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Longevity Risk and Annuitisation Decisions in the Absence of Special-Rate Life Annuities

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Abstract: Longevity risk affecting older adults can be transferred to the insurance market by purchasing a lifetime annuity. Special-rate life annuities, which are priced, among other factors, on the basis of health and lifestyle factors, go beyond traditional considerations of age and sex by using modified mortality tables. However, they are not available in many countries. In regions where life annuities are priced solely via standard mortality tables, retirees with below-average life expectancy may face unfair pricing. This study aims to quantify this actuarial unfairness and proposes an alternative annuitisation strategy for these retirees. The strategy allows them to transfer longevity risk by acquiring a life annuity on the basis of their actual mortality probabilities, thereby mitigating actuarial inequities. Additionally, the paper examines how tax incentives can exacerbate actuarial unfairness and, specifically for Spanish tax regulations, compares different alternatives under two scenarios related to the sources used for purchasing life annuities.

Keywords: annuitisation; actuarial fairness; longevity risk; life annuity; special-rate life annuities; tax incentives

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1. Introduction

The increasing life expectancy (LE) in developed countries presents both a remarkable achievement and a significant challenge. As people live longer than they did a few decades ago, the sustainability of public pension systems may be at risk (Sánchez-Serrano and Peltronen 2020). Extended lifespans lead to longer periods of retirement, placing additional financial strains on public resources designed to support retirees. To address this, several countries have undertaken reforms aimed at adjusting pensions to this new reality (Scott 2023).

On the other hand, the retiree population faces longevity risk, which involves the possibility of outliving the financial resources accumulated during their active years. This risk can be compounded by health issues in retirement, resulting in increased expenses, particularly in advanced age. Yaari (1965) suggested that the optimal strategy for hedging longevity risk in complete markets is to purchase actuarially fair life annuities with lifelong payments. Subsequent studies (see, e.g., Sinha 1986; Mitchell et al. 1999; Davidoff, 2009; Lockwood 2012; Sutcliffe 2015; Atta Mills and Anyomi 2023) expanded on this idea, considering factors such as incomplete life annuity markets and subjective preferences (e.g., bequest motive). They unanimously agree that using a portion of accumulated wealth to purchase life annuities is a natural way to manage longevity risk. However, developed countries exhibit lower voluntary demand for life annuities than expected (Lambregts and Schut 2020), a phenomenon known as the annuity puzzle (Cannon and Tonks 2011), underscoring the disparity between economic theory and individuals' actual behaviours. For example, focusing on Spain, Berges and Manzano (2023) state that saving in pension funds, one of the main instruments to complement the public retirement pension if its accumulated balance is converted into a lifetime annuity, is far from being minimally relevant by desirable standards. Additionally, financial products for pension savings represent a very small share of the financial assets of Spanish households compared to their European counterparts, accounting for barely 3% of the gross wealth of Spanish households. There is no single explanatory reason for the annuity puzzle; rather, it results from a set of factors of different types. Some are attributable to individuals' psychology, such as the bequest motive, the underestimation of longevity risk, or the perception that this risk can be personally managed (Brown et al. 2008; Benartzi et al. 2011; Alexandrova and Gatzert 2019). Others are associated with the culture and economic system to which retirees belong. In systems with more extensive public benefits, such as in Spain, the need for private life annuities is lower (Brown 2001; de Andrés Sánchez and González-Vila Puchades 2020). There are also economic and financial factors, such as low-interest-rate environments, low returns on life annuities (de Andrés Sánchez and González-Vila Puchades 2020), or the fact that life annuities offered in the insurance market are not tailored to the individual circumstances of the person (Alexandrova and Gatzert 2019). This study must be understood within the context of this latter factor, which triggers the annuity puzzle, especially among groups of retirees who are potential buyers of life annuities to complement those provided by public pension systems.

It is well known that mortality varies among individuals of the same age and within the same municipality, region, or country. Considering heterogeneous lifespans involves assuming that a population is composed of subgroups subject to different mortality laws (Pitacco 2019). Several factors influencing longevity heterogeneity have been extensively documented. Clear examples are sociodemographic factors such as sex, income level, education, or marital status. Women tend to have a greater LE than men. Individuals with higher income levels exhibit greater survival rates, partly because they can afford better healthcare services. Moreover, those with higher educational attainment and/or those who are married tend to adopt healthier lifestyles (Chetty et al. 2016; Ayuso et al. 2017; Bravo et al. 2021). On the other hand, other factors, such as congenital characteristics, are harder to observe (Pitacco 2017).

Although longevity heterogeneity among a specific population is a widely documented phenomenon, insurers in most countries offer standard life annuities, i.e., rated at a uniform price for all individuals of the same age and sex (if regulations allow), without additional segmentation among annuitants, assuming that they share the same LE. This approach results in unfairness for those with below-average or substandard LE (SSLE), as the expected value of received payments is much lower than the premiums paid (Hoermann and Ruß 2008). The term "SSLE" refers to a reduced LE relative to the number of years a person of a certain age is expected to live based on statistical averages for a specific population, often due to health conditions such as chronic illnesses, lifestyle factors (e.g., smoking or obesity), or other risk factors that shorten an individual's LE. Different authors may use varying terminology to refer to this concept, such as "impaired life", "substandard life" (as opposed to standard life), or "reduced LE", in both actuarial and health contexts, all conveying the same idea (Kita 2006; Robb and Willets 2006; Ozasa et al. 2008). In an insurance market where only standard life annuities are available, individuals of the same age who pay the same single premium will receive the same periodic payments, regardless of their LE. This means that a person with an SSLE and another with a standard LE (SLE) will receive identical annuity payments, but the SSLE person will likely do so for a shorter duration. As a result, part of the premium paid by this individual effectively subsidises the periodic payments for annuitants with a longer LE. Special-rate life annuities address this by considering various health and lifestyle factors, estimating mortality probabilities and adjusting periodic payments accordingly. This ensures actuarial fairness, providing equitable pricing for individuals with SSLE (Pitacco and Tabakova 2022).

The significant pressure that the ageing populations of developed countries place on public pension systems has led governments to promote the growth or establishment of a third pillar for old-age provision. This initiative encourages individuals to compensate for potential reductions in public retirement pensions by increasing private savings, essentially reallocating a portion of their income during their working years to fund their retirement and providing incentives to consume these savings gradually during retirement. Therefore, some countries offer tax incentives to promote life annuity demand (Brunner and Pech 2008). Tax incentives, when available only for standard life annuities in a market without special-rate life annuities, create a dilemma for retirees with SSLE (Kling et al. 2014). They face the choice of either not using part of their wealth for a life annuity, which is unattractive for tax reasons, or opting for a fiscally appealing annuity with an unjust price. The latter results in the loss of part of the tax benefit, as the annuity is acquired at an extra cost compared with an equitable scenario.

These considerations motivate this work, which explores three issues that, to the best of our knowledge, have not been previously researched:

Issue 1. We measure the unfairness faced by SSLE retirees when contracting a lifetime annuity priced with standard probabilities. To do so, it is considered that purchasing a lifetime annuity is the default strategy adopted in insurance markets where special-rate life annuities are unavailable, regardless of the retiree's LE. This measure is based on the concept of the transfer rate in Ayuso et al. (2021).

Issue 2. Under the assumption of no personal income taxation, we propose an alternative annuitisation strategy for SSLE retirees in markets where special-rate life annuities are unavailable. This alternative involves purchasing a standard temporary annuity designed to cover the expected remaining years of life with a high probability (e.g., 95%), thereby reducing the transfer rate.

Issue 3. We consider personal income taxation to determine the amount of payments that SSLE retirees receive. Focusing on tax regulation in Spain, where special-rate life annuities are not available, life annuity purchases are examined under two possible scenarios that comprehensively cover most cases in which a person might acquire a life annuity, each involving a completely different tax situation. In the first scenario, the life annuity is purchased via the accumulated balance from a pension plan. In the second scenario, the retiree uses a portion of their personal assets to buy the annuity.

Notice that Issues 1 and 2 apply to any country where special-rate life annuities are unavailable, as they address the challenges faced by retirees in markets offering only standard-rate life annuities. Issue 3, which emphasises the role of personal income taxation in annuitisation decisions, is also relevant to personal financial planning worldwide. Thus, although the analysis conducted in this study focuses on Spanish regulation, the methodology applied can be adapted to other countries by taking into account their specific regulations governing life annuities. Likewise, it should not be overlooked that Spain is a significant destination for migrant retirees (Valero-Escandell et al. 2022) as their country for permanent residence. Therefore, the focus on Spanish tax regulation provides valuable insights for actuarial practitioners regarding the tax planning of retirees in similar situations across different countries.

The paper is organised as follows. Section 2 explains the classification and pricing of special-rate life annuities, focusing on some characteristics and a practical pricing method. This section aims to describe the variety of products encompassed by special-rate annuities and determine how we will model the excess mortality of SSLE individuals. This modelling will be performed through the mortality factor, which is derived from a widely used approach in practice (Olivieri 2006). Section 3 begins with a brief discussion of the concept of actuarial fairness and describes a method for measuring the actuarial unfairness experienced by retirees with SSLE in markets where special-rate life annuities are not available. Section 4 presents an alternative annuitisation strategy that partially mitigates this unfairness for such retirees. Section 5 considers tax incentives to promote life annuities demand, when available only for standard life annuities in markets without special-rate options, as a factor that exacerbates the actuarial unfairness for SSLE individuals. It specifically compares, under Spanish tax regulation and for two different scenarios related to the origin of the sources used to purchase life annuities, the payment amount to be received when a standard lifetime annuity is purchased, when the alternative strategy proposed in Section 4 is implemented and when a special-rate lifetime annuity is contracted if possible. Section 6 summarises the main findings of the paper.

2. Special-Rate Life Annuities: Concept and Pricing

2.1. Some Aspects of Special-Rate Annuities

In many countries, retirees have limited options for purchasing life annuities, often based solely on age and sex or only on age, as seen in the European Union, where pricing by sex is prohibited (European Council 2004). However, some nations have innovated their annuity markets with advanced risk-underwriting strategies, introducing special-rate life annuities. These consider factors such as residence, lifestyle, and health status (Pitacco and Tabakova 2022; Olivieri and Tabakova 2024). By incorporating these factors, special-rate life annuities provide higher periodic payments compared to standard annuities, reflecting a shorter predicted LE for the insured (Gatzert and Klotzki 2016).

Underwriting special-rate life annuities differs significantly across countries and insurers. In the UK, for instance, postcode-based life annuities do not require specific underwriting, as insurers utilise mortality data linked to postcodes, which act as proxies for social class and mortality expectations (Telford et al. 2011). Minor risk factors like marital status or smoking habits are similarly assessed, reflecting limited health deterioration. UK insurers, with extensive market experience, often use automated, rule-based systems for pricing based on additional health details such as blood pressure or cholesterol levels. Online platforms facilitate rapid pricing for conditions like hypertension and diabetes, while significant health issues, such as cancer or heart disease, require more personalised underwriting through questionnaires and medical exams (Ridsdale 2012). Complex cases involve expert systems adjusting mortality estimates using disease-specific mortality tables, further refined for combinations of conditions and individual circumstances. Fully individualised underwriting processes are applied for some special-rate life annuities. In contrast, underwriting practices outside the UK are less standardised. In the USA, some insurers employ established systems to process health data and estimate mortality efficiently (Drinkwater et al. 2006). Others rely on medical underwriters or use prediction models and medical databases for diseases like diabetes or multiple sclerosis. Some insurers adhere to life insurance underwriting guides. Regardless of the method, accurately pricing special-rate life annuities depends on estimating mortality probabilities (see Section 2.2) to adjust annuity payments appropriately.

The specific denominations given to the various types of special-rate annuities are not the same in all markets. Therefore, this paper follows the terminology commonly found in markets where they are most prevalent, such as in the UK, as discussed in Pitacco (2017) and Olivieri and Tabakova (2024). Thus, we differentiate the following types[1]:

- Lifestyle annuities. They are priced by considering various risk factors, such as the postcode of residence, smoking habits, alcohol consumption, marital status, professional activity, and physiological characteristics. Mortality probabilities in these annuities are marginally higher compared to standard life annuities.
- Enhanced life annuities are determined based on the individual's medical history, which leads to a reduced LE. In this case, mortality probabilities are greater than those associated with standard and lifestyle annuities.
- Impaired life annuities are specifically designed for individuals whose medical conditions significantly reduce their anticipated lifespan (for example, due to diabetes, chronic asthma, cancer, etc.). In such cases, mortality probabilities are substantially higher than those for standard annuities and greater than those for enhanced annuities.
- Care annuities are intended for individuals experiencing severe impairments or those in a state of senescent disability. Such products can be understood as a form of long-term care insurance. Owing to the individual's critical health circumstances, higher mortality probabilities and a lower LE are adopted in comparison to the aforementioned annuity types.

Although some of these life annuities, particularly lifestyle annuities, may be subject to potential annuitants misrepresenting their personal circumstances, such as smoking status or residence, to obtain greater pay-outs, these circumstances can generally be assessed through a standard underwriting process. For example, smokers, even if in good health, exhibit numerous indicators of their status that can be easily detected through their medical history or even a routine medical examination (Chauhan et al. 2013). Similar considerations apply to individuals who consume alcohol (Abidi et al. 2018). With respect to postcode, it can be verified whether the applicant has only recently moved to a "favourable" postcode or if it is genuinely their usual residence. Both routine medical examinations and the verification of postcode through corresponding official documentation are straightforward and cost-effective procedures that do not compromise the viability of using these factors in life insurance pricing. In fact, in countries where special-rate life annuities are offered, substance consumption and postcode are commonly used factors in pricing (Telford et al. 2011). Nonetheless, the insurer must consider, in the management of special-rate annuities, the potential existence of moral hazard behaviours. This is because a greater annuity payment may enable individuals to invest more in healthcare while simultaneously motivating them to pursue greater longevity (Tricker 2018).

2.2. Pricing Special-Rate Annuities

2.2.1. Actuarial Pricing of Life Annuities

Let us consider a retiree aged x who wants to purchase a life annuity. We denote the annual probability of dying at age $x+j$ as q_{x+j}, $j = 0, 1, \ldots, \omega - x$, where ω is the maximum attainable age. The probability that the retiree will live to age $x+t$, ${}_tp_x$, is as follows:

$$ {}_tp_x = \prod_{j=0}^{t-1} \left(1 - q_{x+j}\right). \tag{1} $$

Suppose that the retiree has Π monetary units (m.u.) to acquire an immediate life annuity whose annual payments are, for simplicity, constant and equal to C. If payments are lifelong, i.e., a lifetime annuity is purchased:

$$ C = \frac{\Pi}{a_x}, \tag{2} $$

where

$$a_x = \sum_{t=1}^{\omega-x} (1+i)^{-t} {}_tp_x, \tag{3}$$

and i is the annual interest rate used to price the annuity, i.e., the so-called technical interest rate.

In the case of purchasing an n-year temporary life annuity, $n < \omega - x$ the payments, C^n, would be as follows:

$$C^n = \frac{\Pi}{a_{x:\overline{n}|}}, \tag{4}$$

being

$$a_{x:\overline{n}|} = \sum_{t=1}^{n} (1+i)^{-t} {}_tp_x. \tag{5}$$

If the amount of the annual payments is determined on the basis of the average life probabilities for the age and sex (when permitted) of the annuitant, the retiree is acquiring a standard life annuity. Alternatively, if the amount of the annual payments is obtained by considering the individual's specific probabilities, they purchase a special-rate life annuity. In this last case, the specific one-year mortality probabilities are denoted as q^*_{x+j}. From these probabilities, the adjusted probability that the retiree will live to age $x+t$, ${}_tp^*_x$ is obtained via (1). If ${}_tp^*_x$ are considered in (3), we will write a^*_x. Moreover, the amount of the annual payment of the special-rate lifetime annuity, C^*, can be obtained by writing a^*_x in (2).

2.2.2. Fitting Mortality Probabilities for Special-Rate Life Annuities

From a theoretical standpoint, an appropriate approach to capturing the mortality of a population subgroup and pricing special-rate life annuities involves constructing specific mortality tables for that subgroup. Such tables are common in actuarial practice (e.g., for smokers or based on postcodes) and have inspired a highly active academic literature. Thus, within a broader population, there may be subgroups for which specific mortality adjustments are relevant, such as those at advanced ages, certain geographic areas within a country (Ahcan et al. 2014), or those associated with a specific insurance portfolio (Atance et al. 2025). In this context, the contribution of Dowd et al. (2011) can be understood as a development of the concept of adjustment based on the existence of a joint centre of gravity for two related populations. Similarly, Jarner and Kryger (2011) apply the SAINT method to adjust the mortality of a relatively small population (Danish people over the age of 70) by referencing the mortality behaviour of that segment of the population to the combined population of 19 OECD countries. Ahcan et al. (2014) propose adjusting the mortality tables of smaller countries, such as Slovakia, by referencing them to geographically larger and relatively close countries, such as those in the European Union. Likewise, the methodologies proposed in the works of Wan and Bertschi (2015) and Atance et al. (2025) aim to adjust specific mortality tables for the management of pension funds and life insurance.

In this work, the approach we use to price special-rate life annuities is based on a less sophisticated method than those mentioned in the previous paragraphs but is frequently observed in actuarial practice: the "numerical rating system" (Olivieri 2006), which is described in the following paragraphs. This approach requires the use of a reference mortality table, which we refer to as the standard mortality table[2]. After analysing various risk factors related to the health and lifestyle of the insured, a mortality table adjusted to their specific situation must be derived from this standard table.

Olivieri (2006) describes two alternatives to determine adjusted mortality probabilities:

$$q^*_{x+j} = q_{x+k+j}, \tag{6}$$

$$q^*_{x+j} = \alpha + \beta q_{x+j}, \tag{7}$$

where, from the standard annual probabilities, q_{x+j}, $j = 0, 1, \ldots, \omega - x$, q^*_{x+j} are the adjusted individual annual probabilities for an SSLE retiree. The adjusted mortality probabilities in (6) are obtained by increasing the retiree's current age by k years based on the estimation that their LE is equal to that of a person aged $x + k$. Similarly, in (7), q^*_{x+j} are obtained by increasing the standard mortality probability with a summative coefficient, α, and a multiplicative coefficient, β, usually called the mortality factor, which depend on the risk factors intended to be reflected.

Equation (7) allows for the inclusion of the potential variability of risk factors over time. There may be risk factors that increase mortality probabilities only during a specific term, with no further influence once this term has passed. These cases are typical of certain diseases that lead to early death or recovery within a specific term. In such cases, the parameters in Equation (7) may be $\alpha > 0$ and $\beta > 1$ for only a certain number of years. Once that term has elapsed, mortality probabilities return to the standard level (Olivieri 2006).

In this article, it is assumed that in (7) $\alpha = 0$ and $\beta > 1$, referred to as the multiplicative model. This is a simplifying assumption that has been used in actuarial assessments (Olivieri 2006). Thus, this approach can be found in studies on life insurance ratemaking (Kita 2006), life settlement valuations (Xu and Hoesch 2018), and, of course, pricing special-rate life annuities for SSLE individuals (e.g., Hoermann and Ruß 2008; Gatzert et al. 2012; Kling et al. 2014; Olivieri and Pitacco 2016; de Andrés-Sánchez et al. 2020; Olivieri and Tabakova 2024). In this framework, each person is characterised by a factor β, and their individual mortality probabilities are given by β times the mortality probabilities from a standard mortality base table. Therefore, an insured person with an individual factor $\beta > 1$ has an above-average mortality probability (or, equivalently, a below-average LE). Given that $q^*_{x+j} = \beta q_{x+j}$, it must be ensured that $0 < \beta < \frac{1}{q_{x+j}}$ since $0 \leq q^*_{x+j} \leq 1$. Nevertheless, this condition may not hold for all ages in the standard mortality table. Therefore, q^*_{x+j} is rewritten as

$$q^*_{x+j} = \min\{1, \beta q_{x+j}\}. \tag{8}$$

The multiplicative model is the foundation of the so-called numerical rating system, which is the model used in all the works cited in the paragraph above. According to the numerical rating system,

$$\beta = 1 + \sum_{i=1}^{n} \rho^{(i)}, \tag{9}$$

where $\rho^{(i)}$ is a coefficient that quantifies the increase or decrease in mortality relative to the standard, depending on the ith risk factor. Thus, if $\rho^{(i)} < 0$, we are dealing with a credit, which reduces the mortality probability, while if $\rho^{(i)} > 0$, it is a debit, increasing the mortality probability (Pitacco 2019). In practice, there is a vast array of risk factors that can generate debits and credits, encompassing physiological, psychological and lifestyle variables, which can be assessed with varying degrees of granularity (Cummins et al. 1983, pp. 131–220). To adjust for the excess mortality generated by impairments, generalised linear models may be particularly useful (Meyricke and Sherris 2013).

Theoretically, determining the mortality multiplier β for a given individual involves summing the coefficients $\rho^{(i)}$ associated with each existing debit and credit, which should be interpreted as "expected values" (Pitacco 2019). These coefficients are linked to specific symptoms of health deterioration or improvement, such as physical or psychological conditions. Sometimes, the adjustment of β may extend beyond simple aggregation, taking into account other relevant factors, such as common comorbidities, the influence of age on the considered factors, or lifestyle. The coefficients and adjustments are typically tabulated by the insurer (Cummins et al. 1983, pp. 131–220). In actuarial practice, it is also common to use LE certificates, which provide detailed information on the impairments that justify the mortality multiplier β and the corresponding applicable mortality table (see, for example, the certificate by TwentyFirst 2024). These certificates are constructed based on the individual judgment of a medical evaluator, considering the personal circumstances of the assessed individual (Xu and Hoesch 2018), and align with the representation of β as a parameter, as suggested by the literature on mortality heterogeneity modeling (Olivieri 2006; Kling et al. 2014; Pitacco 2019). A more academic approach involves the use of generalised linear models, which can be particularly useful for adjusting for the excess mortality generated by impairments (Meyricke and Sherris 2013). In this way, the coefficients adjusted through regression for each type of impairment can be assimilated to the coefficients used in the calculation of $\rho^{(i)}$.

The values of the mortality factor are often divided into intervals that define different risk classes, each with its own specific mortality table. Hence, the final value obtained for β is not applied directly via Equation (7) but rather through a mortality law common to the group in which the individual is classified (Cummins et al. 1983, pp. 113–20; Kita 2006; Olivieri and Pitacco 2016; Olivieri and Tabakova 2024). In life insurance ratemaking for substandard lives, it is common practice to use an SLE table as a reference and, on this basis, establish tables for different levels of SSLE (Cummins et al. 1983, pp. 131–220).

3. Actuarial Unfairness for SSLE Retirees in the Absence of Enhanced Annuities

3.1. Measuring the Unfairness of Standard Annuities for SSLEs

The concept of actuarial fairness, also known as actuarial justice or equity, has been studied extensively and debated alongside actuarial discrimination. According to Frezal and Barry (2020), actuarial fairness can be understood either as solidarity among a group of insured individuals pooling their uncertain results or as a fair contract between an insured person and an insurer. Due to the growing influence of neoliberal theories, the definition and application of actuarial justice have evolved over time. Initially, it focused on the collective pooling approach based on statistical averages, but it later shifted toward individualised assessments to promote fairness (Meyers and Van Hoyweghen 2017; Frezal and Barry 2020; Heras et al. 2020; Frees and Huang 2023). In this paper, we consider actuarial fairness as the principle that insurance products should reflect in the risk profiles of insured individuals, ensuring equitable treatment in terms of pricing and benefits, as discussed by Donnelly (2015), Landes (2015), and Chen and Vanduffel (2023).

Insurance regulators have played a significant role in shaping the application of actuarial fairness. For example, European and American regulators, driven by a desire to avoid unfair discrimination, have reached differing conclusions. Although women generally have a higher LE, European regulations prohibit sex-based pricing to ensure equal treatment between men and women (European Council 2004; Court of Justice of the European Union 2011). This approach defines fairness as “equal treatment”, disregarding statistical arguments and limiting the adjustment of prices on the basis of risk costs.

In recent years, actuarial fairness has faced new challenges with the rise of artificial intelligence (AI) in insurance pricing. Several authors (e.g., Barry 2020; Barry and Charpentier 2023; du Preez et al. 2024; Xin and Huang 2024) note that while AI offers the potential for more precise risk assessments through the analysis of large datasets, it also raises ethical concerns. The use of AI in pricing could inadvertently perpetuate or worsen disparities if it leads to more granular but less transparent discrimination. Therefore, maintaining actuarial justice requires ensuring that AI applications in insurance pricing adhere to standards of fairness and transparency.

Although a detailed examination of actuarial justice and discrimination falls outside the scope of this article, it is important to emphasise that actuarial justice should apply to all insurance products. Special-rate life annuities exemplify this application by adjusting pay-outs on the basis of individuals' health and lifestyle factors. These products provide higher annuity payments to individuals with SSLE, ensuring a fairer income relative to the premium paid. This personalised approach reflects the principle of actuarial fairness by recognising the diverse health conditions of retirees and their implications for life annuity pricing.

According to Gatzert et al. (2012), special-rate life annuities were introduced in the UK in the mid-1990s. They emerged as a response to inequities in the traditional life annuity market. Retirees who benefited from tax exemptions on pension plan contributions were required to purchase life annuities, often resulting in unfair treatment for those in poorer health. Standard life annuities provided the same rates to all annuitants, irrespective of their individual LEs. Special-rate life annuities address this disparity by offering prices tailored to health and lifestyle, thereby promoting a fairer allocation of retirement income (Pitacco and Tabakova 2022).

When special-rate life annuities are unavailable, annuitants with SSLE effectively subsidise those with SLE, as in exchange for the same premium, the latter group benefits from longer payment periods without accounting for health disparities. Thus, individuals with shorter LEs may be financially disadvantaged. This section describes how to measure the unfairness that SSLE annuitants face when purchasing a standard annuity in markets where special-rate life annuities do not exist. This is performed on the basis of the concept of the transfer rate introduced by Ayuso et al. (2021).

Hedging longevity risk is the purpose of purchasing life annuities. Achieving a complete hedge of this risk necessitates the acquisition of a lifetime annuity. In markets where only standard life annuities are available, individuals with SLE who acquire a lifetime annuity do not experience wealth loss as they are offered actuarially fair prices. In other words, the wealth they invest in a life annuity, which is Π, matches the wealth they receive, which is equal to $C{\cdot}a_x$.

However, this is not the case for SSLE annuitants. Following the approach outlined in Ayuso et al. (2021), if SSLE individuals were given actuarially fair annuities, they would not lose wealth and, consequently, $\Pi = C^*{\cdot}a_x^*$. However, since only standard life annuities can be purchased, the actual situation is that they receive $C{\cdot}a_x^*$. Therefore, by considering (2), it can be argued that in such markets, SSLE individuals, upon contracting, transition from wealth Π to wealth $\Pi\frac{a_x^*}{a_x}$. Consequently, SSLE annuitants implicitly transfer a portion of their wealth to SLE annuitants. We define the transfer rate that the SSLE annuitants afford as

$$TR = \frac{\Pi - \Pi\frac{a_x^*}{a_x}}{\Pi} = 1 - \frac{a_x^*}{a_x}. \tag{10}$$

This can be interpreted as the amount that the SSLE annuitants transfer to annuitants who buy standard lifetime annuities for each m.u. they allocate to their acquisition.

Numerical Application 1. Table 1 shows the transfer rate, TR, with a lifetime annuity of $\Pi = 1$ for several technical interest rates and mortality factors in the case of $x = 65$, 75, 80 years. With respect to standard mortality probabilities, we have considered those included in the Human Mortality Database (2021), Spain, and both sexes to avoid potential underestimations introduced by mortality models (Lledó and Atance 2023).

The different mortality factors used in this numerical application can be understood as originating from various mortality tables (Kita 2006; Cummins et al. 1983, pp. 131–220). Thus, β = 1.5 corresponds to a mortality table associated with a subgroup where the standard mortality table is loaded by 50%. Similarly, β = 2 implies the use of a mortality table for a subgroup with a higher mortality level, reflecting a 100% load, and so on.

Table 1. Transfer rate, TR, with a standard lifetime annuity.

	$x = 65$			
i \ β	1.5 $LE = 18.05$	2.0 $LE = 16.02$	5.0 $LE = 10.16$	10 $LE = 6.53$
1.5%	0.128753	0.218884	0.493228	0.674575
3.0%	0.113452	0.195553	0.458465	0.643050
4.0%	0.104416	0.181559	0.436648	0.622649
	$x = 75$			
i \ β	1.5 $LE = 10.92$	2.0 $LE = 9.37$	5.0 $LE = 5.30$	10 $LE = 3.10$
1.5%	0.173109	0.287622	0.601223	0.779901
3.0%	0.159585	0.268154	0.577734	0.762514
4.0%	0.151324	0.256102	0.562641	0.751086
	$x = 80$			
i \ β	1.5 $LE = 7.82$	2.0 $LE = 6.54$	5.0 $LE = 3.36$	10 $LE = 1.78$
1.5%	0.207482	0.339738	0.678156	0.853162
3.0%	0.195245	0.322891	0.661138	0.843064
4.0%	0.187661	0.312319	0.650105	0.836395

Source: Own elaboration.

The sensitivity analysis provided by Table 1 and Figure 1 suggests that both the mortality factor β and the interest rate influence the transfer rate. It is observed to increase with respect to the mortality factor and to decrease with respect to the interest rate. Furthermore, it is more sensitive to β than to the interest rate applied in the pricing of the life annuity. Finally, the results in Table 1 suggest that, under the same conditions for the mortality factor and interest rate, the transfer rate increases with respect to the retiree's age.

Figure 1 shows the variation in the transfer rate for a lifetime annuity, $\Pi = 1$, with respect to the mortality factor and the technical interest rate, specifically for Numerical Application 1 and with $x = 65$.

3.2. Some Properties of the Transfer Rate

3.2.1. Behaviour of the Transfer Rate with Respect to the Mortality Factor

We now assess the behaviour of the transfer rate TR with respect to the mortality factor. Let us first analyse the values of TR in the limiting case where $\beta \to \frac{1}{q_x}$ (the probability of dying in the next year is 1).

If $\beta \to \frac{1}{q_x}$, the life annuity has no payments. Therefore,

$$\lim_{\beta \to \frac{1}{q_x}} TR = 1 - \frac{0}{a_x} = 1. \tag{11}$$

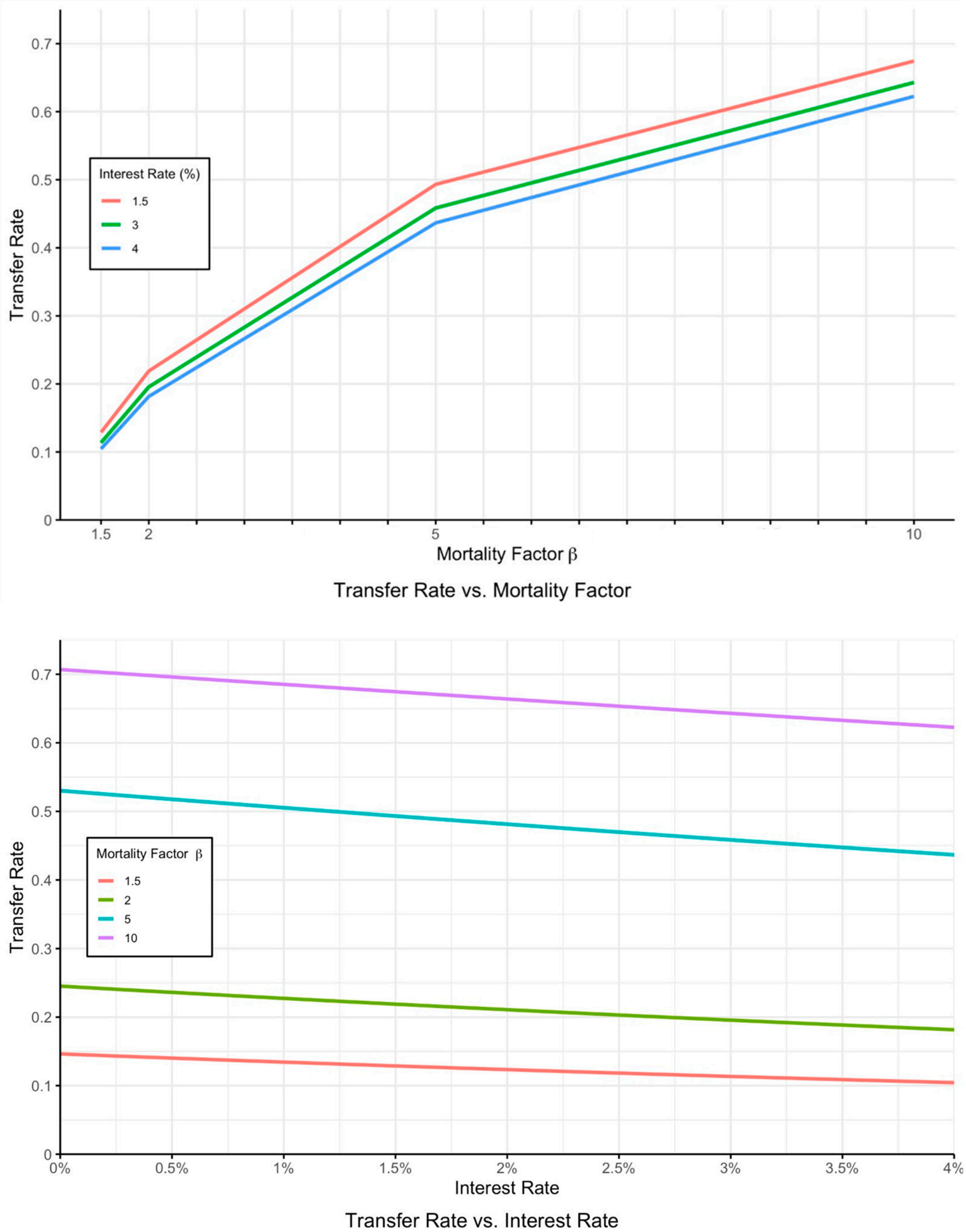

Figure 1. Variation in the transfer rate TR. Source: Own elaboration.

As shown in Numerical Application 1, TR increases with the mortality multiplier, β. By considering (10) and calculating the derivative of TR with respect to β,

$$\frac{\partial TR}{\partial \beta} = \frac{\partial \left(1 - \frac{\sum_{t=1}^{\omega - x} (1+i)^{-t} \ {}_t p_x^*}{a_x} \right)}{\partial \beta}. \tag{12}$$

Given that, from (1), ${}_tp_x^* = \prod_{j=0}^{t-1}\left(1-\min\{1,\beta q_{x+j}\}\right)$ and $p_{x+j}^* = 1-\min\{1,\beta q_{x+j}\}$, then

$$\begin{aligned}\frac{\partial TR}{\partial \beta} &= \frac{\sum_{t=1}^{\omega-x}(1+i)^{-t}\sum_{j=0}^{t-1} q_{x+j}\prod_{\substack{k=0\\k\neq j}}^{t-1}\left(1-\min\{1,\beta q_{x+k}\}\right)}{a_x} \\ &= \frac{\sum_{t=1}^{\omega-x}(1+i)^{-t}\sum_{j=0}^{t-1} q_{x+j}\prod_{\substack{k=0\\k\neq j}}^{t-1} p_{x+k}^*}{a_x} \\ &= \frac{\sum_{t=1}^{\omega-x}(1+i)^{-t}\ {}_tp_x^*\sum_{j=0}^{t-1}\frac{q_{x+j}}{p_{x+j}^*}}{a_x}\end{aligned}. \tag{13}$$

Thus, it is easy to check that $\frac{\partial TR}{\partial \beta} > 0$. Therefore, the shorter the LE of the insured (i.e., the greater β), the greater the wealth transferred since, for the same single premium, they are receiving annual payments for a much shorter term than what would correspond to them if their actual mortality probabilities were considered.

Similarly, by naming ${}_tp_x^{**} = {}_tp_x^*\sum_{j=0}^{t-1}\frac{q_{x+j}}{p_{x+j}^*}$ and $a_x^{**} = \sum_{t=1}^{\omega-x}(1+i)^{-t}\,{}_tp_x^{**}$, (13) can be rewritten as a quotient of two present values:

$$\frac{\partial TR}{\partial \beta} = \frac{a_x^{**}}{a_x}. \tag{14}$$

Note that $\Delta TR \approx \frac{a_x^{**}}{a_x}\Delta\beta$ can be interpreted as a measure of the sensitivity of TR to fluctuations in the annuitant's health status. Thus, $\Delta\beta$ can be understood as the direct result of a fluctuation in debits and credits, as the outcome of a change in the annuitant's classification (such as transitioning from a standard LE to qualifying for a special-rate life annuity), or as moving from one risk group (and thus a specific mortality table) to another, which changes the mortality table used in the pricing process.

3.2.2. Behaviour of the Transfer Rate with Respect to the Interest Rates

Now, let us examine the behaviour in the cases where $i \to 0$, meaning no preference for liquidity, and where $i \to \infty$, indicating an infinite preference for liquidity. Note that the first situation could resemble the conditions in many OECD countries during much of the 2010s and the early 2020s when risk-free interest rates were practically zero.

In the case where $i \to 0$, $a_x = \sum_{t=1}^{\omega-x}(1+i)^{-t}\,{}_tp_x = \sum_{t=1}^{\omega-x}{}_tp_x$ and, analogously, $a_x^* = \sum_{t=1}^{\omega-x}{}_tp_x^*$. Therefore, (10) becomes analogous to that in Ayuso et al. (2021) since a_x and a_x^* turn into the LEs for SLE and SSLE annuitants, respectively. That is to say,

$$\lim_{i\to 0} TR = 1 - \frac{e_x^*}{e_x}. \tag{15}$$

On the other hand, if $i \to \infty$,

$$\lim_{i\to\infty} TR = 1 - \frac{p_x^*}{p_x} = \frac{p_x - p_x^*}{p_x}. \tag{16}$$

Regarding the derivative of TR with respect to i, from (10):

$$\begin{aligned}\frac{\partial TR}{\partial i} &= \frac{\partial\left(1-\frac{\sum_{t=1}^{\omega-x}(1+i)^{-t}\ {}_tp_x^*}{\sum_{t=1}^{\omega-x}(1+i)^{-t}\ {}_tp_x}\right)}{\partial i} \\ &= \frac{\sum_{t=1}^{\omega-x} t(1+i)^{-t-1}\ {}_tp_x^*\cdot\sum_{t=1}^{\omega-x}(1+i)^{-t}\,{}_tp_x}{(a_x)^2} \\ &- \frac{\sum_{t=1}^{\omega-x}(1+i)^{-t}\,{}_tp_x^*\ \cdot\sum_{t=1}^{\omega-x} t(1+i)^{-t-1}\,{}_tp_x}{(a_x)^2}\end{aligned}. \tag{17}$$

Rearranging it, expression (17) becomes

$$\frac{\partial TR}{\partial i} = -\frac{\sum_{t=1}^{\omega-x}\sum_{h=t+1}^{\omega-x}(h-t)(1+i)^{-(t+h)}\left({}_hp_{x\,t}p_x^* - {}_hp_{x\,t}^*p_x\right)}{(1+i)(a_x)^2}. \tag{18}$$

Since ${}_hp_{x\,t}p_x^* - {}_hp_{x\,t}^*p_x$ can be rewritten as ${}_hp_{x\,t}p_x^*({}_{h-t}p_{x+t} - {}_{h-t}p_{x+t}^*)$, ${}_tp_x \geq 0$, ${}_tp_x^* \geq 0$ and ${}_{h-t}p_{x+t} \geq {}_{h-t}p_{x+t}^*$ in the case of an SSLE individual, $\frac{\partial TR}{\partial i} \leq 0$. Thus, this result aligns with the reasoning that a higher interest rate leads to greater life annuity payments being received, which, in turn, results in less wealth being transferred for the same single premium.

Note that (17) is also

$$\frac{\partial TR}{\partial i} = -\frac{(Ia)_x a_x^* - (Ia)_x^* a_x}{(1+i)(a_x)^2}. \tag{19}$$

where $(Ia)_x$ and $(Ia)_x^*$ represent the present value of an increasing life annuity.

Expression (19) can be interpreted similarly to the duration of fixed-income securities, allowing it to be used to approximately determine the sensitivity of TR to changes in the interest rate. Specifically, $\Delta TR \approx -\frac{(Ia)_x a_x^* - (Ia)_x^* a_x}{(1+i)(a_x)^2}\Delta i$. Since the interest rate used to price life annuities is typically closely related to the long-term interest rate on national government debt, fluctuations in Δi can be assessed based on changes in the yields of medium- and long-term government debt, which may be considered structural. These structural changes could be attributed, for instance, to long-term monetary or fiscal policy decisions.

4. Mitigation of Actuarial Unfairness for SSLE Retirees

Let us consider a market where life annuities are rated exclusively using standard mortality probabilities. We propose an alternative strategy to purchase a life annuity for those SSLE retirees who want to hedge their longevity risk and, at the same time, reduce the transfer rate they provide to SLE annuitants, i.e., reduce the actuarial unfairness they face.

This alternative consists of purchasing a standard temporary life annuity covering the remaining number of whole years of life with a probability greater than or equal to $1-\varepsilon$. This probability is determined based on the retiree's degree of aversion to longevity risk, and the determination of such a number of years depends on the mortality factor β through the random variable "remaining number of whole years of life" $\boldsymbol{N}$.

The values of the random variable $\boldsymbol{N}$ are $\{0, 1, 2, \ldots, \omega - x - 1\}$ with respective probabilities $\left\{q_x^*, {}_{1|}q_x^*, {}_{2|}q_x^*, \ldots, {}_{\omega-x-1|}q_x^*\right\}$, where ${}_{t|}q_x^*$ is the modified probability that the retiree aged x dies between ages $[x+t, x+t+1]$, i.e.,

$${}_{t|}q_x^* = {}_{t-1}p_x^* \cdot q_{x+t}^*. \tag{20}$$

The $1-\varepsilon$ percentile of $\boldsymbol{N}$ is as follows:

$$q_{\boldsymbol{N}}^{1-\varepsilon} = \min_j | \sum_{k\leq j} {}_{k|}q_x^* \geq 1-\varepsilon. \tag{21}$$

The temporary life annuity to be purchased under this alternative will have $n^* = q_{\boldsymbol{N}}^{1-\varepsilon}$ payments, and the amount of the payments of the annuity, C^{n^*}, is obtained by substituting n^* in (4):

$$C^{n^*} = \frac{\Pi}{a_{x:\overline{n^*}|}}. \tag{22}$$

It is worth noting that this alternative does not provide a complete hedge against longevity risk since the retiree may survive more than n^* years, and if so, they will not receive annuity payments once this number of years has passed. In other words, the transfer rate to SLE annuitants is reduced at the cost of assuming a certain level of longevity risk.

To calculate the transfer rate in this case, a similar reasoning to that in Section 3.1 is followed but considering temporary life annuities. Therefore, SSLE individuals, at the time of contracting the n^*-year temporary life annuity, transition from wealth Π to wealth $\frac{\Pi}{a_{x:\overline{n^*}|}} a^*_{x:\overline{n^*}|}$. Then, the transfer rate, TR^{n^*}, is now

$$TR^{n^*} = \frac{\Pi - \Pi \frac{a^*_{x:\overline{n^*}|}}{a_{x:\overline{n^*}|}}}{\Pi} = 1 - \frac{a^*_{x:\overline{n^*}|}}{a_{x:\overline{n^*}|}}. \tag{23}$$

Thus, in a standard life annuity market, the SSLE annuitants reduce the transfer rate with this annuitisation strategy. This reduction is measured by combining (10) and (23) as

$$\nabla TR^{n^*} = TR - TR^{n^*} = \frac{a^*_{x:\overline{n^*}|}}{a_{x:\overline{n^*}|}} - \frac{a^*_x}{a_x}, \tag{24}$$

where $TR^{n^*} \geq 0$.

Notice that

$$\frac{a^*_{x:\overline{n}|}}{a_{x:\overline{n}|}} = \frac{\sum_{t=1}^{n} \frac{{}_tp^*_x}{{}_tp_x} w_t}{\sum_{t=1}^{n} w_t}, \tag{25}$$

where $w_t = (1+i)^{-t}\,{}_tp_x$, for $t = 1, 2, \ldots, n$. Therefore, (25) can be understood as a weighted average of the survival probability ratios $\frac{{}_tp^*_x}{{}_tp_x}$ weighted by w_t. Since $\frac{{}_{t+1}p^*_x}{{}_{t+1}p_x} = \frac{{}_tp^*_x}{{}_tp_x} \cdot \frac{p^*_{x+t}}{p_{x+t}}$ and $0 \leq \frac{p^*_{x+t}}{p_{x+t}} \leq 1$, it holds that $\frac{{}_{t+1}p^*_x}{{}_{t+1}p_x} \leq \frac{{}_tp^*_x}{{}_tp_x}$. That is, when a new payment is added to the temporary annuities $a^*_{x:\overline{n}|}$ and $a_{x:\overline{n}|}$, the new survival probability ratio is lower than the existing ones, thereby decreasing the weighted average $\frac{a^*_{x:\overline{n}|}}{a_{x:\overline{n}|}}$. And, considering (23), it can be stated that TR^{n^*} increases respect to the number of payments.

Analogously,

$$\frac{a^*_x}{a_x} = \frac{\sum_{t=1}^{\omega-x} \frac{{}_tp^*_x}{{}_tp_x} w_t}{\sum_{t=1}^{\omega-x} w_t}, \tag{26}$$

can be seen as a weighted mean. It averages the rates $\frac{{}_tp^*_x}{{}_tp_x}$, for $t = 1, 2, \ldots, n$, and, additionally, $\frac{{}_tp^*_x}{{}_tp_x}$ for $t = n+1, n+2, \ldots, \omega - x$. Since these last survival probability ratios weighted in $\frac{a^*_x}{a_x}$ are lower than those common to both quotients of present values, it turns out that $\frac{a^*_{x:\overline{n}|}}{a_{x:\overline{n}|}} \geq \frac{a^*_x}{a_x}$.

In the limiting case where $n^* \to \infty$, Equation (23) turns into Equation (10) and, consequently, the reduction in the transfer rate is zero. Therefore, this reduction decreases with the number of payments of the temporary life annuity. Additionally, since a higher degree of longevity risk hedge (i.e., a smaller ε) represents a greater n^*, the corresponding temporary annuity implies a smaller reduction in the transfer rate. Conversely, the transfer rate can be reduced at the cost of reducing the longevity risk hedge. Finally, the highest possible hedge of the longevity risk corresponds to $\varepsilon = 0$ and, accordingly, to the minimum reduction in the transfer rate.

Numerical Application 2. Table 2 lists the transfer rate with an n^*-year temporary life annuity, TR^{n^*}, for $\varepsilon = 0.05$. The technical interest rates, mortality factors, standard mortality probabilities, and ages are the same as those in Numerical Application 1. Table 3 shows the reduction in the transfer rate with that temporary annuity, ∇TR^{n^*}.

Table 2. Transfer rate with a standard temporary life annuity is TR^{n^*}, for $\varepsilon = 0.05$.

i \ β	1.5	2.0	5.0	10
$x = 65$				
	$n^* = 30$	$n^* = 27$	$n^* = 20$	$n^* = 14$
1.5%	0.119250	0.194294	0.405526	0.516776
3.0%	0.106437	0.176602	0.384307	0.501590
4.0%	0.098714	0.165711	0.370673	0.491646
$x = 75$				
	$n^* = 20$	$n^* = 18$	$n^* = 11$	$n^* = 7$
1.5%	0.155431	0.252573	0.468436	0.589064
3.0%	0.145230	0.238817	0.456924	0.582067
4.0%	0.138852	0.230097	0.449401	0.577459
$x = 80$				
	$n^* = 16$	$n^* = 13$	$n^* = 8$	$n^* = 4$
1.5%	0.187305	0.286425	0.547460	0.650207
3.0%	0.178241	0.276161	0.539165	0.646859
4.0%	0.172503	0.269559	0.533721	0.644653

Source: Own elaboration.

Table 3. Reduction of the transfer rate with a standard temporary life annuity, ∇TR^{n^*}, for $\varepsilon = 0.05$.

i \ β	1.5	2.0	5.0	10
$x = 65$				
	$n^* = 20$	$n^* = 18$	$n^* = 11$	$n^* = 7$
1.5%	0.009503	0.024590	0.087702	0.157799
3.0%	0.007015	0.018951	0.074158	0.141460
4.0%	0.005702	0.015848	0.065975	0.131004
$x = 75$				
	$n^* = 20$	$n^* = 18$	$n^* = 11$	$n^* = 7$
1.5%	0.017678	0.035049	0.132787	0.190837
3.0%	0.014356	0.029338	0.120810	0.180447
4.0%	0.012473	0.026004	0.113240	0.173627
$x = 80$				
	$n^* = 16$	$n^* = 13$	$n^* = 8$	$n^* = 4$
1.5%	0.020177	0.053313	0.130696	0.202954
3.0%	0.007015	0.018951	0.074158	0.141460
4.0%	0.015159	0.042760	0.116384	0.191742

Source: Own elaboration.

Table 2 implements Equation (23) and shows the transfer rate for different considered ages and mortality factors. As observed, a higher mortality factor results in a higher transfer rate. Additionally, the transfer rate decreases as the interest rate increases.

Table 3 illustrates the reduction in the transfer rate achieved by applying the strategy introduced in this section. For each specific scenario (age, mortality factor, and interest rate), the reduction is calculated via Equation (24), which determines the difference between the values in Tables 1 and 2. As shown, the use of temporary life annuities as a means to reduce the transfer rate is more advantageous for individuals with higher β.

5. Tax Incentives, Annuitisation and SSLE Retirees

The tax regulations of various countries include a range of incentives designed to encourage the demand for life annuities that supplement public retirement payments, thereby enabling retirees to transfer their longevity risk to the private insurance sector. In countries where special-rate life annuities do not exist, these incentives create a disadvantage for SSLE individuals because they can only benefit from them at unfair prices (Kling et al. 2014).

Tax incentives for life annuities vary by country. Since it is not possible to cover all scenarios, we limit our work to the tax incentives in Spain, focusing on two different scenarios that are comprehensive enough to encompass the situations a Spanish retiree might encounter:

1. Life annuities are purchased using the accumulated balance from a pension plan.
2. Life annuities are acquired using a portion of the SSLE retiree's personal assets.

For these two scenarios, we compare the annual payment amounts received by the SSLE retirees in three cases:

- Purchasing a standard lifetime annuity.
- Implementing the alternative strategy proposed in Section 4.
- Contracting a special-rate lifetime annuity if possible.

5.1. Using the Accumulated Balance from a Pension Plan

Like in other countries, pension plans in Spain allow workers to reduce their tax liability by contributing to them when the marginal tax rates they have are high and withdrawing funds from the plans' accumulated balance when their marginal tax rates are low. This provides workers with incentives to save for their own retirement. According to Rydqvist et al. (2014), these incentives are based on two key principles: the tax exemption of pension plan contributions during working years and the smoothing of tax payments over retirement, which together reduce lifetime tax liability.

According to Jefatura del Estado (2006), payments from an annuity funded with a pension plan's accumulated balance are classified as labour earnings. Consequently, the taxable amount of annuity payments is determined by the marginal tax rate associated with this category of earnings, denoted as g_L, which fluctuates on the basis of the retiree's income due to progressive taxation. This work assumes that the marginal tax rate on labour earnings for the retiree, g_L, remains constant throughout the period during which life annuity payments are received.

If the accumulated balance in the pension plan equals Π:

- When a standard lifetime annuity is purchased, the after-tax amount of annual payments for SSLE retirees is, bearing in mind (2), as follows:

$$C_{AT} = \frac{\Pi}{a_x}(1 - g_L). \tag{27}$$

- The alternative strategy consists of purchasing a standard temporary annuity with n^* payments, where n^* is obtained as described in Section 4. The payment to be received by an SSLE retiree after taxes, $C_{AT}^{n^*}$, is, considering (22):

$$C_{AT}^{n^*} = \frac{\Pi}{a_{x:\overline{n^*}|}}(1 - g_L). \tag{28}$$

- Similarly, if special-rate lifetime annuities were available, the annual payment to be received after taxes, C^*_{AT}, would be as follows:

$$C^*_{AT} = \frac{\Pi}{a^*_x}(1 - g_L). \tag{29}$$

Numerical Application 3. The marginal tax rate associated with labour earnings in Spain currently takes different values depending on the annual amount of income. These values are 0%, 19%, 24%, 30%, 37%, 45%, and 47%, respectively. To avoid unnecessarily lengthening the extension of the results, only $g_L = 19\%,\ 30\%,\ 45\%$ is considered. The same ages, technical interest rates, mortality factors, and standard mortality probabilities as in Numerical Application 1 are used and also $\varepsilon = 0.05$. For every 100 m.u. of accumulated balance in the pension plan, Table 4 shows the amount of the annual payment after taxes to be received with a standard lifetime annuity; Table 5 depicts the amount of the annual payment after taxes with a standard temporary life annuity; Table 6 contains the amount of the annual payment after taxes to be received if special-rate lifetime annuities were offered.

Table 4. Annual payment after taxes with a standard lifetime annuity for every 100 m.u. of accumulated balance in the SSLE retiree's pension plan.

	$x = 65$		
i	$g_L = 19\%$	$g_L = 30\%$	$g_L = 45\%$
1.5%	4.72	4.08	3.20
3%	5.56	4.81	3.78
4%	6.16	5.32	4.18
	$x = 75$		
i	$g_L = 19\%$	$g_L = 30\%$	$g_L = 45\%$
1.5%	7.15	6.18	4.86
3%	8.04	6.95	5.46
4%	8.65	7.47	5.87
	$x = 80$		
i	$g_L = 19\%$	$g_L = 30\%$	$g_L = 45\%$
1.5%	9.52	8.23	6.47
3%	10.46	9.04	7.10
4%	11.10	9.59	7.54

Source: Own elaboration.

Table 5. Annual payment after taxes with a standard temporary life annuity for every 100 m.u. of accumulated balance in the SSLE retiree's pension plan.

	$x = 65$											
	$g_L = 19\%$				$g_L = 30\%$				$g_L = 45\%$			
i \ β	1.5	2.0	5.0	10	1.5	2.0	5.0	10	1.5	2.0	5.0	10
1.5%	4.79	4.89	5.57	7.12	4.14	4.23	4.82	6.15	3.25	3.32	3.78	4.83
3%	5.62	5.72	6.36	7.87	4.86	4.94	5.50	6.80	3.82	3.88	4.32	5.34
4%	6.21	6.30	6.91	8.39	5.37	5.44	5.97	7.25	4.22	4.28	4.69	5.70
	$x = 75$											
	$g_L = 19\%$				$g_L = 30\%$				$g_L = 45\%$			
i \ β	1.5	2.0	5.0	10	1.5	2.0	5.0	10	1.5	2.0	5.0	10
1.5%	7.37	7.56	9.69	13.66	6.37	6.54	8.38	11.80	5.00	5.14	6.58	9.27
3%	8.23	8.41	10.49	14.44	7.11	7.27	9.06	12.48	5.59	5.71	7.12	9.80
4%	8.82	9.00	11.03	14.97	7.63	7.78	9.53	12.94	5.99	6.11	7.49	10.16

Table 5. *Cont*.

$x = 80$												
	$g_L = 19\%$				$g_L = 30\%$				$g_L = 45\%$			
i \ β	1.5	2.0	5.0	10	1.5	2.0	5.0	10	1.5	2.0	5.0	10
1.5%	9.85	10.45	13.56	23.41	8.51	9.03	11.72	20.23	6.69	7.10	9.21	15.90
3%	10.76	11.33	14.39	24.25	9.30	9.79	12.43	20.96	7.31	7.69	9.77	16.47
4%	11.38	11.93	14.95	24.82	9.84	10.31	12.92	21.45	7.73	8.10	10.15	16.85

Note: Values of n^* for $\beta = 1.5, 2.0, 5.0, 10$ are for $x = 65$, $n^* = 30, 27, 20, 14$; for $x = 75$, $n^* = 20, 18, 11, 7$; and for $x = 80$, $n^* = 16, 13, 8, 4$. Source: Own elaboration.

Table 6. Annual payment after taxes with a special-rate lifetime annuity for every 100 m.u. of accumulated balance in the SSLE retiree's pension plan.

$x = 65$												
	$g_L = 19\%$				$g_L = 30\%$				$g_L = 45\%$			
i \ β	1.5	2.0	5.0	10	1.5	2.0	5.0	10	1.5	2.0	5.0	10
1.5%	5.42	6.04	9.31	14.50	4.68	5.22	8.05	12.53	3.68	4.10	6.32	9.85
3%	6.27	6.92	10.27	15.58	5.42	5.98	8.88	13.47	4.26	4.70	6.98	10.58
4%	6.88	7.52	10.93	16.32	5.94	6.50	9.45	14.10	4.67	5.11	7.42	11.08
$x = 75$												
	$g_L = 19\%$				$g_L = 30\%$				$g_L = 45\%$			
i \ β	1.5	2.0	5.0	10	1.5	2.0	5.0	10	1.5	2.0	5.0	10
1.5%	8.65	10.04	17.94	32.50	7.48	8.68	15.50	28.09	5.87	6.82	12.18	22.07
3%	9.56	10.98	19.04	33.85	8.27	9.49	16.45	29.25	6.49	7.46	12.93	22.98
4%	10.19	11.63	19.77	34.75	8.81	10.05	17.09	30.03	6.92	7.89	13.43	23.59
$x = 80$												
	$g_L = 19\%$				$g_L = 30\%$				$g_L = 45\%$			
i \ β	1.5	2.0	5.0	10	1.5	2.0	5.0	10	1.5	2.0	5.0	10
1.5%	12.02	14.42	29.59	64.85	10.38	12.46	25.57	56.04	8.16	9.79	20.09	44.04
3%	13.00	15.45	30.87	66.65	11.23	13.35	26.68	57.60	8.83	10.49	20.96	45.26
4%	13.67	16.14	31.73	67.86	11.81	13.95	27.42	58.64	9.28	10.96	21.54	46.08

Note: LEs for $\beta = 1,\ 1.5,\ 2.0,\ 5.0,\ 10$ are $x = 65$, 21.06, 18.05, 16.02, 10.16 and 6.53; for $x = 75$, 13.33, 10.92, 9.37, 5.30 and 3.10; and for $x = 80$, 9.90, 7.82, 6.54, 3.36 and 1.78. Source: Own elaboration.

Numerical Application 3 allows two conclusions to be drawn:

- In all cases where the retiree has an SSLE $(\beta > 1)$, given that $a_x > a_{x:\overline{n^*}|}$, the strategy of purchasing a temporary life annuity is preferable to acquiring a standard lifetime annuity, even when it is calculated with mortality probabilities not adjusted to the individual's situation. This strategy only covers the number of years set according to the probability of longevity risk ε, resulting in an increased annual annuity payment.
- When a fair actuarial annuity can be purchased, i.e., a special-rate lifetime annuity, the annual payment amount is always greater than that of a standard lifetime annuity. Furthermore, if the annuitant's LE is significantly reduced, this amount can be more than six times greater than that obtained with a temporary life annuity.

5.2. Using a Portion of the SSLE Retiree's Personal Assets

In this case, and again focusing on Spanish taxation, it is assumed that the life annuity is purchased using a portion of the SSLE retiree's assets, which may require the monetisation or liquidation of those assets. Depending on the nature of these assets, their liquidation may or may not be subject to taxation. For example, using the available balance in a bank account does not incur income tax. However, winnings from lotteries or gambling may be subject to taxes. Additionally, the sale of real estate that has appreciated in value since the purchase or the sale of equity securities for more than their acquisition cost may also be taxable, as these are capital gains resulting from asset transfers. These capital gains could be exempt under certain conditions. These exemptions, applicable when specific requirements are met, include the following:

- Capital gains resulting from the sale of assets by individuals over 65 years, if the total amount obtained from that selling is used to purchase a lifetime annuity for which the individual is the beneficiary.
- Capital gains derived from the sale of the habitual residence by individuals over 65 years of age or those in severe or significant dependency situations when the proceeds are received as a lump sum or exchanged for a temporary or lifetime annuity.
- Capital gains arise from the difference between contributions made to individual systematic savings plans[3] and their final accumulated value at the time of purchasing lifetime annuities.

In summary, the total amount obtained from liquidating a retiree's assets, which will be used to buy a life annuity, may or may not be subject to taxes. Thus, the percentage of this amount that is taxable, p, can be equal to or different from zero. When $p \neq 0$, the retiree's marginal tax rate on capital gains must be considered.

Spanish tax legislation provides incentives for contracting a life annuity using funds that do not come from the accumulated balance of a pension plan. These incentives allow only a percentage of the annual payments received to be subject to taxes. For a lifetime annuity, the taxable percentage is based on the retiree's age, whereas for a temporary life annuity, it is based on the term of the annuity. Once the appropriate percentage is applied, the resulting amount must be included in the savings taxable base.

To compare, as in Section 5.1, the amount of the annual payment to be received by an SSLE retiree when purchasing a standard lifetime annuity, an n^*-year temporary life annuity, and a special-rate lifetime annuity, we define the following variables:

g_K : Retiree's marginal tax rate on capital gains.
k_x : Percentage of the annual amount of a lifetime annuity subject to tax in the savings taxable base, depending on the retiree's age at the time of purchasing. For the age ranges relevant to this paper, Spanish taxation specifies $k_x = 24\%$ for those aged between 60 and 65 years, $k_x = 20\%$ for those aged between 66 and 69 years and $k_x = 8\%$ for those aged 70 years or older.
k_n: Percentage of the annual amount of a temporary life annuity subject to tax in the savings taxable base, depending on the term of the annuity, n, measured in years. In Spain, for $n \leq 5$, $k_n = 12\%$; for $5 < n \leq 10$, $k_n = 16\%$; for $10 < n \leq 15$, $k_n = 20\%$; and for $n > 15$, $k_n = 25\%$.
g_S : Retiree's marginal tax rate on saving. It is assumed that it remains constant throughout the period in which life annuity payments are received.

Every one m.u. obtained from liquidating a retiree's assets is subject to taxes amounting to $p\, g_K$. Therefore, the amount available to purchase a life annuity is $1 - p\, g_K$. If the amount obtained from the assets' liquidation is equal to Π, the single premium to purchase a life annuity is $\Pi(1 - p\, g_K)$.

Following similar reasoning as in Section 5.1, the results below are obtained.

- The amount before taxes of annual payments when a standard lifetime annuity is purchased by an SSLE retiree is as follows:

$$C_{BT} = \Pi \, \frac{1 - p\, g_K}{a_x}. \tag{30}$$

Therefore, considering the retiree's marginal tax rate on savings, g_S, and (30), the after-tax amount is as follows:

$$C_{AT} = \Pi \frac{1 - p\, g_K}{a_x} (1 - k_x\, g_S). \tag{31}$$

- The alternative strategy consists of purchasing a standard n^*-year temporary life annuity. The amount of payments to be received by an SSLE retiree before taxes is

$$C_{BT}^{n^*} = \Pi \, \frac{1 - p\, g_K}{a_{x:\overline{n^*}|}}, \tag{32}$$

and, consequently, after taxes,

$$C_{AT}^{n^*} = \Pi \frac{1 - p\, g_K}{a_{x:\overline{n^*}|}} (1 - k_{n^*} g_S). \tag{33}$$

- If special-rate lifetime annuities were offered, the amount of the annual payment after taxes were received, C_{AT}^*, would be as follows:

$$C_{AT}^* = \Pi \frac{1 - p\, g_K}{a_x^*} (1 - k_x\, g_S). \tag{34}$$

Numerical Application 4. The marginal tax rate on savings in Spain currently takes different values depending on the annual amount of savings. These values are 0%, 19%, 21%, 23%, 27%, and 28%, respectively. For every 100 m.u. obtained from liquidating the SSLE retiree's assets, Table 7 shows the amount after taxes of the annual payment to be received with a standard lifetime annuity; Table 8 contains the amount of the annual payment after taxes with a standard temporary life annuity; Table 9 lists the amount of the annual payment after taxes to be received if special-rate lifetime annuities were offered. The same technical interest rates, mortality factors, and standard mortality probabilities as in Numerical Application 1 are used and also $\varepsilon = 0.05$. Additionally, to avoid lengthening the extension of the results, only $g_S = 19\%,\ 23\%,\ 28\%$, $x = 75$ and $i = 3\%$ are considered.

Table 7. Annual payment after taxes with a standard lifetime annuity for every 100 m.u. obtained from liquidating the SSLE retiree's assets, $x = 75$, $i = 3\%$.

	$p\, g_K = 0\%$			$p\, g_K = 5\%$		
g_S	19%	23%	28%	19%	23%	28%
	9.77	9.74	9.70	9.28	9.25	9.22
	$p\, g_K = 10\%$			$p\, g_K = 20\%$		
g_S	19%	23%	28%	19%	23%	28%
	8.80	8.77	8.73	7.82	7.79	7.76

Note: For $x = 75$, $k_x = 8\%$. Source: Own elaboration.

Table 8. Annual payment after taxes with a standard temporary life annuity for every 100 m.u. obtained from liquidating the SSLE retiree's assets, $x = 75$, $i = 3\%$.

		$p\ g_K = 0\%$			$p\ g_K = 5\%$		
β	n^* \ g_S	19%	23%	28%	19%	23%	28%
1.5	20	9.68	9.57	9.45	9.19	9.10	8.97
2.0	18	9.89	9.79	9.66	9.40	9.30	9.17
5.0	11	12.45	12.35	12.22	11.83	11.73	11.61
10.0	7	17.28	17.17	17.03	16.42	16.31	16.18
		$p\ g_K = 10\%$			$p\ g_K = 20\%$		
β	n^* \ g_S	19%	23%	28%	19%	23%	28%
1.5	20	8.71	8.62	8.50	7.74	7.66	7.56
2.0	18	8.90	8.81	8.69	7.91	7.83	7.73
5.0	11	11.21	11.12	11.00	9.96	9.88	9.78
10.0	7	15.56	15.45	15.32	13.83	13.74	13.62

Note: For $n^* = 20,\ 18$, $k_{n^*} = 25\%$; for $n^* = 11$, $k_{n^*} = 20\%$; for $n^* = 7$, $k_{n^*} = 16\%$. Source: Own elaboration.

Table 9. Annual payment after taxes with a special-rate lifetime annuity for every 100 m.u. obtained from liquidating the SSLE retiree's assets, $x = 75$, $i = 3\%$.

	$p\ g_K = 0\%$			$p\ g_K = 5\%$		
β \ g_S	19%	21%	23%	19%	21%	23%
1.5	11.63	11.59	11.54	11.05	11.01	10.97
2.0	13.35	13.31	13.26	12.69	12.64	12.59
5.0	23.14	23.07	22.97	21.99	21.91	21.83
10.0	41.15	41.02	40.85	39.09	38.97	38.81
	$p\ g_K = 10\%$			$p\ g_K = 20\%$		
β \ g_S	19%	21%	23%	19%	21%	23%
1.5	10.47	10.43	10.39	9.30	9.27	9.23
2.0	12.02	11.98	11.93	10.68	10.65	10.60
5.0	20.83	20.76	20.68	18.51	18.45	18.38
10.0	37.03	36.91	36.76	32.92	32.81	32.68

Note: For $x = 75$, $k_x = 8\%$. Source: Own elaboration.

From Numerical Application 4, the following conclusions can be drawn:

- When $\beta > 1$, purchasing a temporary life annuity results, for the given age, in a percentage k_{n^*} higher than that of a lifetime annuity, k_x. Furthermore, the potential for capital gains exemptions when purchasing a lifetime annuity, which may not apply to a temporary annuity, is also significant. Thus, in situations where $\beta = 1.5$, the payment for the temporary annuity is slightly lower than that of a lifetime annuity for the same value of $p\ g_K$. However, for $\beta = 2$, the payment for the temporary life annuity is, in most cases, slightly higher than that for a lifetime annuity. This difference becomes more significant in the simulation analysed for $\beta \geq 5$. Conversely, in situations where a capital gain exemption applies to acquiring a lifetime annuity, the SSLE retiree would be in the case where $p\ g_K = 0$, with a lifetime annuity, but might be in a situation where $p\ g_K \neq 0\%$, when a temporary life annuity is purchased, making the lifetime annuity clearly more advantageous.

- Once again, if it could be contracted, the annual payment amount of a special-rate lifetime annuity would be greater than that of any other alternative, and this difference would increase with β. Additionally, since these annuities are lifelong, they could benefit from the capital gain exemptions described.

6. Conclusions

This paper sheds light on the inherent actuarial injustice faced by retirees with substandard life expectancy (SSLE) in markets where special-rate annuities are unavailable. The concept of actuarial fairness, which emphasises equitable pricing based on individual risk profiles, is compromised when life annuities are priced uniformly using standard mortality tables. In such markets, retirees with SSLE end up subsidising those with standard life expectancy (SLE). This dynamic, where SSLE individuals receive, for the same single premium, the same annuity payments as their healthier counterparts but over a potentially shorter period, represents significant actuarial unfairness.

With respect to Issue 1, the analysis provided in this study quantifies this unfairness through the concept of the transfer rate, a metric that captures the implicit wealth transfer from SSLE retirees to those with SLE when a standard lifetime annuity is purchased. The results underscore the extent of this transfer, highlighting the disproportionate burden placed on SSLE individuals who are unable to access life annuities tailored to their actual mortality risk.

As far as Issue 2 is concerned, the study proposes an alternative annuitisation strategy for SSLE retirees to mitigate the unfairness associated with the lack of enhanced annuities under the hypothesis of no personal income taxation. This strategy involves purchasing a standard temporary life annuity designed to cover the expected remaining years of life with a high probability, thereby reducing the wealth transfer to SLE retirees. While this approach does not entirely eliminate actuarial unfairness, it offers a pragmatic solution in the absence of special-rate annuities, allowing SSLE retirees to manage their longevity risk without excessively subsidising others.

Issue 3 examines how tax incentives can exacerbate actuarial unfairness. In countries such as Spain, where tax incentives are offered to encourage the purchase of life annuities, SSLE retirees face a difficult decision. They must decide between forfeiting tax benefits by not purchasing a life annuity or accepting a standard life annuity at an unfair price to gain a tax advantage. This dilemma further intensifies the inequity faced by SSLE individuals, as the tax incentives intended to promote annuitisation inadvertently push them toward financially disadvantageous decisions. The article explores two scenarios based on the source used by SSLE retirees to purchase the life annuity: either the accumulated balance from a pension plan or a portion of their personal assets. While acquiring a standard temporary life annuity is the best strategy to minimise actuarial unfairness when a pension plan balance is used, this may not be the case when the source is personal assets.

The findings of this work have significant implications for policymakers and insurance regulators in many countries. The absence of special-rate annuities in the insurance markets of many countries reveals a critical gap in the financial products available to SSLE retirees. Introducing special-rate annuities that account for individual mortality risk would promote actuarial fairness and ensure a more equitable distribution of retirement income. Additionally, policymakers should carefully design tax incentives to avoid worsening the disadvantages faced by SSLE retirees. Adjusting tax policies to address the actuarial inequities inherent in standard life annuities could help alleviate the financial burden on these individuals and support more equitable retirement planning.

The analysis presented also has significant implications for personal financial advisors. This study demonstrates that having an accurate estimate of an individual's actual LE is crucial in retirement financial planning. This becomes even more important for retirees with an SSLE when there are no life annuities priced according to their actual mortality probabilities. Therefore, when determining the optimal type of life annuity to hedge longevity risk, the annuitant's LE is just as important as personal income taxes.

We are aware that certain legal principles, such as non-discrimination based on sex, may influence the design of special-rate life annuities. In such cases, although a particular risk may affect men and women differently, the excess mortality derived from this risk should not be reflected differently between both sexes in markets where sex discrimination is prohibited, such as the European Union (European Council 2004). An example of this situation could be special-rate life annuities offered in a 'standardised' manner (for instance, impaired life annuities for people with diabetes). On the other hand, for a care annuity, where the risk evaluation is individualised, the in-depth consideration and analysis of all the factors associated with the retiree could ultimately implicitly account for sex differences.

The simulations regarding the measurement of unfairness in standard life annuities for SSLE individuals have been implemented by modelling their mortality probabilities using the numerical rating system. This system involves applying a factor to a standard mortality table and is widely used in actuarial practice (Olivieri 2006). As a practical method, it may present some inaccuracies (Atance et al. 2025). However, its simplicity allows for the incorporation of various factors that can increase or decrease mortality probabilities relative to the standard ones in the form of "credits" and "debits". This provides, for our purposes, an adequate way to measure the extent to which the retiree presents an SSLE, as the interpretation is intuitive.

Anyway, both the measurement of the transfer rate in (10) and, for a given ε, the determination of the number of payments of a temporary life annuity that allows for a reduction in that transfer rate can be implemented without major difficulties using mortality tables adjusted to the reference population through more sophisticated methods (Dowd et al. 2011; Jarner and Kryger 2011; Ahcan et al. 2014; Wan and Bertschi 2015; Atance et al. 2025). In fact, the application of these methodologies and the numerical rating system are not mutually exclusive. For example, a mortality table tailored to a specific company (Atance et al. 2025) or adjusted to a particular age group in a specific geographic area (Jarner and Kryger 2011) can serve as a basis for estimating the mortality of individuals with a significant SSLE by applying a mortality factor obtained through the numerical rating system.

Although Issue 3 is developed with Spanish tax regulations, its applicability extends beyond Spain. The analysis can be adapted to the tax characteristics of life annuities in other countries. In any context, taxation considerations share two common elements. The first is the potential existence of tax incentives related to the source of purchasing a life annuity, which can increase the annuity payments. The second is that the annuity payments are subject to personal income tax in such a way that the difference between countries lies in the after-tax annuity payments. Moreover, it is worth noting that Spain is one of the largest recipients of migrant retirees in the European Union (Valero-Escandell et al. 2022), making Spanish life annuity regulations particularly relevant for retirees considering relocation to Spain.

Finally, our work models mortality using a static standard mortality table, which is common in practice. Nevertheless, a dynamic standard mortality table could also be employed to account for different LEs linked to SSLE individuals. Such a table can be adjusted using one of the Lee–Carter-based methodologies proposed in the literature (see, e.g., Bravo et al. 2021; Atance and Navarro 2024) or directly obtained from legal resources, as in the case of the PERM/F 2020 mortality tables for Spain (Ministerio de Asuntos Económicos y Transformación Digital 2020).

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Notes

1 In the actuarial literature, there is no single classification for this type of annuities. Moreover, their denomination is not uniform either. For instance, they may be referred to as "underwritten annuities", or "substandard annuities" (as opposed to standard annuities).

2 Although the remainder of this work refers only to standard mortality tables, all the developments presented can be applied, without loss of generality, to cases where a different mortality table is used as a baseline. That baseline mortality table may be derived using the methodologies reviewed in the first paragraph of this Subsection.

3 Individual systematic savings plans are long-term life savings insurance products that allow individuals to obtain a life annuity using their contributed resources. The Personal Income Tax Law in Spain offers tax incentives for these plans when certain requirements are met.

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