard GARCH model does well when q is positive, only 15% of additional capital is required to pass the conditional coverage and independence test, lowest within GARCH family models.

	sGARCH	eGARCH	gjrGARCH	tGARCH	historical	Normal	Average_GARCH
\bar{q}	-0.22%	-0.20%	-0.23%	-0.23%	-0.21%	-0.26%	-0.22%
$ \bar{q} $	0.24%	0.24%	0.24%	0.25%	0.41%	0.44%	0.24%
$\bar{q}, \text{if } q > 0$	0.15%	0.22%	0.20%	0.20%	0.41%	0.36%	0.12%
$\bar{q}, \text{if } q < 0$	-0.24%	-0.25%	-0.24%	-0.26%	-0.42%	-0.47%	-0.25%
σ_q	0.17%	0.20%	0.17%	0.18%	0.48%	0.48%	0.16%

Table 8

According to the AIC and BIC statistics derived in previous chapter, the asymmetric GARCH family model should have made a better estimation than the standard symmetric GARCH model. In general, the result of the optimal capital adjustment q is aligned with the AIC and BIC statistics, though to our surprise, standard GARCH model performs better than other models when capital adjustment is positive. As explained for DJI index, the reason of such inconsistency is that the optimal capital adjustment q in this paper takes into account frequency of the hit and the independence of the hit, while the magnitude of the hit is neglected.

Summary - model statistics						
		Akaike	Bayes	Shibata	hannan-Quinr	LogLikelihood
1	eGARCH(1,1),norm	-6.8365	-6.8337	-6.8365	-6.8356	57904.19
2	Tgarch(1,1), norm	-6.8363	-6.8335	-6.8363	-6.8354	57902.38
3	gjrGARCH(1,1),norm	-6.8341	-6.8314	-6.8341	-6.8332	57884.35
4	sGARCH(1,1),norm	-6.8107	-6.8089	-6.8107	-6.8101	57684.1

Table 9

As shown in the table below, the gjr GARCH model gives the lowest average number of hits with a one year moving window, while the magnitude of the hit is only -0.51%. The average number of hit of standard GARCH model is 13.545, highest among all models. The reason that the standard GARCH model results in relatively good q adjustment is that it performs well in the independence test, while other GARCH family methods get penalized for violation clustering. Here, the optimal adjustment q takes a different perspective than traditional method.

	sgarch	egarch	gjrgarch	tgarch	Historical	Norm
\bar{hit}	13.545	14.265	13.225	13.522	14.199	13.092
σ_{hit}	5.496	6.536	5.291	5.916	11.488	11.401
$mag.(hit)$	-0.53%	-0.51%	-0.51%	-0.51%	-0.56%	-0.58%

Table 10

Quantification of Model Risk with Bootstrapping Method

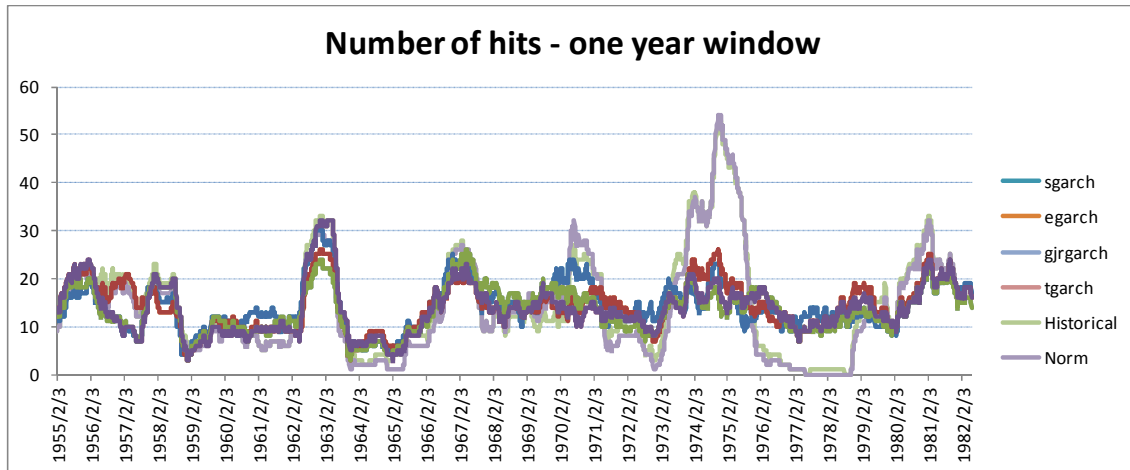


Figure 72

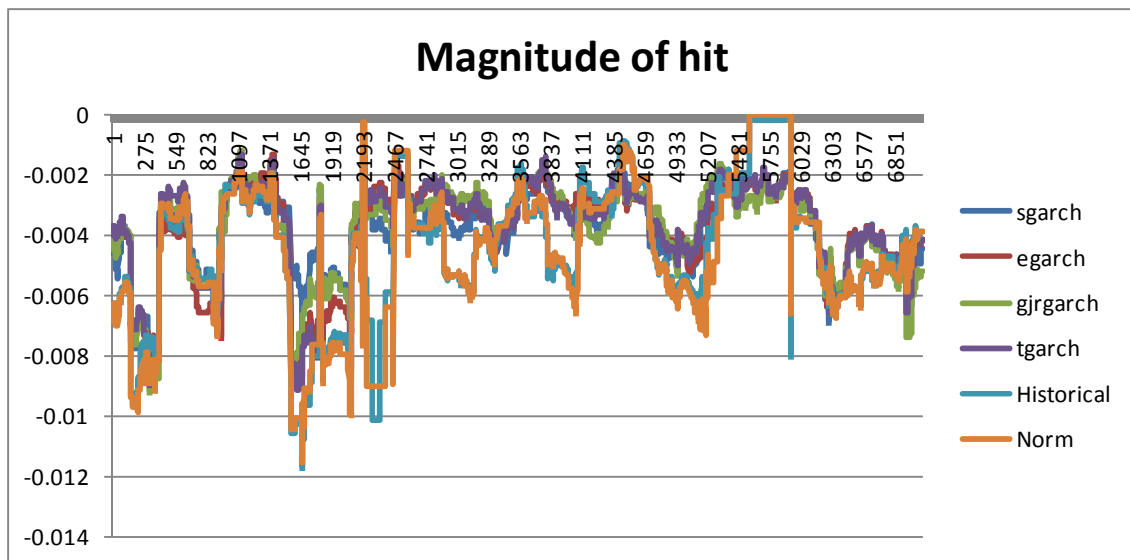


Figure 73

5.5 Model risk and significance level

Compared with the optimal capital adjustment calculated for VaR at significance level 5%, VaR at significance level 1% requires more capital adjustment to pass the unconditional coverage and independence test. In other words, the model risk boosts when significance level decreases. This suggests that the extreme quantiles proposed by some authorities are not necessary the best solution since the model risk also soars.

For the both indexes, the optimal capital adjustment q of VaR (99%) is almost 4 times of the adjustment of VaR(95%). Boucher et al.(2014) suggest that the relationship between the capital adjustment q and probability of VaR is not linear. Further study could focus on the components that impact such increase. Further study could focus on the components that impact such increase.

Quantification of Model Risk with Bootstrapping Method

DJI VaR(95%)

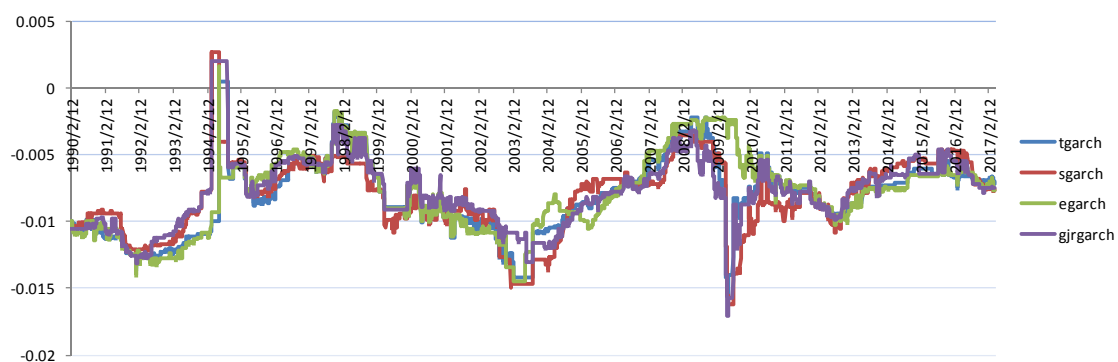
	sGARCH	eGARCH	gjrGARCH	tGARCH	historical	Normal	Average_GARCH
\bar{q}	-0.23%	-0.22%	-0.23%	-0.26%	-0.38%	-0.47%	-0.24%
$ \bar{q} $	0.27%	0.31%	0.28%	0.30%	0.56%	0.65%	0.28%
$\bar{q}, \text{if } q > 0$	0.19%	0.35%	0.23%	0.21%	0.47%	0.51%	0.18%
$\bar{q}, \text{if } q < 0$	-0.28%	-0.31%	-0.28%	-0.31%	-0.59%	-0.68%	-0.29%
σ_q	0.20%	0.29%	0.22%	0.22%	0.60%	0.64%	0.21%

Table 11

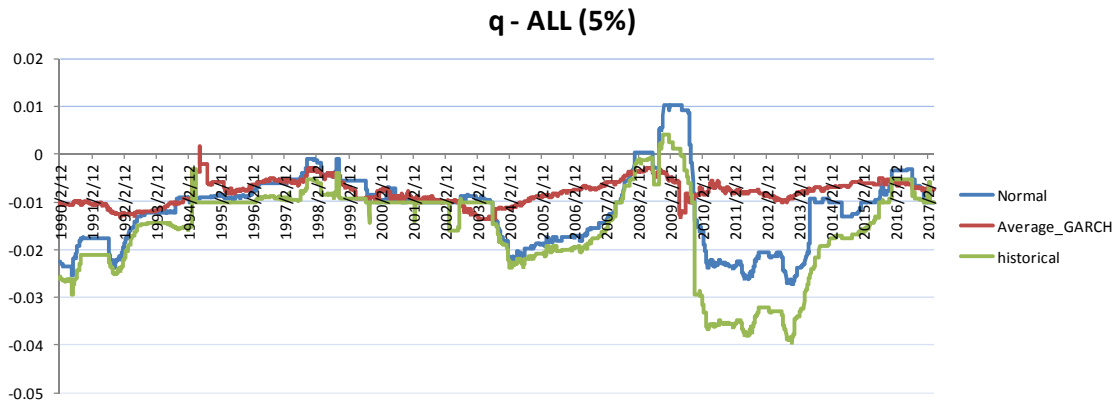
DJI VaR(99%)

For VaR at significance level 99%, as usual, the GARCH family models result in less capital adjustment q than historical simulation and normal distribution method. The historical simulation and normal distribution method are not desirable since they require too much capital when it is unnecessary and not sufficient capital in the times of crisis. In general, for all modes, the capital adjustment is mostly negative and can reach -4%. The excessive requirement is not recommended since it undermines the profitability of financial institutions.

	sGARCH	eGARCH	gjrGARCH	tGARCH	historical	Normal	Average_GARCH
\bar{q}	-0.81%	-0.79%	-0.78%	-0.81%	-1.56%	-1.16%	-0.80%
$ \bar{q} $	0.82%	0.79%	0.79%	0.81%	1.57%	1.22%	0.80%
$\bar{q}, \text{if } q > 0$	0.27%	0.16%	0.20%	0.05%	0.24%	0.59%	0.17%
$\bar{q}, \text{if } q < 0$	-0.82%	-0.79%	-0.80%	-0.82%	-1.60%	-1.26%	-0.80%
σ_q	0.28%	0.29%	0.26%	0.28%	0.95%	0.77%	0.25%

Table 12**q - GARCH FAMILY (5%)**Figure 74

Quantification of Model Risk with Bootstrapping Method

Figure 75

GSPC VaR(95%)

	sGARCH	eGARCH	gjrGARCH	tGARCH	historical	Normal	Average_GARCH
\bar{q}	-0.22%	-0.20%	-0.23%	-0.23%	-0.21%	-0.26%	-0.22%
$ \bar{q} $	0.24%	0.24%	0.24%	0.25%	0.41%	0.44%	0.24%
$\bar{q}, \text{if } q > 0$	0.15%	0.22%	0.20%	0.20%	0.41%	0.36%	0.12%
$\bar{q}, \text{if } q < 0$	-0.24%	-0.25%	-0.24%	-0.26%	-0.42%	-0.47%	-0.25%
σ_q	0.17%	0.20%	0.17%	0.18%	0.48%	0.48%	0.16%

Table 13

GSPC VaR(99%)

For GSPC index, similar conclusions can be drawn. For VaR at significance level 99%, the GARCH family models outperforms historical simulation and normal distribution method. The historical simulation and normal distribution method are not desirable since they require too much capital when it is unnecessary and not sufficient capital in the times of crisis. In general, for all modes, the capital adjustment is mostly negative and can reach -4%. The excessive requirement is not recommended since it undermines the profitability of financial institutions.

	sGARCH	eGARCH	gjrGARCH	tGARCH	historical	Normal	Average_GARCH
\bar{q}	-0.75%	-0.71%	-0.74%	-0.74%	-1.14%	-0.92%	-0.74%
$ \bar{q} $	0.75%	0.71%	0.74%	0.74%	1.15%	0.97%	0.74%
$\bar{q}, \text{if } q > 0$	2.57%	0.08%	2.55%	0.00%	0.17%	0.56%	0.46%
$\bar{q}, \text{if } q < 0$	-0.75%	-0.72%	-0.74%	-0.74%	-1.17%	-0.99%	-0.74%
σ_q	0.25%	0.25%	0.23%	0.25%	0.81%	0.65%	0.23%

Table 14

Quantification of Model Risk with Bootstrapping Method

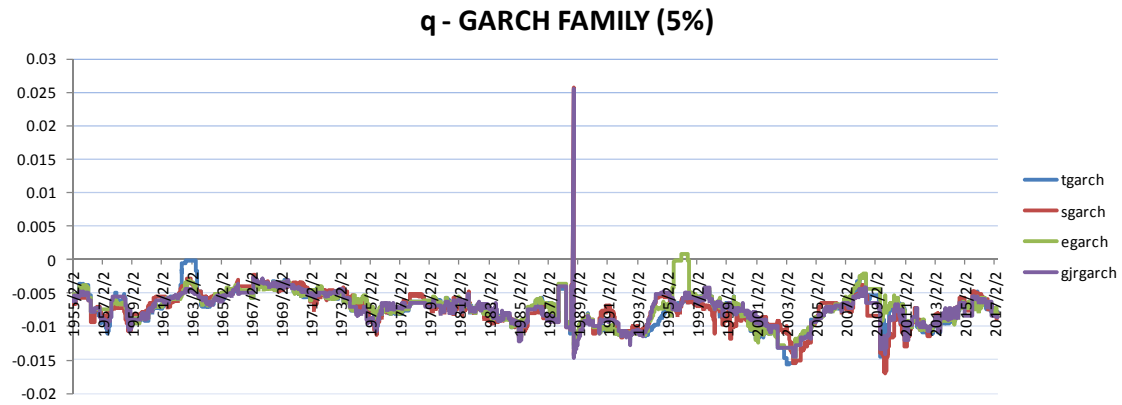


Figure 76

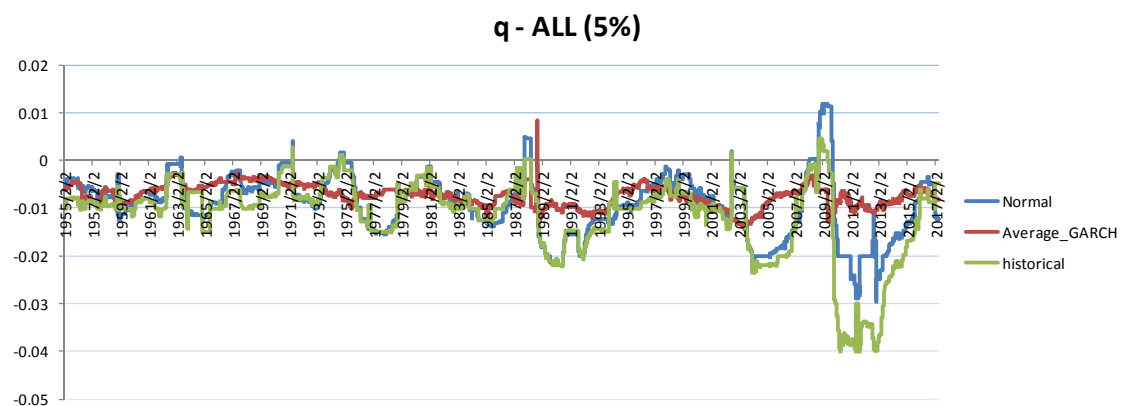


Figure 77

5.6 Adjustment q – influence of window size for historical simulations

As discussed in the methodology part, the results of the VaR estimation depends a lot on the window size of the historical simulations. The selection of the correct window size is a delicate issue since the quantile estimator only is consistent when the window size is infinite.

In this paper, VaR forecasts and optimal capital adjustment are measured for four-year window and ten-year window. The results and analysis are presented below.

DJI (5%)

For VaR estimates at significance level 5%, the simulation results of ten-year window shows less volatility than four-year window and the VaR forecasts result is also less aggressive.

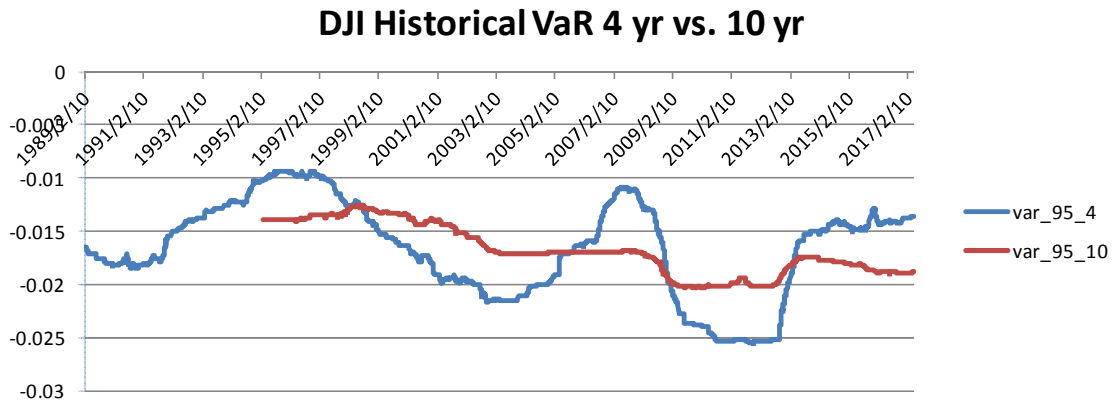


Figure 78

In general, the VaR forecasts of ten-year window requires less capital adjustment than the four year window, which demonstrates better stability and requires less capital.

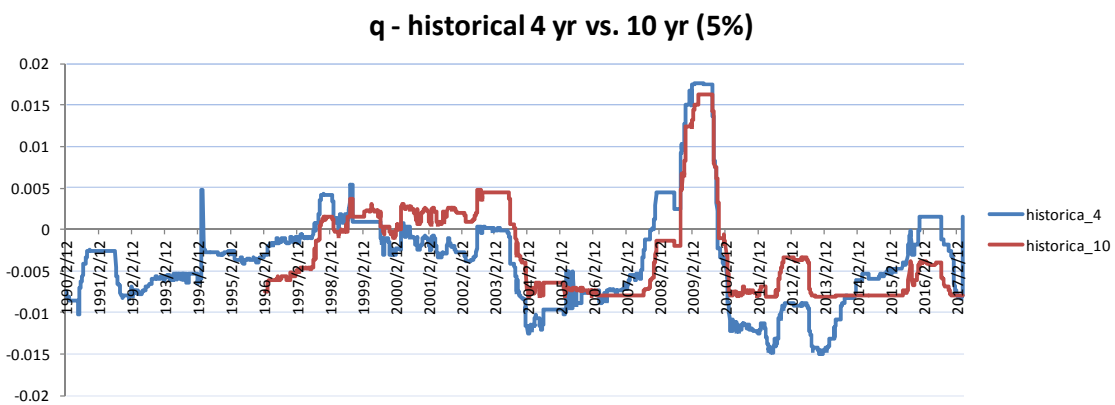


Figure 79

q adjustment (total)		q adjustment (>0)		q adjustment (<0)	
Historical_4	-0.38%	Historical_4	0.47%	Historical_4	-0.59%
Historical_10	-0.30%	Historical_10	0.41%	Historical_10	-0.61%

Table 15

GSPC (5%)

For the GSPC index, the simulation results of ten-year window shows less volatility than four-year window and the VaR forecasts result is also less aggressive. The results tend to differ more along the time.

Quantification of Model Risk with Bootstrapping Method

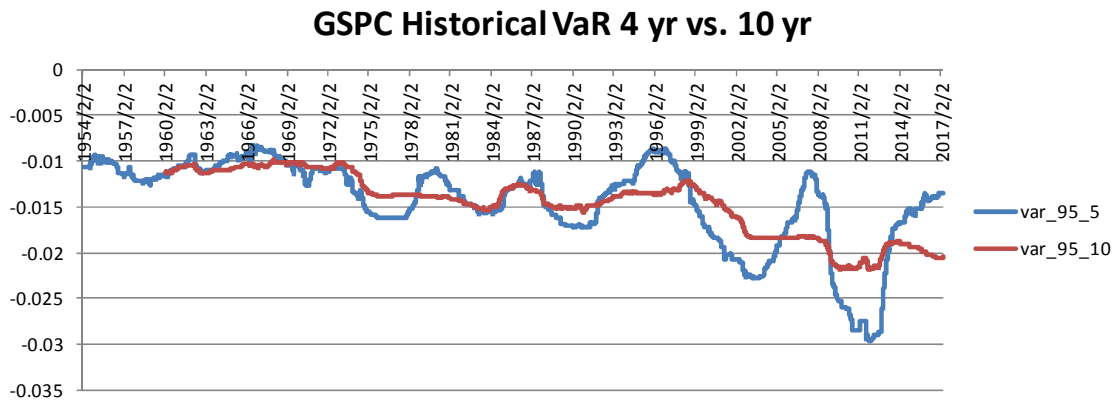


Figure 80

For the GSPC index, the VaR estimates with ten-year window require more optimal capital adjustment when more capital is required to pass the test. In this case the four-year window is better and result in less volatile capital adjustment.

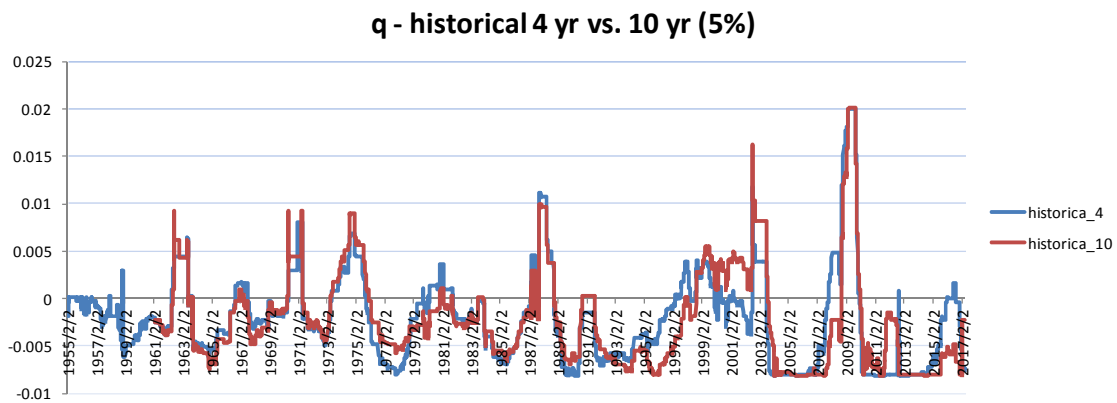


Figure 81

q adjustment (total)		q adjustment (>0)		q adjustment (<0)	
Historical_4	-0.14%	Historical_4	0.26%	Historical_4	-0.31%
Historical_10	-0.13%	Historical_10	0.39%	Historical_10	-0.32%

Table 16

Part VI Conclusion

After the 2007 crisis, it has been widely acknowledged that insufficient understanding of model risk is one of the most critical reasons contributing to the failures of risk model. The concept of model risk intrigues both academic world and financial institutions. Inspired by Boucher et al.(2014), this paper is devoted to finding an applicable method to quantify the model risk. In this paper, model risk is regarded as the excessive frequency of violation and clustering of the violations. Backtesting method with one-year window is used to search for an optimal capital adjustment for market prevalent VaR models, including GARCH family models, historical simulation and normal distribution method. The properties of the optimal capital adjustment are compared and analyzed and it offers a perspective different from pure statistics measures.

This paper uses Dow Jones Industrial Average (DJI) and S&P 500 (GSPC) indexes as sample Log return of each index is used as input for modeling and forecast process.

After a series of analysis and comparison, GARCH (1,1) – ARMA(0,0) is adopted for DJI and GARCH (1,1) – ARMA(0,1) is employed for GSPC. AIC and BIC statistics suggest that asymmetric GARCH models are better than standard GARCH model given the empirically observed asymmetric volatility.

The VaR forecasts are then calculated using a four-year moving window (1020 points). The parameters of the GARCH models were calibrated every single year (255 points) since it is common in market practice. The frequency of hits (actual return < VaR forecasts) increases with significance level α . Moreover, hits tend to cluster together especially when volatility is relatively high (e.g. in crisis era).

The results of the non-parametric historical simulation and normal distribution are estimated with a four-year moving window as well. The frequency of hits increases with significance level α and clustering of hits is observable in times of crisis. Although more restricted α level significantly lowers the frequency of hits, such improvement comes at the expense of excessive capital requirement when volatility is relatively low. Compared with the dynamic GARCH family models, the historical simulation method is insensitive to the sudden change of volatility.

The results of VaR forecasts vary with different methodologies. Hence, it is crucial to select appropriate model which does not require unnecessary excessive capital and responds fast to the sudden rise of volatility.

The concept of the optimal capital adjustment q is introduced in order to quantify the model selection criteria mentioned before. In this paper, an ideal model should not allow excessive hits of VaR forecasts and the hits should be independent to each other. In order to quantify these criteria, the backtesting technique is applied to

Quantification of Model Risk with Bootstrapping Method

calculate the optimal capital adjustment q . The algorithm programmed is using a loop function to exhaustively search for the minimum q that can pass the conditional coverage and independence test.

Generally speaking, the GARCH family VaR estimates work efficiently to predict the potential risk as the adjustment is usually negative and immaterial, while the adjustment q for historical simulation and normal distribution method demonstrates more volatility. Historical simulation and normal distribution methods adapt very slow to the sudden exchange of volatility. As a result, insufficient VaR forecasts are made during crisis while excessive capital is required after crisis.

To our surprise, the standard GARCH model performs relatively well within GARCH family models. For the DJI index, it requires least capital adjustment q . For the GPSC index, it also performs well and results in stable capital adjustment q . The reason is that the optimal capital adjustment q in this paper takes into account frequency of the hit and the independence of the hit, while the magnitude of the hit is neglected. Further study could incorporate the magnitude criteria to study the its impact on the capital adjustment q . Also, the standard GARCH model performs relatively well in independence test, while other GARCH family methods get penalized for violation clustering.

This paper also studies the relationship between model risk and significance level of VaR estimates. Compared with the optimal capital adjustment calculated for VaR at significance level 5%, VaR at significance level 1% requires more capital adjustment to pass the unconditional coverage and independence test. In other words, the model risk boosts when significance level decreases. Evidence suggests that the relationship is not lineal.

Last but not least, the influence of window size for historical simulation is also studied, since the VaR forecasts vary a lot with distinct window size. For DJI index, the simulation results of ten-year window shows less volatility than four-year window and the VaR forecasts result is also less aggressive. Also, the VaR forecasts of ten-year window requires less capital adjustment than the four year window. For GPSC index, the four-year window is better since the VaR estimates with ten-year window require more optimal capital adjustment when more capital is required to pass the test.

In summary, the optimal capital adjustment method offers an applicable method to quantify model risk. By tailoring the tests used in backtesting, it offers a subjective method to evaluate model risk and the choice can be made based on different aspects of the model performance. Also, by studying evolution of the optimal capital adjustment over time, the model owner can acquire a clear idea of the performance of various models in times of crisis.

Part VII Reference

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Quantification of Model Risk with Bootstrapping Method

PART VIII Annex – R Program

01. Data Collection

```

#install.packages('quantmod')

library('quantmod')

setwd("C:/Users/Administrator/Dropbox/TFM/TFM/01.Data Collection")

#####

#Start with Dow Jones and sp500#

#####

#####DOW JONES - ticker ^DJI

getSymbols("^DJI", from="1900-01-01", to="2017-04-27")

chartSeries(Ad(DJI))

ld.DJI <- dailyReturn(Ad(DJI), type='log')

chartSeries(ld.DJI)

write.zoo(ld.DJI,"ld.DJI.csv",index.name="Date",sep=",")

#####S&P500 - ticker ^GSPC

getSymbols("^GSPC", from="1900-01-01", to="2017-04-27")

chartSeries(Ad(GSPC))

ld.GSPC <- dailyReturn(Ad(GSPC), type='log')

chartSeries(ld.GSPC)

write.zoo(ld.GSPC,"ld.GSPC.csv",index.name="Date",sep=",")

##### PLOT #####

plot(y = DJI$'DJI.Adjusted',x = index(DJI$'DJI.Adjusted'),main = "DJI.Adjusted",type = "l",
      ylab = "Index", xlab = "Date")

plot(y = ld.DJI,x = index(ld.DJI),main = "DJI Log Return",type = "l",
      ylab = "Index", xlab = "Date")

```

02.1 Visualize data

```
#####
```

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```
#####LOAD DATA FROM 01.DATA COLLECTION###
```

```
#####
```

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/01.Data Collection")
```

```
ld.GSPC <- read.table("ld.GSPC.csv",header = TRUE,sep = ",")
```

```
ld.DJI <- read.table("ld.DJI.csv",header = TRUE,sep = ",")
```

```
#####
```

```
##### RESET WORKING DIRECTORY FOR OUTPUT###
```

```
#####
```

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/02.VaR - GARCH Family/02.1 Visualize  
data output")
```

```
##### Get The Graph of Autoregressive feature #####
```

```
ld.GSPC <- ld.GSPC$daily.returns
```

```
ld.DJI <- ld.DJI$daily.returns
```

```
#####
```

```
#####DJI####
```

```
#####
```

```
par(mfrow=c(2,2)) #we overwrite the default plotting parameters of R which is mfrow=c(1,1)
```

```
#to organize the four diagrams of interest in
```

```
#a convenient table format:
```

```
acf(ld.DJI, main="Return ACF"); ##### tiene pinta de AR(1)
```

```
pacf(ld.DJI, main="Return PACF");
```

Quantification of Model Risk with Bootstrapping Method

```

acf(ld.DJI^2, main="Squared return ACF");

pacf(ld.DJI^2, main="Squared return PACF")

par(mfrow=c(1,1))

#Compare the empirical distribution with normal

m=mean(ld.DJI);s=sd(ld.DJI);

par(mfrow=c(1,2))

hist(ld.DJI, nclass=40, freq=FALSE, main='Return histogram');curve(dnorm(x,
mean=m,sd=0.007), from = -0.3, to = 0.2, add=TRUE, col="red")

plot(density(ld.DJI), main='Return empirical distribution');curve(dnorm(x, mean=m,sd=0.007),
from = -0.3, to = 0.2, add=TRUE, col="red")

par(mfrow=c(1,1)) ##### El empirico parece que tiene la cola mas larga que normal

#test of normality

ks.test(ld.DJI,"pnorm",mean(ld.DJI),sd(ld.DJI))

library('moments')

# tail zoom

plot(density(ld.DJI), main='Return EDF - upper tail', xlim = c(0, 0.2), ylim=c(0,2));

curve(dnorm(x, mean=m,sd=s), from = -0.3, to = 0.2, add=TRUE, col="red")

plot(density(ld.DJI), main='Return EDF - lower tail', xlim = c(-0.2, 0), ylim=c(0,2));

curve(dnorm(x, mean=m,sd=s), from = -0.3, to = 0.2, add=TRUE, col="red")

#Common 2 tools to comparing density

# density plots on log-scale

plot(density(ld.DJI), xlim=c(-5*s,5*s),log='y', main='Density on log-scale')

curve(dnorm(x, mean=m,sd=s), from=-5*s, to=5*s, log="y", add=TRUE, col="red")

```


Quantification of Model Risk with Bootstrapping Method

```

# QQ-plot

qqnorm(ld.DJI);qqline(ld.DJI)

#####

#####GSPC####

#####

par(mfrow=c(2,2)) #we overwrite the default plotting parameters of R which is mfrow=c(1,1)
#to organize the four diagrams of interest in
#a convenient table format:
acf(ld.GSPC, main="Return ACF"); ##### tiene pinta de AR(1)
pacf(ld.GSPC, main="Return PACF");
acf(ld.GSPC^2, main="Squared return ACF");
pacf(ld.GSPC^2, main="Squared return PACF")
par(mfrow=c(1,1))

#Compare the empirical distribution with normal
m=mean(ld.GSPC);s=sd(ld.GSPC);
par(mfrow=c(1,2))
hist(ld.GSPC, nclass=40, freq=FALSE, main='Return histogram');curve(dnorm(x,
mean=m,sd=0.007), from = -0.3, to = 0.2, add=TRUE, col="red")
plot(density(ld.GSPC), main='Return empirical distribution');curve(dnorm(x,
mean=m,sd=0.007), from = -0.3, to = 0.2, add=TRUE, col="red")
par(mfrow=c(1,1)) ##### El empirico parece que tiene la cola mas larga que normal

#test of normality
ks.test(ld.GSPC,"pnorm",mean(ld.GSPC),sd(ld.GSPC))

```

Quantification of Model Risk with Bootstrapping Method

```

# tail zoom

plot(density(ld.GSPC), main='Return EDF - upper tail', xlim = c(0, 0.2), ylim=c(0,2));
curve(dnorm(x, mean=m,sd=s), from = -0.3, to = 0.2, add=TRUE, col="red")

plot(density(ld.GSPC), main='Return EDF - lower tail', xlim = c(-0.2, 0), ylim=c(0,2));
curve(dnorm(x, mean=m,sd=s), from = -0.3, to = 0.2, add=TRUE, col="red")

#Common 2 tools to comparing density

# density plots on log-scale
plot(density(ld.GSPC), xlim=c(-5*s,5*s),log='y', main='Density on log-scale')
curve(dnorm(x, mean=m,sd=s), from=-5*s, to=5*s, log="y", add=TRUE, col="red")

# QQ-plot
qqnorm(ld.GSPC);qqline(ld.GSPC)

plot(y = GSPC$'GSPC.Adjusted',x = index(GSPC$'GSPC.Adjusted'),main =
"GSPC.Adjusted",type = "l",
      ylab = "Index", xlab = "Date")

plot(y = ld.GSPC,x = index(ld.GSPC),main = "GSPC Log Return",type = "l",
      ylab = "Index", xlab = "Date")

02.2 & 02.3 GARCH FAMILY VAR

#####
#####LOAD DATA FROM 01.DATA COLLECTION###
#####

```

Quantification of Model Risk with Bootstrapping Method

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/01.Data Collection")
```

```
ld.DJI <- read.table("ld.DJI.csv",header = TRUE,sep = ",")
```

```
ld.GSPC <- read.table("ld.GSPC.csv",header = TRUE,sep = ",")
```

```
#colnames(ld.DJI)[2] <- "ld.DJI"
```

```
#colnames(ld.GSPC)[2] <- "ld.GSPC"
```

```
#####
```

```
##### RESET WORKING DIRECTORY FOR OUTPUT###
```

```
#####
```

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/02.VaR - GARCH Family/02.2GF VAR DJI  
output")
```

```
#install.packages("rugarch")
```

```
library("rugarch")
```

```
library("scales")
```

```
#####
```

```
#first, we have to specify a model as a system object (variable),#
```

```
#which in turn will be inserted into the respective function.#####
```

```
#Models can be specified by calling ugarchspec().#####
```

```
#####
```

```
#####
```

```
##### ar1, garch (1,1), normal, standard #####
```

```
#####
```

Quantification of Model Risk with Bootstrapping Method

```
garch11_ar1.spec = ugarchspec(variance.model = list(model="sGARCH", garchOrder=c(1,1)),
                             mean.model = list(armaOrder=c(1,0))) #Tiene la pinta de ar(1)
```

```
DJI.garch11_ar1.fit = ugarchfit(spec=garch11_ar1.spec, data=ld.DJI$daily.returns)
```

```
DJI.garch11_ar1.fit ##result, the AR1 component is not significant.
```

```
#####
```

```
##### arma(0,0), garch (1,1), normal, standard #####
```

```
#####
```

```
garch11.spec = ugarchspec(variance.model = list(model="sGARCH", garchOrder=c(1,1)),
                           mean.model = list(armaOrder=c(0,0)))
```

```
DJI.garch11.fit = ugarchfit(spec=garch11.spec, data=ld.DJI$daily.returns)
```

```
DJI.garch11.fit
```

```
#save(DJI.garch11.fit, file = "DJI.garch11.fit.rda")
```

```
plot(residuals(DJI.garch11.fit),col=alpha("red", 0.5))
```

```
##plot the change in conditional volatility against shocks in different sizes,
```

```
##and can concisely express the asymmetric effects in volatility.
```

```
ni.garch11 <- newsimpact(DJI.garch11.fit)
```

```
plot(ni.garch11$zx, ni.garch11$zy, type="l", lwd=2,
```

```
col="blue", main="GARCH(1,1) - News Impact",
```

```
ylab=ni.garch11$yexpr, xlab=ni.garch11$xexpr)
```

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```
#####
##### igarch #####
#####

garch11i.spec = ugarchspec(variance.model = list(model="iGARCH", garchOrder=c(1,1)),
                           mean.model = list(armaOrder=c(0,0)))

DJl.garch11i.fit = ugarchfit(spec=garch11i.spec, data=ld.DJl$daily.returns)

DJl.garch11i.fit

#save(DJl.garch11i.fit, file = "DJl.garch11i.fit.rda")

plot(residuals(DJl.garch11i.fit),col=alpha("red", 0.5))

##unable to plot the change in conditional volatility against shocks in different sizes,

#####
##### egarch #####
#####

garch11e.spec = ugarchspec(variance.model = list(model="eGARCH", garchOrder=c(1,1)),
                           mean.model = list(armaOrder=c(0,0)))

DJl.garch11e.fit = ugarchfit(spec=garch11e.spec, data=ld.DJl$daily.returns)

DJl.garch11e.fit

#save(DJl.garch11e.fit, file = "DJl.garch11e.fit.rda")
```

Quantification of Model Risk with Bootstrapping Method

```
plot(residuals(DJI.garch11e.fit),col=alpha("red", 0.5))
```

```
##plot the change in conditional volatility against shocks in different sizes,
```

```
##and can concisely express the asymmetric effects in volatility.
```

```
ni.garch11 <- newsimpact(DJI.garch11e.fit)
```

```
plot(ni.garch11$zx, ni.garch11$zy, type="l", lwd=2,
```

```
col="blue", main="GARCH(1,1) - News Impact",
```

```
ylab=ni.garch11$yexpr, xlab=ni.garch11$xexpr)
```

```
#####
```

```
##### gjr garch #####
```

```
#####
```

```
#gjrgarch
```

```
garch11gjr.spec = ugarchspec(variance.model = list(model="gjrGARCH", garchOrder=c(1,1)),
```

```
mean.model = list(armaOrder=c(0,0)))
```

```
DJI.garch11gjr.fit = ugarchfit(spec=garch11gjr.spec, data=ld.DJI$daily.returns)
```

```
DJI.garch11gjr.fit
```

```
#save(DJI.garch11gjr.fit, file = "DJI.garch11gjr.fit.rda")
```

```
plot(residuals(DJI.garch11gjr.fit),col=alpha("red", 0.5))
```

```
##plot the change in conditional volatility against shocks in different sizes,
```

```
##and can concisely express the asymmetric effects in volatility.
```

```
ni.garch11 <- newsimpact(DJI.garch11gjr.fit)
```

```
plot(ni.garch11$zx, ni.garch11$zy, type="l", lwd=2,
```

Quantification of Model Risk with Bootstrapping Method

```

col="blue", main="GARCH(1,1) - News Impact",
ylab=ni.garch11$yexpr, xlab=ni.garch11$xexpr)

#gjr garch is restricted form of sgarch, so likelihood ratio test can be applied here

LRst <- 2*(DJI.garch11.fit@fit$LLH-DJI.garch11gjr.fit@fit$LLH)
pchisq(q=LRst,df=1) #gjrgarch is better here!! Intuitively make sense

#####
##### threshold garch #####
#####

#tgarch

garch11t.spec = ugarchspec(variance.model = list(model="fGARCH",
garchOrder=c(1,1),submodel='TGARCH'),

                        mean.model = list(armaOrder=c(0,0)))

DJI.garch11t.fit = ugarchfit(spec=garch11t.spec, data=ld.DJI$daily.returns)

DJI.garch11t.fit

#save(DJI.garch11t.fit, file = "DJI.garch11t.fit.rda")

plot(residuals(DJI.garch11t.fit),col=alpha("red", 0.5))

##plot the change in conditional volatility against shocks in different sizes,
##and can concisely express the asymmetric effects in volatility.

ni.garch11 <- newsimpact(DJI.garch11t.fit)

plot(ni.garch11$zx, ni.garch11$zy, type="l", lwd=2,
col="blue", main="GARCH(1,1) - News Impact",

```

Quantification of Model Risk with Bootstrapping Method

```
ylab=ni.garch11$yexpr, xlab=ni.garch11$xexpr)
```

```
#####
```

```
##### Get various GARCH MODEL FIT DIRECTLY #####
```

```
#####
```

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/02.VaR - GARCH Family/02.2GF VAR DJI  
output")
```

```
install.packages("scales")
```

```
#sgarch
```

```
load(file = "DJI.garch11.fit.rda")
```

```
#igarch
```

```
load(file = "DJI.garch11i.fit.rda")
```

```
#egarch
```

```
load(file = "DJI.garch11e.fit.rda")
```

```
#GJR GARCH
```

```
load(file = "DJI.garch11gjr.fit.rda")
```

```
#TGARCH
```

```
load(file = "DJI.garch11t.fit.rda")
```

```
#####
```

```
##### SOME USEFUL COMMANDS #####
```


Quantification of Model Risk with Bootstrapping Method

#####

```

coef(DJI.garch11.fit)    #estimated coefficients
vcov(DJI.garch11.fit)    #covariance matrix of param estimates
infocriteria(DJI.garch11.fit) #common information criteria list
newsimpact(DJI.garch11.fit) #calculate news impact curve
signbias(DJI.garch11.fit) #Engle - Ng sign bias test
fitted(DJI.garch11.fit)  #obtain the fitted data series
residuals(DJI.garch11.fit) #obtain the residuals
uncvariance(DJI.garch11.fit) #unconditional (long-run) variance
uncmean(DJI.garch11.fit)  #unconditional (long-run) mean

```

#####

#####LOAD DATA FROM 01.DATA COLLECTION###

#####

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/01.Data Collection")
```

```
ld.DJI <- read.table("ld.DJI.csv",header = TRUE,sep = ",")
```

```
ld.GSPC <- read.table("ld.GSPC.csv",header = TRUE,sep = ",")
```

#####

RESET WORKING DIRECTORY FOR OUTPUT###

#####

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/02.Var - GARCH Family/02.3GF VAR
GSPC output")
```

Quantification of Model Risk with Bootstrapping Method

```

#install.packages("rugarch")

library("rugarch")

library("scales")

#####

##### arma(0,1), garch (1,1), normal, standard #####

#####

garch11.spec = ugarchspec(variance.model = list(model="sGARCH", garchOrder=c(1,1)),
                          mean.model = list(armaOrder=c(0,1)))

GSPC.garch11.fit = ugarchfit(spec=garch11.spec, data=ld.GSPC$daily.returns,solver =
"hybrid")

GSPC.garch11.fit

#save(GSPC.garch11.fit, file = "GSPC.garch11.fit.rda")

plot(residuals(GSPC.garch11.fit),col=alpha("red", 0.5))

##plot the change in conditional volatility against shocks in different sizes,
##and can concisely express the asymmetric effects in volatility.

ni.garch11 <- newsimpact(GSPC.garch11.fit)

plot(ni.garch11$zx, ni.garch11$zy, type="l", lwd=2,
     col="blue", main="GARCH(1,1) - News Impact",
     ylab=ni.garch11$yexpr, xlab=ni.garch11$xexpr)

```

Quantification of Model Risk with Bootstrapping Method

```
#####
##### igarch #####
#####

garch11i.spec = ugarchspec(variance.model = list(model="iGARCH", garchOrder=c(1,1)),
                           mean.model = list(armaOrder=c(0,1)))
GSPC.garch11i.fit = ugarchfit(spec=garch11i.spec, data=ld.GSPC$daily.returns)
GSPC.garch11i.fit

#save(GSPC.garch11i.fit, file = "GSPC.garch11i.fit.rda")
plot(residuals(GSPC.garch11i.fit),col=alpha("red", 0.5))

##unable to plot the change in conditional volatility against shocks in different sizes,

#####
##### egarch #####
#####

garch11e.spec = ugarchspec(variance.model = list(model="eGARCH", garchOrder=c(1,1)),
                           mean.model = list(armaOrder=c(0,1)))
GSPC.garch11e.fit = ugarchfit(spec=garch11e.spec, data=ld.GSPC$daily.returns)
GSPC.garch11e.fit

#save(GSPC.garch11e.fit, file = "GSPC.garch11e.fit.rda")

plot(residuals(GSPC.garch11e.fit),col=alpha("red", 0.5))
```

Quantification of Model Risk with Bootstrapping Method

```

##plot the change in conditional volatility against shocks in different sizes,
##and can concisely express the asymmetric effects in volatility.
ni.garch11 <- newsimpact(GSPC.garch11e.fit)
plot(ni.garch11$zx, ni.garch11$zy, type="l", lwd=2,
     col="blue", main="GARCH(1,1) - News Impact",
     ylab=ni.garch11$yexpr, xlab=ni.garch11$xexpr)

#####
##### gjr garch #####
#####
#gjrgarch
garch11gjr.spec = ugarchspec(variance.model = list(model="gjrGARCH", garchOrder=c(1,1)),
                             mean.model = list(armaOrder=c(0,1)))

GSPC.garch11gjr.fit = ugarchfit(spec=garch11gjr.spec, data=ld.GSPC$daily.returns)
GSPC.garch11gjr.fit

#save(GSPC.garch11gjr.fit, file = "GSPC.garch11gjr.fit.rda")
plot(residuals(GSPC.garch11gjr.fit),col=alpha("red", 0.5))

##plot the change in conditional volatility against shocks in different sizes,
##and can concisely express the asymmetric effects in volatility.
ni.garch11 <- newsimpact(GSPC.garch11gjr.fit)
plot(ni.garch11$zx, ni.garch11$zy, type="l", lwd=2,
     col="blue", main="GARCH(1,1) - News Impact",

```

Quantification of Model Risk with Bootstrapping Method

```

ylab=ni.garch11$yexpr, xlab=ni.garch11$xexpr)

#gjr garch is restricted form of sgarch, so likelihood ratio test can be applied here

LRst <- 2*(GSPC.garch11.fit@fit$LLH-GSPC.garch11gjr.fit@fit$LLH)
pchisq(q=LRst,df=1) #gjrgarch is better here!! Intuitively make sense

#####
##### threshold garch #####
#####

#tgarch

garch11t.spec = ugarchspec(variance.model = list(model="fGARCH",
garchOrder=c(1,1),submodel='TGARCH'),
mean.model = list(armaOrder=c(0,1)))

GSPC.garch11t.fit = ugarchfit(spec=garch11t.spec, data=ld.GSPC$daily.returns)

GSPC.garch11t.fit

#save(GSPC.garch11t.fit, file = "GSPC.garch11t.fit.rda")

plot(residuals(GSPC.garch11t.fit),col=alpha("red", 0.5))

##plot the change in conditional volatility against shocks in different sizes,
##and can concisely express the asymmetric effects in volatility.

ni.garch11 <- newsimpact(GSPC.garch11t.fit)

plot(ni.garch11$zx, ni.garch11$zy, type="l", lwd=2,
col="blue", main="GARCH(1,1) - News Impact",
ylab=ni.garch11$yexpr, xlab=ni.garch11$xexpr)

```

Quantification of Model Risk with Bootstrapping Method

02.4 4 YR rolling VaR

#####

#####LOAD DATA FROM internet #####

#####

library('quantmod')

getSymbols("^DJI", from="1900-01-01", to="2017-04-27")

chartSeries(Ad(DJI))

ld.DJI <- dailyReturn(Ad(DJI), type='log')

chartSeries(ld.DJI)

getSymbols("^GSPC", from="1900-01-01", to="2017-04-27")

chartSeries(Ad(GSPC))

ld.GSPC <- dailyReturn(Ad(GSPC), type='log')

chartSeries(ld.GSPC)

#####

SET WORKING DIRECTORY FOR OUTPUT###

#####

setwd("C:/Users/Administrator/Dropbox/TFM/TFM/02.VaR - GARCH Family/02.4 4YR Rolling
VAR output")

#####

ROLLING ESTIMATION

Quantification of Model Risk with Bootstrapping Method

```
#####
```

```
library("rugarch")
```

```
#install.packages("snowfall")
```

```
library("snowfall")
```

```
#####
```

```
##### DJI #####
```

```
#####
```

```
####sGARCH####
```

```
spec = ugarchspec(variance.model = list(model="sGARCH", garchOrder=c(1,1)),
```

```
          mean.model = list(armaOrder=c(0,0)))
```

```
#moving window: 4 years, refit every year, days per year:255
```

```
roll_DJI_s = ugarchroll(spec, data = ld.DJI, n.ahead = 1,
```

```
          n.start = 1020, refit.every = 255, refit.window = "moving",
```

```
          solver = "hybrid", fit.control = list(), parallel = TRUE,
```

```
          parallel.control = list(pkg = "snowfall", cores = 6),
```

```
          calculate.VaR = TRUE, VaR.alpha = c(0.01, 0.025, 0.05),
```

```
          keep.coef = FALSE)
```

```
#report(roll_DJI_s, type = "VaR", VaR.alpha = 0.01, conf.level = 0.95)
```

```
save(roll_DJI_s, file = "roll_DJI_s.rda")
```

```
DJI_s_var<-roll_DJI_s@forecast$VaR
```

Quantification of Model Risk with Bootstrapping Method

```
save(DJI_s_var, file = "DJI_s_var.rda")

#report(roll_DJI_s, type="fpm")

# Fit Diagnostics

plot(roll_DJI_s, which=4, VaR.alpha=0.01)
plot(roll_DJI_s, which=4, VaR.alpha=0.025)
plot(roll_DJI_s, which=4, VaR.alpha=0.05)

####eGARCH####
spec = ugarchspec(variance.model = list(model="eGARCH", garchOrder=c(1,1)),
                  mean.model = list(armaOrder=c(0,0)))

#moving window: 4 years, refit every year, days per year:255
roll_DJI_e = ugarchroll(spec, data = ld.DJI, n.ahead = 1,
                       n.start = 1020, refit.every = 255, refit.window = "moving",
                       solver = "hybrid", fit.control = list(), parallel = TRUE,
                       parallel.control = list(pkg = "snowfall", cores = 6),
                       calculate.VaR = TRUE, VaR.alpha = c(0.01, 0.025, 0.05),
                       keep.coef = FALSE)

#report(roll_DJI_e, type = "VaR", VaR.alpha = 0.01, conf.level = 0.95)
save(roll_DJI_e, file = "roll_DJI_e.rda")

DJI_e_var <- roll_DJI_e@forecast$VaR
save(DJI_e_var, file = "DJI_e_var.rda")
```


Quantification of Model Risk with Bootstrapping Method

```
#report(roll_DJI_e, type="fpm")

# Fit Diagnostics

plot(roll_DJI_e,which=4,VaR.alpha=0.01)
plot(roll_DJI_e,which=4,VaR.alpha=0.025)
plot(roll_DJI_e,which=4,VaR.alpha=0.05)

####gjrGARCH####

spec = ugarchspec(variance.model = list(model="gjrGARCH", garchOrder=c(1,1)),
                  mean.model = list(armaOrder=c(0,0)))

#moving window: 4 years, refit every year, days per year:255
roll_DJI_gjr = ugarchroll(spec, data = ld.DJI, n.ahead = 1,
                          n.start = 1020, refit.every = 255, refit.window = "moving",
                          solver = "hybrid", fit.control = list(), parallel = TRUE,
                          parallel.control = list(pkg = "snowfall", cores = 6),
                          calculate.VaR = TRUE, VaR.alpha = c(0.01, 0.025, 0.05),
                          keep.coef = FALSE)

#report(roll_DJI_e, type = "VaR", VaR.alpha = 0.01, conf.level = 0.95)

save(roll_DJI_gjr, file = "roll_DJI_gjr.rda")

DJI_gjr_var<-roll_DJI_gjr@forecast$VaR

save(DJI_gjr_var, file = "DJI_gjr_var.rda")
```

Quantification of Model Risk with Bootstrapping Method

```
#report(roll_DJI_gjr, type="fpm")

# Fit Diagnostics

plot(roll_DJI_gjr,which=4,VaR.alpha=0.01)
plot(roll_DJI_gjr,which=4,VaR.alpha=0.025)
plot(roll_DJI_gjr,which=4,VaR.alpha=0.05)

#### threshold GARCH #####

spec = ugarchspec(variance.model = list(model="fGARCH",
garchOrder=c(1,1),submodel='TGARCH'),

                mean.model = list(armaOrder=c(0,0)))

#moving window: 4 years, refit every year, days per year:255
roll_DJI_t = ugarchroll(spec, data = ld.DJI, n.ahead = 1,

                n.start = 1020, refit.every = 255, refit.window = "moving",

                solver = "hybrid", fit.control = list(), parallel = TRUE,

                parallel.control = list(pkg = "snowfall", cores = 6),

                calculate.VaR = TRUE, VaR.alpha = c(0.01, 0.025, 0.05),

                keep.coef = FALSE)

#report(roll_DJI_t, type = "VaR", VaR.alpha = 0.01, conf.level = 0.95)

save(roll_DJI_t, file = "roll_DJI_t.rda")

DJI_t_var<-roll_DJI_t@forecast$VaR

save(DJI_t_var, file = "DJI_t_var.rda")

#report(roll_DJI_t, type="fpm")
```

Quantification of Model Risk with Bootstrapping Method

```
# Fit Diagnostics
```

```
plot(roll_DJI_t,which=4,VaR.alpha=0.01)
```

```
plot(roll_DJI_t,which=4,VaR.alpha=0.025)
```

```
plot(roll_DJI_t,which=4,VaR.alpha=0.05)
```

```
#####
```

```
##### GSPC #####
```

```
#####
```

```
####sGARCH####
```

```
spec = ugarchspec(variance.model = list(model="sGARCH", garchOrder=c(1,1)),
  mean.model = list(armaOrder=c(0,1)))
```

```
#moving window: 4 years, refit every year, days per year:255
```

```
roll_GSPC_s = ugarchroll(spec, data = ld.GSPC, n.ahead = 1,
  n.start = 1020, refit.every = 255, refit.window = "moving",
  solver = "hybrid", fit.control = list(), parallel = TRUE,
  parallel.control = list(pkg = "snowfall", cores = 6),
  calculate.VaR = TRUE, VaR.alpha = c(0.01, 0.025, 0.05),
  keep.coef = FALSE)
```

```
#report(roll_GSPC_s, type = "VaR", VaR.alpha = 0.01, conf.level = 0.95)
```

```
save(roll_GSPC_s, file = "roll_GSPC_s.rda")
```

Quantification of Model Risk with Bootstrapping Method

```
GSPC_s_var<-roll_GSPC_s@forecast$VaR
save(GSPC_s_var, file = "GSPC_s_var.rda")

#report(roll_GSPC_s, type="fpm")

# Fit Diagnostics

plot(roll_GSPC_s,which=4,VaR.alpha=0.01)
plot(roll_GSPC_s,which=4,VaR.alpha=0.025)
plot(roll_GSPC_s,which=4,VaR.alpha=0.05)

####eGARCH####

spec = ugarchspec(variance.model = list(model="eGARCH", garchOrder=c(1,1)),
                  mean.model = list(armaOrder=c(0,1)))

#moving window: 4 years, refit every year, days per year:255
roll_GSPC_e = ugarchroll(spec, data = ld.GSPC, n.ahead = 1,
                        n.start = 1020, refit.every = 255, refit.window = "moving",
                        solver = "hybrid", fit.control = list(), parallel = TRUE,
                        parallel.control = list(pkg = "snowfall", cores = 6),
                        calculate.VaR = TRUE, VaR.alpha = c(0.01, 0.025, 0.05),
                        keep.coef = FALSE)

#report(roll_GSPC_e, type = "VaR", VaR.alpha = 0.01, conf.level = 0.95)
save(roll_GSPC_e, file = "roll_GSPC_e.rda")
```

Quantification of Model Risk with Bootstrapping Method

```

GSPC_e_var<-roll_GSPC_e@forecast$VaR

save(GSPC_e_var, file = "GSPC_e_var.rda")

#report(roll_GSPC_e, type="fpm")

# Fit Diagnostics

plot(roll_GSPC_e,which=4,VaR.alpha=0.01)
plot(roll_GSPC_e,which=4,VaR.alpha=0.025)
plot(roll_GSPC_e,which=4,VaR.alpha=0.05)

####gjrGARCH####

spec = ugarchspec(variance.model = list(model="gjrGARCH", garchOrder=c(1,1)),
                  mean.model = list(armaOrder=c(0,1)))

#moving window: 4 years, refit every year, days per year:255
roll_GSPC_gjr = ugarchroll(spec, data = ld.GSPC, n.ahead = 1,
                          n.start = 1020, refit.every = 255, refit.window = "moving",
                          solver = "hybrid", fit.control = list(), parallel = TRUE,
                          parallel.control = list(pkg = "snowfall", cores = 6),
                          calculate.VaR = TRUE, VaR.alpha = c(0.01, 0.025, 0.05),
                          keep.coef = FALSE)

#report(roll_GSPC_e, type = "VaR", VaR.alpha = 0.01, conf.level = 0.95)
save(roll_GSPC_gjr, file = "roll_GSPC_gjr.rda")

GSPC_gjr_var<-roll_GSPC_gjr@forecast$VaR

```

Quantification of Model Risk with Bootstrapping Method

```

save(GSPC_gjr_var, file = "GSPC_gjr_var.rda")

#report(roll_GSPC_gjr, type="fpm")

# Fit Diagnostics

plot(roll_GSPC_gjr,which=4,VaR.alpha=0.01)
plot(roll_GSPC_gjr,which=4,VaR.alpha=0.025)
plot(roll_GSPC_gjr,which=4,VaR.alpha=0.05)

#### threshold GARCH #####
spec = ugarchspec(variance.model = list(model="fGARCH",
garchOrder=c(1,1),submodel='TGARCH'),
                mean.model = list(armaOrder=c(0,1)))

#moving window: 4 years, refit every year, days per year:255
roll_GSPC_t = ugarchroll(spec, data = ld.GSPC, n.ahead = 1,
                n.start = 1020, refit.every = 255, refit.window = "moving",
                solver = "hybrid", fit.control = list(), parallel = TRUE,
                parallel.control = list(pkg = "snowfall", cores = 6),
                calculate.VaR = TRUE, VaR.alpha = c(0.01, 0.025, 0.05),
                keep.coef = FALSE)

#report(roll_GSPC_t, type = "VaR", VaR.alpha = 0.01, conf.level = 0.95)
save(roll_GSPC_t, file = "roll_GSPC_t.rda")

GSPC_t_var<-roll_GSPC_t@forecast$VaR
save(GSPC_t_var, file = "GSPC_t_var.rda")

```

Quantification of Model Risk with Bootstrapping Method

```

#report(roll_GSPC_t, type="fpm")

# Fit Diagnostics

plot(roll_GSPC_t,which=4,VaR.alpha=0.01)
plot(roll_GSPC_t,which=4,VaR.alpha=0.025)
plot(roll_GSPC_t,which=4,VaR.alpha=0.05)

03.1 HIT&DEPENDENCE

#####

##### LOAD WORKING DIRECTORY FOR INPUT#####

#####

setwd("C:/Users/Administrator/Dropbox/TFM/TFM/02.VaR - GARCH Family/02.4 4YR Rolling
VAR output")

load(file = "DJI_s_var.rda")
load(file = "DJI_e_var.rda")
load(file = "DJI_gjr_var.rda")
load(file = "DJI_t_var.rda")

load(file = "GSPC_s_var.rda")
load(file = "GSPC_e_var.rda")
load(file = "GSPC_gjr_var.rda")
load(file = "GSPC_t_var.rda")

#####

##### SET WORKING DIRECTORY FOR OUTPUT#####

#####

```

Quantification of Model Risk with Bootstrapping Method

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/03. q - hit and independence/03.1
Hit&Independence Output")
```

```
#####
```

```
##### CALCULATIN OF Q ADJUSTMENT #####
```

```
#####
```

```
library("rugarch")
```

```
#####
```

```
##### LOAD UP FUNCTION THAT WILL BE USED LATER #####
```

```
#####
```

```
#### OPEN R SCRIPT AND RUN:
```

```
### 03.2 CCLRp LOOP FUNCTION
```

```
##### DJI_s #####
```

```
q_DJI_s_1 <- cclrp_rolling(255,DJI_s_var$`alpha(1%)`,DJI_s_var$realized,0.01)
```

```
q_DJI_s_1 <- q_DJI_s_1[1:6855]
```

```
write.csv(q_DJI_s_1,file = "q_DJI_s_1.csv")
```

```
q_DJI_s_25 <- cclrp_rolling(255,DJI_s_var$`alpha(2%)`,DJI_s_var$realized,0.025)
```

```
q_DJI_s_25 <- q_DJI_s_25[1:6855]
```

```
write.csv(q_DJI_s_25,file = "q_DJI_s_25.csv")
```

```
q_DJI_s_5 <- cclrp_rolling(255,DJI_s_var$`alpha(5%)`,DJI_s_var$realized,0.05)
```

```
q_DJI_s_5 <- q_DJI_s_5[1:6855]
```


Quantification of Model Risk with Bootstrapping Method

```
write.csv(q_DJI_s_5,file = "q_DJI_s_5.csv")
```

```
##### DJI_e #####
```

```
q_DJI_e_1 <- cclrp_rolling(255,DJI_e_var$`alpha(1%)`,DJI_e_var$realized,0.01)
```

```
q_DJI_e_1 <- q_DJI_e_1[1:6855]
```

```
write.csv(q_DJI_e_1,file = "q_DJI_e_1.csv")
```

```
q_DJI_e_25 <- cclrp_rolling(255,DJI_s_var$`alpha(2%)`,DJI_e_var$realized,0.025)
```

```
q_DJI_e_25 <- q_DJI_e_25[1:6855]
```

```
write.csv(q_DJI_e_25,file = "q_DJI_e_25.csv")
```

```
q_DJI_e_5 <- cclrp_rolling(255,DJI_s_var$`alpha(5%)`,DJI_e_var$realized,0.05)
```

```
q_DJI_e_5 <- q_DJI_e_5[1:6855]
```

```
write.csv(q_DJI_e_5,file = "q_DJI_e_5.csv")
```

```
##### DJI_gjr #####
```

```
q_DJI_gjr_1 <- cclrp_rolling(255,DJI_gjr_var$`alpha(1%)`,DJI_gjr_var$realized,0.01)
```

```
q_DJI_gjr_1 <- q_DJI_gjr_1[1:6855]
```

```
write.csv(q_DJI_gjr_1,file = "q_DJI_gjr_1.csv")
```

```
q_DJI_gjr_25 <- cclrp_rolling(255,DJI_s_var$`alpha(2%)`,DJI_gjr_var$realized,0.025)
```

```
q_DJI_gjr_25 <- q_DJI_gjr_25[1:6855]
```

```
write.csv(q_DJI_gjr_25,file = "q_DJI_gjr_25.csv")
```

```
q_DJI_gjr_5 <- cclrp_rolling(255,DJI_s_var$`alpha(5%)`,DJI_gjr_var$realized,0.05)
```

```
q_DJI_gjr_5 <- q_DJI_gjr_5[1:6855]
```

```
write.csv(q_DJI_gjr_5,file = "q_DJI_gjr_5.csv")
```

Quantification of Model Risk with Bootstrapping Method

```
##### DJI_t #####
```

```
q_DJI_t_1 <- cclrp_rolling(255,DJI_t_var$`alpha(1%)`,DJI_t_var$realized,0.01)
```

```
q_DJI_t_1 <- q_DJI_t_1[1:6855]
```

```
write.csv(q_DJI_t_1,file = "q_DJI_t_1.csv")
```

```
q_DJI_t_25 <- cclrp_rolling(255,DJI_s_var$`alpha(2%)`,DJI_t_var$realized,0.025)
```

```
q_DJI_t_25 <- q_DJI_t_25[1:6855]
```

```
write.csv(q_DJI_t_25,file = "q_DJI_t_25.csv")
```

```
q_DJI_t_5 <- cclrp_rolling(255,DJI_s_var$`alpha(5%)`,DJI_t_var$realized,0.05)
```

```
q_DJI_t_5 <- q_DJI_t_5[1:6855]
```

```
write.csv(q_DJI_t_5,file = "q_DJI_t_5.csv")
```

```
##### GSPC_s #####
```

```
q_GSPC_s_1 <- cclrp_rolling(255,GSPC_s_var$`alpha(1%)`,GSPC_s_var$realized,0.01)
```

```
q_GSPC_s_1 <- q_GSPC_s_1[1:15665]
```

```
write.csv(q_GSPC_s_1,file = "q_GSPC_s_1.csv")
```

```
q_GSPC_s_25 <- cclrp_rolling(255,GSPC_s_var$`alpha(2%)`,GSPC_s_var$realized,0.025)
```

```
q_GSPC_s_25 <- q_GSPC_s_25[1:15665]
```

```
write.csv(q_GSPC_s_25,file = "q_GSPC_s_25.csv")
```

```
q_GSPC_s_5 <- cclrp_rolling(255,GSPC_s_var$`alpha(5%)`,GSPC_s_var$realized,0.05)
```

```
q_GSPC_s_5 <- q_GSPC_s_5[1:15665]
```

```
write.csv(q_GSPC_s_5,file = "q_GSPC_s_5.csv")
```

Quantification of Model Risk with Bootstrapping Method

```
##### GSPC_e #####  
q_GSPC_e_1 <- cclrp_rolling(255,GSPC_e_var$`alpha(1%)`,GSPC_e_var$realized,0.01)  
q_GSPC_e_1 <- q_GSPC_e_1[1:15665]  
write.csv(q_GSPC_e_1,file = "q_GSPC_e_1.csv")  
  
q_GSPC_e_25 <- cclrp_rolling(255,GSPC_s_var$`alpha(2%)`,GSPC_e_var$realized,0.025)  
q_GSPC_e_25 <- q_GSPC_e_25[1:15665]  
write.csv(q_GSPC_e_25,file = "q_GSPC_e_25.csv")  
  
q_GSPC_e_5 <- cclrp_rolling(255,GSPC_s_var$`alpha(5%)`,GSPC_e_var$realized,0.05)  
q_GSPC_e_5 <- q_GSPC_e_5[1:15665]  
write.csv(q_GSPC_e_5,file = "q_GSPC_e_5.csv")  
  
##### GSPC_gjr #####  
q_GSPC_gjr_1 <- cclrp_rolling(255,GSPC_gjr_var$`alpha(1%)`,GSPC_gjr_var$realized,0.01)  
q_GSPC_gjr_1 <- q_GSPC_gjr_1[1:15665]  
write.csv(q_GSPC_gjr_1,file = "q_GSPC_gjr_1.csv")  
  
q_GSPC_gjr_25 <- cclrp_rolling(255,GSPC_s_var$`alpha(2%)`,GSPC_gjr_var$realized,0.025)  
q_GSPC_gjr_25 <- q_GSPC_gjr_25[1:15665]  
write.csv(q_GSPC_gjr_25,file = "q_GSPC_gjr_25.csv")  
  
q_GSPC_gjr_5 <- cclrp_rolling(255,GSPC_s_var$`alpha(5%)`,GSPC_gjr_var$realized,0.05)  
q_GSPC_gjr_5 <- q_GSPC_gjr_5[1:15665]  
write.csv(q_GSPC_gjr_5,file = "q_GSPC_gjr_5.csv")
```

Quantification of Model Risk with Bootstrapping Method

```
##### GSPC_t #####

q_GSPC_t_1 <- cclrp_rolling(255,GSPC_t_var$`alpha(1%)`,GSPC_t_var$realized,0.01)
q_GSPC_t_1 <- q_GSPC_t_1[1:15665]
write.csv(q_GSPC_t_1,file = "q_GSPC_t_1.csv")

q_GSPC_t_25 <- cclrp_rolling(255,GSPC_s_var$`alpha(2%)`,GSPC_t_var$realized,0.025)
q_GSPC_t_25 <- q_GSPC_t_25[1:15665]
write.csv(q_GSPC_t_25,file = "q_GSPC_t_25.csv")

q_GSPC_t_5 <- cclrp_rolling(255,GSPC_s_var$`alpha(5%)`,GSPC_t_var$realized,0.05)
q_GSPC_t_5 <- q_GSPC_t_5[1:15665]
write.csv(q_GSPC_t_5,file = "q_GSPC_t_5.csv")

cclrp_loop <- function(data,data_r,alpha_v){

  for (q in -200:5000){r <- VaRTest(alpha = alpha_v, VaR=data-q/10000, actual = data_r,
  conf.level = 0.95)

  if(is.na(r$cc.LRp)){

    return((q-1)/10000)

    break

  }

  if(r$cc.LRp > 0.05 ){

    return(q/10000)

    break

  }

}

}
```

Quantification of Model Risk with Bootstrapping Method

#Only adjust Var with result of previous 1 yr, which means period = 255

```
cclrp_rolling <- function(period,data,data_r,alpha_v){
  n <- length(data)
  result_q <- vector(mode="numeric", length=n-period)
  for(i in period+1:n){
    a <- i-period
    b <- i-1
    result_q[a] <- cclrp_loop(data[a:b],data_r[a:b],alpha_v)
  }
  return(result_q)
}
```

04.1&04.2 Non parametric VaR

```
#####
#####LOAD DATA FROM 01.DATA COLLECTION###
#####
```

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/01.Data Collection")
```

```
ld.DJI <- read.table("ld.DJI.csv",header = TRUE,sep = ",")
```

```
ld.GSPC <- read.table("ld.GSPC.csv",header = TRUE,sep = ",")
```

```
#####
##### RESET WORKING DIRECTORY FOR OUTPUT###
#####
```

Quantification of Model Risk with Bootstrapping Method

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/04.VaR - Non-parametric/04.1 Non
parametric VaR output")
```

```
#install.packages("PerformanceAnalytics")
```

```
library("PerformanceAnalytics")
```

```
#####
```

```
##### LOAD THE HISTORICAL VAR CALCULATION FUNCTION ###
```

```
#####
```

```
#DJI
```

```
var_his <- function(data,p,year){
```

```
  n <- length(data)
```

```
  result <- vector(mode = "numeric",length = n - 255*year)
```

```
  for (i in 1: (n-255*year)){
```

```
    result[i] <- VaR(data[i:(255*year+i-1)],p=p,method ="historical",portfolio_method =
"single")
```

```
  }
```

```
  return(result)
```

```
}
```

```
#Historical Return 4 yrs & 10 yrs rolling window
```

```
VaR_his_95_4 <- var_his(ld.DJI$daily.returns,p=0.95,year=4)
```

```
VaR_his_975_4 <- var_his(ld.DJI$daily.returns,p=0.975,year=4)
```

```
VaR_his_99_4 <- var_his(ld.DJI$daily.returns,p=0.99,year=4)
```

```
VaR_his_4 <- as.data.frame(cbind(VaR_his_95_4,VaR_his_975_4,VaR_his_99_4))
```

Quantification of Model Risk with Bootstrapping Method

```

VaR_his_4 <-
cbind(ld.DJI$Date[(255*4+1):length(ld.DJI$daily.returns)],VaR_his_4,ld.DJI$daily.returns[(25
5*4+1):length(ld.DJI$daily.returns)])

colnames(VaR_his_4) <- c("date","var_95","var_975","var_99","realized")

#write.csv(VaR_his_4, file="DJI_VaR_his_4.csv")

```

```

VaR_his_95_10 <- var_his(ld.DJI$daily.returns,p=0.95,year=10)

```

```

VaR_his_975_10 <- var_his(ld.DJI$daily.returns,p=0.975,year=10)

```

```

VaR_his_99_10 <- var_his(ld.DJI$daily.returns,p=0.99,year=10)

```

```

VaR_his_10 <- as.data.frame(cbind(VaR_his_95_10,VaR_his_975_10,VaR_his_99_10))

```

```

VaR_his_10 <-
cbind(ld.DJI$Date[(255*10+1):length(ld.DJI$daily.returns)],VaR_his_10,ld.DJI$daily.returns[(
255*10+1):length(ld.DJI$daily.returns)])

colnames(VaR_his_10) <- c("date","var_95","var_975","var_99","realized")

#write.csv(VaR_his_10, file="DJI_VaR_his_10.csv")

```

```

#####

```

```

##### LOAD THE NORMAL VAR CALCULATION FUNCTION ###

```

```

#####

```

```

var_norm <- function(data,p,year){
  n <-length(data)
  result <- vector(mode = "numeric",length = n - 255*year)
  for (i in 1: (n-255*year)){
    result[i] <- VaR(data[i:(255*year+i-1)],p=p,method ="gaussian",portfolio_method =
"single",mu = mean(data[i:(255*year+i-1)]),sigma=var(data[i:(255*year+i-1)]))
  }
}

```

Quantification of Model Risk with Bootstrapping Method

```

return(result)
}

#Normal return VaR

VaR_norm_95_4 <- var_norm(ld.DJI$daily.returns,p=0.95,year=4)
VaR_norm_975_4 <- var_norm(ld.DJI$daily.returns,p=0.975,year=4)
VaR_norm_99_4 <- var_norm(ld.DJI$daily.returns,p=0.99,year=4)

VaR_norm_4 <- as.data.frame(cbind(VaR_norm_95_4,VaR_norm_975_4,VaR_norm_99_4))

VaR_norm_4 <-
cbind(ld.DJI$Date[(255*4+1):length(ld.DJI$daily.returns)],VaR_norm_4,ld.DJI$daily.returns[
(255*4+1):length(ld.DJI$daily.returns)])

colnames(VaR_norm_4) <- c("date","var_95","var_975","var_99","realized")

#write.csv(VaR_norm_4, file="DJI_VaR_norm_4.csv")

#GSPC

#Historical Return 4 yrs & 10 yrs rolling window

VaR_his_95_4 <- var_his(ld.GSPC$daily.returns,p=0.95,year=4)
VaR_his_975_4 <- var_his(ld.GSPC$daily.returns,p=0.975,year=4)
VaR_his_99_4 <- var_his(ld.GSPC$daily.returns,p=0.99,year=4)

VaR_his_4 <- as.data.frame(cbind(VaR_his_95_4,VaR_his_975_4,VaR_his_99_4))

VaR_his_4 <-
cbind(ld.GSPC$Date[(255*4+1):length(ld.GSPC$daily.returns)],VaR_his_4,ld.GSPC$daily.returns[
(255*4+1):length(ld.GSPC$daily.returns)])

colnames(VaR_his_4) <- c("date","var_95","var_975","var_99","realized")

#write.csv(VaR_his_4, file="GSPC_VaR_his_4.csv")

```


Quantification of Model Risk with Bootstrapping Method

```

VaR_his_95_10 <- var_his(ld.GSPC$daily.returns,p=0.95,year=10)
VaR_his_975_10 <- var_his(ld.GSPC$daily.returns,p=0.975,year=10)
VaR_his_99_10 <- var_his(ld.GSPC$daily.returns,p=0.99,year=10)

VaR_his_10 <- as.data.frame(cbind(VaR_his_95_10,VaR_his_975_10,VaR_his_99_10))

VaR_his_10 <-
cbind(ld.GSPC$Date[(255*10+1):length(ld.GSPC$daily.returns)],VaR_his_10,ld.GSPC$daily.re
turns[(255*10+1):length(ld.GSPC$daily.returns)])

colnames(VaR_his_10) <- c("date","var_95","var_975","var_99","realized")

#write.csv(VaR_his_10, file="GSPC_VaR_his_10.csv")

```

```
#Normal return VaR
```

```

VaR_norm_95_4 <- var_norm(ld.GSPC$daily.returns,p=0.95,year=4)
VaR_norm_975_4 <- var_norm(ld.GSPC$daily.returns,p=0.975,year=4)
VaR_norm_99_4 <- var_norm(ld.GSPC$daily.returns,p=0.99,year=4)

VaR_norm_4 <- as.data.frame(cbind(VaR_norm_95_4,VaR_norm_975_4,VaR_norm_99_4))

VaR_norm_4 <-
cbind(ld.GSPC$Date[(255*4+1):length(ld.GSPC$daily.returns)],VaR_norm_4,ld.GSPC$daily.re
turns[(255*4+1):length(ld.GSPC$daily.returns)])

colnames(VaR_norm_4) <- c("date","var_95","var_975","var_99","realized")

#write.csv(VaR_norm_4, file="GSPC_VaR_norm_4.csv")

```

04.3 HIT&INDEPENDENCE

```

#####

##### LOAD WORKING DIRECTORY FOR INPUT#####

#####

```

Quantification of Model Risk with Bootstrapping Method

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/04.VaR - Non-parametric/04.1 Non
parametric VaR output")
```

```
DJI_VaR_his_4 <- read.csv(file="DJI_VaR_his_4.csv")
```

```
DJI_VaR_his_4$X <- NULL
```

```
DJI_VaR_his_10 <- read.csv(file="DJI_VaR_his_10.csv")
```

```
DJI_VaR_his_10$X <- NULL
```

```
DJI_VaR_norm_4 <- read.csv(file="DJI_VaR_norm_4.csv")
```

```
DJI_VaR_norm_4$X <- NULL
```

```
GSPC_VaR_his_4 <- read.csv(file="GSPC_VaR_his_4.csv")
```

```
GSPC_VaR_his_4$X <- NULL
```

```
GSPC_VaR_his_10 <- read.csv(file="GSPC_VaR_his_10.csv")
```

```
GSPC_VaR_his_10$X <- NULL
```

```
GSPC_VaR_norm_4 <- read.csv(file="GSPC_VaR_norm_4.csv")
```

```
GSPC_VaR_norm_4$X <- NULL
```

```
#####
```

```
##### SET WORKING DIRECTORY FOR OUTPUT#####
```

```
#####
```

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/04.VaR - Non-parametric/04.3 Hit
Independence output")
```

Quantification of Model Risk with Bootstrapping Method

#####

CALCULATIN OF Q ADJUSTMENT

#####

library("rugarch")

#####

LOAD UP FUNCTION THAT WILL BE USED LATER

#####

OPEN R SCRIPT AND RUN:

04.4 CCLRp LOOP FUNCTION

DJI

```
q_DJI_VaR_his_4_99 <-
cclrp_rolling(255,DJI_VaR_his_4$var_99,DJI_VaR_his_4$realized,0.01)
```

```
q_DJI_VaR_his_4_975 <-
cclrp_rolling(255,DJI_VaR_his_4$var_975,DJI_VaR_his_4$realized,0.025)
```

```
q_DJI_VaR_his_4_95 <-
cclrp_rolling(255,DJI_VaR_his_4$var_95,DJI_VaR_his_4$realized,0.05)
```

```
write.csv(q_DJI_VaR_his_4_99,file = "q_DJI_VaR_his_4_99")
```

```
write.csv(q_DJI_VaR_his_4_975,file = "q_DJI_VaR_his_4_975")
```

```
write.csv(q_DJI_VaR_his_4_95,file = "q_DJI_VaR_his_4_95")
```

```
q_DJI_VaR_his_10_99 <-
cclrp_rolling(255,DJI_VaR_his_10$var_99,DJI_VaR_his_10$realized,0.01)
```

Quantification of Model Risk with Bootstrapping Method

```

q_DJI_VaR_his_10_975 <-
cclrp_rolling(255,DJI_VaR_his_10$var_975,DJI_VaR_his_10$realized,0.025)

q_DJI_VaR_his_10_95 <-
cclrp_rolling(255,DJI_VaR_his_10$var_95,DJI_VaR_his_10$realized,0.05)

write.csv(q_DJI_VaR_his_10_99,file = "q_DJI_VaR_his_10_99")
write.csv(q_DJI_VaR_his_10_975,file = "q_DJI_VaR_his_10_975")
write.csv(q_DJI_VaR_his_10_95,file = "q_DJI_VaR_his_10_95")

q_DJI_VaR_norm_4_99 <-
cclrp_rolling(255,DJI_VaR_norm_4$var_99,DJI_VaR_norm_4$realized,0.01)

q_DJI_VaR_norm_4_975 <-
cclrp_rolling(255,DJI_VaR_norm_4$var_975,DJI_VaR_norm_4$realized,0.025)

q_DJI_VaR_norm_4_95 <-
cclrp_rolling(255,DJI_VaR_norm_4$var_95,DJI_VaR_norm_4$realized,0.05)

write.csv(q_DJI_VaR_norm_4_99,file = "q_DJI_VaR_norm_4_99")
write.csv(q_DJI_VaR_norm_4_975,file = "q_DJI_VaR_norm_4_975")
write.csv(q_DJI_VaR_norm_4_95,file = "q_DJI_VaR_norm_4_95")

##### GSPC #####

q_GSPC_VaR_his_4_99 <-
cclrp_rolling(255,GSPC_VaR_his_4$var_99,GSPC_VaR_his_4$realized,0.01)

q_GSPC_VaR_his_4_975 <-
cclrp_rolling(255,GSPC_VaR_his_4$var_975,GSPC_VaR_his_4$realized,0.025)

q_GSPC_VaR_his_4_95 <-
cclrp_rolling(255,GSPC_VaR_his_4$var_95,GSPC_VaR_his_4$realized,0.05)

write.csv(q_GSPC_VaR_his_4_99,file = "q_GSPC_VaR_his_4_99")
write.csv(q_GSPC_VaR_his_4_975,file = "q_GSPC_VaR_his_4_975")

```

Quantification of Model Risk with Bootstrapping Method

```

write.csv(q_GSPC_VaR_his_4_95,file = "q_GSPC_VaR_his_4_95")

q_GSPC_VaR_his_10_99 <-
cclrp_rolling(255,GSPC_VaR_his_10$var_99,GSPC_VaR_his_10$realized,0.01)

q_GSPC_VaR_his_10_975 <-
cclrp_rolling(255,GSPC_VaR_his_10$var_975,GSPC_VaR_his_10$realized,0.025)

q_GSPC_VaR_his_10_95 <-
cclrp_rolling(255,GSPC_VaR_his_10$var_95,GSPC_VaR_his_10$realized,0.05)

write.csv(q_GSPC_VaR_his_10_99,file = "q_GSPC_VaR_his_10_99")

write.csv(q_GSPC_VaR_his_10_975,file = "q_GSPC_VaR_his_10_975")

write.csv(q_GSPC_VaR_his_10_95,file = "q_GSPC_VaR_his_10_95")

q_GSPC_VaR_norm_4_99 <-
cclrp_rolling(255,GSPC_VaR_norm_4$var_99,GSPC_VaR_norm_4$realized,0.01)

q_GSPC_VaR_norm_4_975 <-
cclrp_rolling(255,GSPC_VaR_norm_4$var_975,GSPC_VaR_norm_4$realized,0.025)

q_GSPC_VaR_norm_4_95 <-
cclrp_rolling(255,GSPC_VaR_norm_4$var_95,GSPC_VaR_norm_4$realized,0.05)

write.csv(q_GSPC_VaR_norm_4_99,file = "q_GSPC_VaR_norm_4_99")

write.csv(q_GSPC_VaR_norm_4_975,file = "q_GSPC_VaR_norm_4_975")

write.csv(q_GSPC_VaR_norm_4_95,file = "q_GSPC_VaR_norm_4_95")

cclrp_loop <- function(data,data_r,alpha_v){

  for (q in -200:10000){r <- VaRTest(alpha = alpha_v, VaR=data-q/10000, actual = data_r,
  conf.level = 0.95)

  if(is.na(r$cc.LRp)){

    return((q-1)/10000)

    break
  }
}

```

Quantification of Model Risk with Bootstrapping Method

```

}
if(r$cc.LRp > 0.05 ){
  return(q/10000)
  break
}
}
}
}

```

#Only adjust Var with result of previous 1 yr, which means period = 255

```

cclrp_rolling <- function(period,data,data_r,alpha_v){
  n <- 2200 #length(data)
  result_q <- vector(mode="numeric", length=n-period)
  for(i in period+1:n){
    a <- i-period
    b <- i-1
    result_q[a] <- cclrp_loop(data[a:b],data_r[a:b],alpha_v)
  }
  return(result_q)
}

```

04.5 VaR Graphics

```

#####
##### LOAD VaR forecasts results #####
#####

```

Quantification of Model Risk with Bootstrapping Method

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/04.VaR - Non-parametric/04.1 Non  
parametric VaR output")
```

```
DJI_VaR_his_4 <- read.csv(file="DJI_VaR_his_4.csv")
```

```
DJI_VaR_his_4$X <- NULL
```

```
DJI_VaR_his_10 <- read.csv(file="DJI_VaR_his_10.csv")
```

```
DJI_VaR_his_10$X <- NULL
```

```
DJI_VaR_norm_4 <- read.csv(file="DJI_VaR_norm_4.csv")
```

```
DJI_VaR_norm_4$X <- NULL
```

```
GSPC_VaR_his_4 <- read.csv(file="GSPC_VaR_his_4.csv")
```

```
GSPC_VaR_his_4$X <- NULL
```

```
GSPC_VaR_his_10 <- read.csv(file="GSPC_VaR_his_10.csv")
```

```
GSPC_VaR_his_10$X <- NULL
```

```
GSPC_VaR_norm_4 <- read.csv(file="GSPC_VaR_norm_4.csv")
```

```
GSPC_VaR_norm_4$X <- NULL
```

```
#####
```

```
##### RESET WORKING DIRECTORY FOR OUTPUT###
```

```
#####
```

```
setwd("C:/Users/Administrator/Dropbox/TFM/TFM/04.VaR - Non-parametric/04.5 VaR  
Graphics output")
```

Quantification of Model Risk with Bootstrapping Method

```
#DJI
```

```
# Set new column values to appropriate colours
```

```
DJI_VaR_his_4$Colour[DJI_VaR_his_4$realized>=DJI_VaR_his_4$var_95]="gray"
```

```
DJI_VaR_his_4$Colour[DJI_VaR_his_4$realized<DJI_VaR_his_4$var_95]="red"
```

```
plot(DJI_VaR_his_4$realized, type = "p",pch=20,col = DJI_VaR_his_4$Colour,xlab = "DJI  
95%",ylab = "Var")
```

```
lines(DJI_VaR_his_4$var_95,type = "l", col = "black",lwd=3)
```

```
# Set new column values to appropriate colours
```

```
DJI_VaR_his_4$Colour[DJI_VaR_his_4$realized>=DJI_VaR_his_4$var_975]="gray"
```

```
DJI_VaR_his_4$Colour[DJI_VaR_his_4$realized<DJI_VaR_his_4$var_975]="red"
```

```
plot(DJI_VaR_his_4$realized, type = "p",pch=20,col = DJI_VaR_his_4$Colour,xlab = "DJI  
97.5%",ylab = "Var")
```

```
lines(DJI_VaR_his_4$var_975,type = "l", col = "black",lwd=3)
```

```
# Set new column values to appropriate colours
```

```
DJI_VaR_his_4$Colour[DJI_VaR_his_4$realized>=DJI_VaR_his_4$var_99]="gray"
```

```
DJI_VaR_his_4$Colour[DJI_VaR_his_4$realized<DJI_VaR_his_4$var_99]="red"
```

```
plot(DJI_VaR_his_4$realized, type = "p",pch=20,col = DJI_VaR_his_4$Colour,xlab = "DJI  
99%",ylab = "Var")
```

```
lines(DJI_VaR_his_4$var_99,type = "l", col = "black",lwd=3)
```

```
# Set new column values to appropriate colours
```

```
DJI_VaR_norm_4$Colour[DJI_VaR_norm_4$realized>=DJI_VaR_norm_4$var_95]="gray"
```


Quantification of Model Risk with Bootstrapping Method

```

DJI_VaR_norm_4$Colour[DJI_VaR_norm_4$realized<DJI_VaR_norm_4$var_95]="red"

plot(DJI_VaR_norm_4$realized, type = "p",pch=20,col = DJI_VaR_norm_4$Colour,xlab = "DJI
95%",ylab = "Var")

lines(DJI_VaR_norm_4$var_95,type = "l", col = "black",lwd=3)

# Set new column values to appropriate colours

DJI_VaR_norm_4$Colour[DJI_VaR_norm_4$realized>=DJI_VaR_norm_4$var_975]="gray"
DJI_VaR_norm_4$Colour[DJI_VaR_norm_4$realized<DJI_VaR_norm_4$var_975]="red"

plot(DJI_VaR_norm_4$realized, type = "p",pch=20,col = DJI_VaR_norm_4$Colour,xlab = "DJI
97.5%",ylab = "Var")

lines(DJI_VaR_norm_4$var_975,type = "l", col = "black",lwd=3)

# Set new column values to appropriate colours

DJI_VaR_norm_4$Colour[DJI_VaR_norm_4$realized>=DJI_VaR_norm_4$var_99]="gray"
DJI_VaR_norm_4$Colour[DJI_VaR_norm_4$realized<DJI_VaR_norm_4$var_99]="red"

plot(DJI_VaR_norm_4$realized, type = "p",pch=20,col = DJI_VaR_norm_4$Colour,xlab = "DJI
99%",ylab = "Var")

lines(DJI_VaR_norm_4$var_99,type = "l", col = "black",lwd=3)

#GSPC

# Set new column values to appropriate colours

GSPC_VaR_his_4$Colour[GSPC_VaR_his_4$realized>=GSPC_VaR_his_4$var_95]="gray"
GSPC_VaR_his_4$Colour[GSPC_VaR_his_4$realized<GSPC_VaR_his_4$var_95]="red"

```

Quantification of Model Risk with Bootstrapping Method

```
plot(GSPC_VaR_his_4$realized, type = "p",pch=20,col = GSPC_VaR_his_4$Colour,xlab =
"GSPC 95%",ylab = "Var")
```

```
lines(GSPC_VaR_his_4$var_95,type = "l", col = "black",lwd=3)
```

```
# Set new column values to appropriate colours
```

```
GSPC_VaR_his_4$Colour[GSPC_VaR_his_4$realized>=GSPC_VaR_his_4$var_975]="gray"
```

```
GSPC_VaR_his_4$Colour[GSPC_VaR_his_4$realized<GSPC_VaR_his_4$var_975]="red"
```

```
plot(GSPC_VaR_his_4$realized, type = "p",pch=20,col = GSPC_VaR_his_4$Colour,xlab =
"GSPC 97.5%",ylab = "Var")
```

```
lines(GSPC_VaR_his_4$var_975,type = "l", col = "black",lwd=3)
```

```
# Set new column values to appropriate colours
```

```
GSPC_VaR_his_4$Colour[GSPC_VaR_his_4$realized>=GSPC_VaR_his_4$var_99]="gray"
```

```
GSPC_VaR_his_4$Colour[GSPC_VaR_his_4$realized<GSPC_VaR_his_4$var_99]="red"
```

```
plot(GSPC_VaR_his_4$realized, type = "p",pch=20,col = GSPC_VaR_his_4$Colour,xlab =
"GSPC 99%",ylab = "Var")
```

```
lines(GSPC_VaR_his_4$var_99,type = "l", col = "black",lwd=3)
```

```
# Set new column values to appropriate colours
```

```
GSPC_VaR_norm_4$Colour[GSPC_VaR_norm_4$realized>=GSPC_VaR_norm_4$var_95]="gr
ay"
```

```
GSPC_VaR_norm_4$Colour[GSPC_VaR_norm_4$realized<GSPC_VaR_norm_4$var_95]="red
"
```

```
plot(GSPC_VaR_norm_4$realized, type = "p",pch=20,col = GSPC_VaR_norm_4$Colour,xlab =
"GSPC 95%",ylab = "Var")
```

```
lines(GSPC_VaR_norm_4$var_95,type = "l", col = "black",lwd=3)
```

Quantification of Model Risk with Bootstrapping Method

```
# Set new column values to appropriate colours
```

```
GSPC_VaR_norm_4$Colour[GSPC_VaR_norm_4$realized>=GSPC_VaR_norm_4$var_975]="gray"
```

```
GSPC_VaR_norm_4$Colour[GSPC_VaR_norm_4$realized<GSPC_VaR_norm_4$var_975]="red"
```

```
plot(GSPC_VaR_norm_4$realized, type = "p",pch=20,col = GSPC_VaR_norm_4$Colour,xlab = "GSPC 97.5%",ylab = "Var")
```

```
lines(GSPC_VaR_norm_4$var_975,type = "l", col = "black",lwd=3)
```

```
# Set new column values to appropriate colours
```

```
GSPC_VaR_norm_4$Colour[GSPC_VaR_norm_4$realized>=GSPC_VaR_norm_4$var_99]="gray"
```

```
GSPC_VaR_norm_4$Colour[GSPC_VaR_norm_4$realized<GSPC_VaR_norm_4$var_99]="red"
```

```
plot(GSPC_VaR_norm_4$realized, type = "p",pch=20,col = GSPC_VaR_norm_4$Colour,xlab = "GSPC 99%",ylab = "Var")
```

```
lines(GSPC_VaR_norm_4$var_99,type = "l", col = "black",lwd=3)
```