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Implications of Risk Management:

The Cases of Options Valuation and of Pricing Insurance Contracts

A Tutorial

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Doron Kliger and Benny Levikson

Detailed Description of the Tutorial

We demonstrate pricing methods of two financial assets via risk management. First we valuate a Call Option and a Warrant (the latter being a variant of a Call Option). Then we price several Insurance Contracts. In both cases we start with a review of known results and then develop more sophisticated pricing methods.

1. *Call Option & Warrant Valuation*

The Black & Scholes (B&S) Option Pricing Model (1973) determines the value of a Call Option by creating *a hedged portfolio*, consisting of a long position in the underlying stock and a short position in the option.

The portfolio is "hedged" in the following sense: Its future value will not depend on the price of the stock, but will depend only on time and on the value of known constants. If the hedge is maintained continuously, then its return becomes certain. In other words: the risk in the hedged position is zero.

One of the assumptions of the B&S model is that the stock price follows a geometric random-walk in continuous time, where the variance rate of return on the stock is constant.

A Warrant can be viewed as a version of a Call Option. Some of the differences between a Warrant and a Call Option are:

(1) A warrant is issued by the same firm that issued the underlying stock, and therefore affects the firm's future cash-flow.

(2) Warrants are usually issued for a long period while Call Options' life times are usually no longer than one year. As the maturity date goes far into the future, the assumption about the fixture of the volatility becomes less realistic. Even if volatilities of the the assets' returns are constant, capital structures that contain Warrants cause the volatilities of the stocks' returns to vary with the stocks' prices, because of the "Negative Leverage" effect of the Warrants.

We will present a new methodology of *Warrant Valuation*, which is based on the B&S formula and considers the special features of Warrants: The behaviour of the elasticity of the volatility due to the suggested model is constant in all manners but time, so the model is to be called a model of "Constant Elasticity of Variance plus Time Effect Correction" or a "CEV+TEC Model" in short.

Next we present a set of stages, by which the CEV+TEC Model performance is empirically compared to the performance of a commonly used version of the B&S Model.* The conclusion is that the CEV+TEC Model is significantly better.

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* The tests were conducted on a data base of the Israeli Stock Exchange (ISE) securities prices.

II. *Pricing Insurance Contracts*

In this part of the tutorial we show how to price several types of insurance contracts. We start with finding the maximal premium, a risk averse individual is ready to pay for a fixed term insurance contract. This idea enabled Arrow and Pratt to introduce their criteria for absolute and relative risk aversion. Then we show how the insurer finds the risk premium for such an insurance, by using a loading factor. This loading factor guarantees, with high probability, that the insurer will be able to pay all claims using only the stream of premium payments.

Next we consider pricing contracts for disability, long term care and life insurance. First we find the net single premium for these contracts using standard actuarial techniques. Then we introduce some tools from probability theory that enable us to find the distribution of the discounted future benefits the insurer will pay the insured. Knowing this distribution enables us (i) to find the net single premium in complex situations, (ii) to find the probability of ruin for different values of risk premiums. The tools needed here are backward induction, the probabilistic approach to Potential Theory and dynamic programming.

Requirements I. A basic course in Finance/Investments

II. A first course in Probability

Some tools which are needed in our analysis and are not covered in the above courses are presented within the tutorial.

Time Schedule: 4 lectures, each of 90 minutes

Handouts will be delivered prior to the tutorial.

References

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